Math 450/586: Quiz # 1

Fall 2009

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 50 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask, 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)

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1. Give the definition of the following objects. (30 points)

1.a) An equivalence class:

1.b) Image of a subset $C \subseteq A$ under a function $f : A \to B$:

1.c) Domain of a function $f : A \to B$:

1.d) A subspace of a vector space:

1.e) Span of a subset of a vector space:

1.f) A (vector space) isomorphism:

2. Prove the following statements.

2.a) Let V be a real vector space and U be a subspace of V. If $A \subseteq U$, then $\mathrm{Span}(A) \subseteq U$. (15 points)

2.b) Let Let V and W be vector spaces and $\phi : V \to W$ be a linear map. Then $\mathrm{Ran}(\phi)$ is a subspace of W. (20 points)

3. Let V be the vector space of real polynomials of degree at most 2, and $\phi : V \to V$ be defined by

$$\forall p \in V,\ \forall x \in \mathbb{R}, \quad (\phi p)(x) := xp'(x) - 2p(x),$$

where p' denotes the derivative of p.

3.a) Find $\mathrm{Ker}(\phi)$. (15 points)

3.b) Let $\mathscr{B} := \{p_1, p_2, p_3\}$ where for all $x \in \mathbb{R}$, $p_1(x) := 1$, $p_2(x) := x$, $p_3(x) := x^2$. Find the matrix representation of ϕ in the basis $\mathscr{B}$. (20 points)