Math 450/586: Quiz # 3

Fall 2009

- <u>Write your name</u> and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have <u>45 minutes</u>.

- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.

- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask, 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)

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1. Give the definition of the following objects.   (20 points)

1.a) Boundary of a subset of a metric space:

1.b) Basis of a topological space:

1.c) Hausdorff topological space:

1.d) A disconnected subspace of a topological space:

2. Let X be a nonempty set and $d_0 : X^2 \to [0, \infty)$ is the discrete metric on X. Show that every subset of X is an open subset of the metric space (X, d_0). (30 points)

Hint: Is $\{p\}$ open for every $p \in X$?

3. Let (M_1, d_1) and (M_2, d_2) be metric spaces. Show that if $f : M_1 \to M_2$ is an isometry, then it is one-to-one. (20 points)

4. Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ and $a, b \in \mathbb{R}^3$ be defined as

$$f\begin{pmatrix} x \\ y \\ z \end{pmatrix} := \begin{pmatrix} x^2 - 2y^2 + z^3 \\ x^3 + y^2 - 3z^2 \end{pmatrix}, \quad a := \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}.$$

Compute $Df(a)b$, where $Df(a)$ is the derivative of f at a and $Df(a)b$ stands for the value of $Df(a)$ at b.   (30 points)