

Suggested Exercise Problems 1

Prove that if $a \in \mathbf{F}$, $v \in V$, and $av = 0$, then $a = 0$ or $v = 0$.

Give an example of a nonempty subset U of \mathbf{R}^2 such that U is closed under addition and under taking additive inverses (meaning $-u \in U$ whenever $u \in U$), but U is not a subspace of \mathbf{R}^2 .

Give an example of a nonempty subset U of \mathbf{R}^2 such that U is closed under scalar multiplication, but U is not a subspace of \mathbf{R}^2 .

Prove that the intersection of any collection of subspaces of V is a subspace of V .

Does the operation of addition on the subspaces of V have an additive identity? Which subspaces have additive inverses?

Prove or give a counterexample: if U_1, U_2, W are subspaces of V such that

$$U_1 + W = U_2 + W,$$

then $U_1 = U_2$.

Prove or give a counterexample: if U_1, U_2, W are subspaces of V such that

$$V = U_1 \oplus W \quad \text{and} \quad V = U_2 \oplus W,$$

then $U_1 = U_2$.