1 Consider a system with phase space \mathbb{R}^2 and let $\mathscr{T} : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\mathscr{T}(q, p) := (\tilde{q}, \tilde{p})$ where \tilde{p} and \tilde{q} satisfy

$$\begin{bmatrix} \tilde{q} \\ \tilde{p} \end{bmatrix} = \begin{bmatrix} a(t) & b(t) \\ c(t) & d(t) \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} + \begin{bmatrix} e(t) \\ f(t) \end{bmatrix},$$

and a(t), b(t), c(t), d(t), e(t), and f(t) are real-valued functions of t.

1.a (10 points) Find the necessary and sufficient conditions on these functions such that \mathscr{T} is a canonical transformation.

1.b (15 points) Consider the infinitesimal canonical transformation that belong to those you find in part a of the problem. Show that their generator is a quadratic polynomial with time-dependent coefficients.

- 2 (10 points) Do Exercise 2.8.1 on Page 100 of Shankar's book.
- **3** (15 points) Do Exercise 2.8.3 on Page 100 of Shankar's book.
- 4 (15 points) Do Exercise 2.8.4 on Page 101 of Shankar's book.
- **5** (10 points) Do Exercise 1.1.3 on Page 4 of Shankar's book.
- 6 (10 points) Do Exercise 1.3.1 on Page 15 of Shankar's book.
- 7 (15 points) Do Exercise 1.4.1 on Page 18 of Shankar's book.