Math 503, Fall 2007
Assignment for Nov. 29 - Dec. 03

- Read pages 782-796 of Riley-Hobson-Bence.
- Solve Problems 22.20 and 22.22 on page 799 of Riley-Hobson-Bence.
- Solve the following problems.

1. Find the stationary points of the following functionals.

$$\mathcal{F}[y(x)] = \int_a^b \sqrt{1 + \frac{y'^2}{y^2}}\, dx,$$
$$\mathcal{G}[y(x)] = \int_a^b \frac{\sqrt{1 + y'^2}}{1 + y}\, dx,$$

2. Let S be the surface of revolution of the curve $z = x^2$ about z-axis. Find the differential equation determining the geodesics on S and obtain its solution.

3. Let $\mathcal{F}$ and $\mathcal{G}$ be functionals. Prove that

$$\frac{\delta}{\delta y(t)} \left(\mathcal{F}[y(s)]\mathcal{G}[y(s)]\right) = \frac{\delta \mathcal{F}[y(s)]}{\delta y(t)}\, \mathcal{G}[y(s)] + \mathcal{F}[y(s)]\, \frac{\delta \mathcal{G}[y(s)]}{\delta y(t)}.$$

4. Show that if $\mathcal{F}[y(x)] := \int_a^b y'^2(x)\, dx$. The second functional derivative of $\mathcal{F}[y(s)]$ is given by

$$\frac{\delta}{\delta y(u)} \frac{\delta}{\delta y(t)}\, \mathcal{F}[y(s)] = -\frac{\partial^2}{\partial t^2}\, \delta(t - u).$$