

Math 450/586: Quiz # 4

Fall 2009

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 45 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask, 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)

1. Give the definition of the following objects. (30 points)

1.a) A C^∞ -structure on a topological manifold:

1.b) A C^∞ -function $f : M \rightarrow N$ for a pair of C^∞ -manifolds M and N :

1.c) Two compatible C^∞ -atlases:

1.d) A product manifold:

2. Show that $M := \{(x, y, z) \in \mathbb{R}^3 \mid 2 < e^{xyz} < 3\}$ endowed with the subspace (induced) topology is a topological manifold. (30 points)

3. Let M be a 2-dimensional C^∞ -manifold, (U_α, ϕ_α) be a coordinate chart of M , (x^1, x^2) denote the coordinates assigned to points q of U_α by ϕ_α , i.e., $\phi_\alpha(q) = (x^1, x^2)$, $f : U_\alpha \rightarrow \mathbb{R}$ be a function given by

$$f(q) = f(\phi_\alpha^{-1}(x^1, x^2)) := x^1 + (x^2)^2,$$

and

$$X^1 = (x^1)^2 - (x^2)^2, \quad X^2 = (x^1)^2 + (x^2)^2, \quad \omega_1 = x^1 + x^2, \quad \omega_2 = 1 + x^1 - x^2,$$

be the components of $X \in T_q M$ and $\omega \in T_q M^*$ in the coordinate chart (U_α, ϕ_α) , i.e., $X = X^i \frac{\partial}{\partial x^i}$ and $\omega = \omega_i dx^i$. Compute the following quantities for $p := \phi_\alpha^{-1}(1, 2) \in U_\alpha$. (40 points)

3.a) $X(f(p)) =$

3.b) $\omega(X) =$