Math 303, Fall 2010
Assignment for Dec. 15-21

- Read pages 824-839 of the textbook (Riley-Hobson-Bence, 3rd Edition).
- Solve Problems 24.1, 24.3, and 24.4 on Page 867 of the textbook.
- Solve the following problems.

1. Find the real and imaginary parts $u(x, y)$ and $v(x, y)$ of the following functions and determine if there is a region in $\mathbb{C}$ where they are analytic.

$$\begin{aligned} f(z) &= \frac{2z+3}{z^2+2}, \\ g(z) &= \frac{\sinh(z)}{z^*}, \end{aligned}$$

2. Let $f : \mathbb{C} \to \mathbb{C}$ and $g : \mathbb{C} \to \mathbb{C}$ be analytic functions in a region $D \subseteq \mathbb{C}$. Prove that their product is also analytic in this domain and that

$$[f(z)g(z)]' = f'(z)g(z) + f(z)g'(z).$$

3. Find the analogues of the Cauchy-Riemann conditions in the polar coordinates (r, θ), i.e., let $u(r, \theta) = \Re[f(r\, e^{i\theta})]$ and $v(r, \theta) = \Im[f(r\, e^{i\theta})]$, where $\Re$ and $\Im$ respectively stand for the real and imaginary parts of their argument, and find conditions on u and v such that f is analytic at some $z \in \mathbb{C}$.
4. Show that $u(x, y) = \cosh y \cos x$ satisfies the Laplace's equation. Find an analytic function $f(z)$ such that $u(x, y)$ is the real part of $f(x + iy)$.
5. Repeat the previous problem for $u(x, y) = \dfrac{y}{(1-x)^2 + y^2}$.

Note: You do not need to turn in your solution to the above problems.