1 Find the radius of convergence of the following power series.

$$\sum_{n=0}^{\infty} 3^n z^n, \qquad \sum_{n=0}^{\infty} \frac{3^n z^n}{2^n + 4^n}, \qquad \sum_{n=0}^{\infty} \frac{2^n z^{2n}}{n^2 + n + 1},$$

$$\sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)^n + 1}, \qquad \sum_{n=0}^{\infty} z^{2^n}, \qquad \sum_{\text{p prime}} z^p.$$

2 Consider the power series $\sum_{n=0}^{\infty} z^{n!}$ and let $U(1) := \{w \in \mathbb{C} \mid |w| = 1\}$

a) Show that the radius of convergence of $\sum_{n=0}^{\infty} z^{n!}$ is 1.

b) Show that for every $\delta \in \mathbb{R}^+$ and every $w \in U(1)$ there are infinitely many $z \in U(1)$ such that $|z - w| < \delta$ and $\sum_{n=0}^{\infty} z^{n!}$ does not converge.

Hint: Let $f(z)$ be the sum of $\sum_{n=0}^{\infty} z^{n!}$ for $|z| < 1$ and evaluate $\lim_{r \to 1^-} f(re^{2\pi i/m})$ where m is a positive integer.

3 Show that the power series $\sum_{n=0}^{\infty} c_n (z-a)^n$ and $\sum_{n=0}^{\infty} \frac{c_n}{n+1}(z-a)^{n+1}$ have the same radius of convergence.