Math 303, Spring 2020
Assignment for March 12-April 01

Read P**ages 406-408** & **825-853** of the textbook (Riley, Hobson, & Bence, 3rd Edition).

Homework Set # 4 (Due on Thyrsday Apr. 02, 16:00):

Solve the following problems from the textbook (10 points each):

Pages 409-412: 11.17**,** 11.19, 11.27

Pages 867-868: 24.1, 24.7

Solve the following 5 problems (10 points each):

1. Find the real and imaginary parts $u(x, y)$ and $v(x, y)$ of the $f(x + iy)$ for following functions and determine if there is a region in $\mathbb{C}$ where they are holomorphic.

$$f(z) := \frac{2z + 3}{z^2 + 2}, \qquad g(z) := \frac{\sinh(z)}{z^*}.$$

2. Show that $u(x, y) = \cosh(y)\cos(x)$ satisfies the Laplace's equation. Find a differentiable function $f(z)$ such that $u(x, y)$ is the real part of $f(x + iy)$. Express $f(z)$ in terms of z.

3. Let $f : \mathbb{C} \to \mathbb{C}$ and $g : \mathbb{C} \to \mathbb{C}$ be differentiable functions at some $z \in \mathbb{C}$. Prove that their product is also differentiable at z and

$$[f(z)g(z)]' = f'(z)g(z) + f(z)g'(z).$$

4. Use Cauchy's Theorem or Integral Formula to evaluate $\displaystyle\oint_C \frac{\sin z \, dz}{2z - \pi}$ for the following choices of C:

   a) C is the circle defined by $|z| = 1$;

   b) C is the circle defined by $|z| = 2$.

5. Use Cauchy's Integral formula for the derivatives of a holomorphic function to evaluate $\displaystyle\oint_C \frac{\sin(2z) \, dz}{(6z - \pi)^3}$ where C is the circle defined by $|z| = 3$.