Math 503: Midterm Exam # 2

Fall 2006

- Write your name and Student ID number in the space provided below and sign.

Student's Name:	
ID Number:	
Signature:	

- You have 80 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Let $\vec{F} : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\vec{F}(x, y) = (3x^4y + x^2y^3, -3xy^4 - x^3y^2)$ and $C := \{(x, y) \in \mathbb{R}^2 \mid y \geq 0, x^2 + y^2 = 1\}$. Evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$ where C is oriented counterclockwise. (20 points)

Problem 2.

a) Find $y : [0,1] \to \mathbb{R}$ such that $y(0) = 0$, $y(1) = 1$, and the functional $\mathcal{F}[y] = \int_0^1 y^2(x) y'^2(x)\, dx$ has a stationary value. (20 points)

Hint: Try to express x as a function of y.

b) Calculate the value of $\mathcal{F}[y]$ for the function y you find in part a). (5 points)

Problem 3. Find the solution $u(x, y)$ of the following initial-value problem for all $(x, y) \in \mathbb{R}^2$.

$$u_x + e^x u_y = u^2, \quad u(0, y) = y^2. \qquad \text{(30 points)}$$

Problem 4. Find $u(x,t)$ for all $x \in [0,\pi]$ and $t \in [0,\infty)$ such that

$$\begin{aligned} &u_t - u_{xx} = u, && \forall x \in [0,\pi],\ \forall t \in [0,\infty), \\ &u(0,t) = u(\pi,t) = 0, && \forall t \in [0,\infty), \\ &u(x,0) = \sin x, && \forall x \in [0,\pi]. \end{aligned}$$

(25 points)