Math 303: Midterm Exam 1

Fall 2008

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have One hour and 45 minutes (105 minutes).
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Find the stationary points of $f(x, y) = (x^2 + 2)^2 + (y^2 + 2)^2 - 6x^2y^2 - 8$ in $\mathbb{R}^2$ and apply the second derivative test to determine if they are minimum, maximum, or saddle points. (20 points)

Problem 2. Find the points on the paraboloid $z = (x-1)^2 + (y-2)^2 + \frac{3}{2}$ that are closest to the point $(1, 2, 3)$. (20 points)

Problem 3.

a) Let ϵ_{ijk} denote the Levi Civita tensor. Show that for every 3×3 matrix M with entries M_{ij} we have $\displaystyle\sum_{i,j,k=1}^{3} \epsilon_{ijk} M_{ij} M_{ik} = 0.$ (10 points)

b) Let $\mathbf{A} : \mathbb{R}^3 \to \mathbb{R}^3$ and $\phi : \mathbb{R}^3 \to \mathbb{R}$ be twice differentiable functions. Use properties of the Levi Civita symbol to show that $\nabla \cdot (\nabla f \times \mathbf{A}) = -\nabla f \cdot (\nabla \times \mathbf{A})$. (10 points)

Problem 4. Let $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be the force field given by

$$\mathbf{F} = [3x^2y + f(y)]\mathbf{i} + [-4y^3x + g(x)]\mathbf{j} + \mathbf{k}$$

where $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable functions.

a) Find the general form of f and g so that $\mathbf{F}$ is a conservative force. (10 points)

b) For the case that $f(1) = g(1) = 0$ and $g'(0) = 1$ compute the work done by $\mathbf{F}$ on a particle that moves from the point $(1, 0, 0)$ to the point $(0, 1, 0)$ along the semicircle $y = \sqrt{1 - x^2}$ in the x-y-plane counterclockwise. (20 points)

Problem 5.

Let r be a positive real number and z be a nonzero complex number that satisfies

$$(z - r)^2 = r^2 e^z.$$

Show that z must have a positive real part. (10 points)

Hint: Compute the modulus of both sides of the above equation.