1 Let $\mathcal{D}$ be an open disc with center $z_0 \in \mathbb{C}$, $f : \mathbb{C} \to \mathbb{C}$ be a function that is holomorphic in the closed disc $\overline{\mathcal{D}}$, and for every pair of points $w_1, w_2 \in \mathbb{C}$, $[w_1, w_2]$ denote the oriented line segment joining w_1 to w_2. Let $x_0 := \mathrm{Re}(z_0)$, $y_0 := \mathrm{Im}(z_0)$, $z = x + iy$ with $x, y \in \mathbb{R}$ be an arbitrary point of $\mathcal{D}$, $\mathcal{C}_1$ and $\mathcal{C}_2$ be the curves defined by

$$\mathcal{C}_1 := [z_0, x_0 + iy] \cup [x_0 + iy, x + iy], \qquad \mathcal{C}_2 := [z_0, x + iy_0] \cup [x + iy_0, x + iy],$$

and $F, G : \mathcal{D} \to \mathbb{C}$ be the functions defined by $F(z) := \int_{\mathcal{C}_1} f(z)dz$ and $G(z) := \int_{\mathcal{C}_2} f(z)dz$.

1.a) (5 points) Use Cauchy's theorem for polygonal contours to show that $F = G$.

1.b) (10 points) Show that $\frac{\partial}{\partial x} F(x + iy) = f(x + iy)$.

1.c) (10 points) Show that $\frac{\partial}{\partial y} F(x + iy) = if(x + iy)$.

1.d) (10 points) Show that the functions $U, V : \mathcal{D} \to \mathbb{R}$ defined by $U(x, y) := \mathrm{Re}(F(x+iy))$ and $V(x, y) := \mathrm{Im}(F(x + iy))$ satisfy the Cauchy-Riemann conditions in $\mathcal{D}$.

1.e) (10 points) Show that F is differentiable in $\mathcal{D}$ and $F'(z) = f(z)$.

1.g) (5 points) Use the statement you prove in 1.e to conclude that Cauchy's theorem holds for contours lying in $\mathcal{D}$.

2 Solve the following exercise problems from the textbook (Howie's Complex Analysis).

2.a) (5 Points) Problem 7.1 b on page 125

2.b) (10 Points) Problem 7.2 on page 125

2.c) (10 Points) Problem 7.3 on page 125

2.d) (15 Points) Problem 7.4 on page 125

2.e) (10 Points) Problem 7.6 on page 126

Instruction for submitting homework papers: Prepare a PDF of your homework paper and send it by email to Math 401's teaching assistant, Mr. Keremcan Doğan, before the deadline. His email address is kedogan@ku.edu.tr. You should also upload a copy of this PDF to Math 401's Blackboard page.