- Solve problems 5, 10, 12, and 21 on Pages 104-106 of Jose and Saletan.
- Solve the following problems.

1) Consider a particle of mass m that is constrained to move on a sphere of radius a and subject to a constant gravitation field. Suppose that the particle is attached to a massless spring with spring constant k and equilibrium length L. The other end of the spring is attached to the north pole of the sphere. The spring always lies inside the sphere and does not bend.

a. Give the expression for the Lagrangian of the system in a spherical coordinate system whose origin is the center of the sphere.

b. Find the value of polar angle θ corresponding to the equilibrium state of the system.

c. Derive and simplify the equations of motion as much as possible.

d. Find special exact solutions for these equations and describe the corresponding initial conditions.

e. For the case that the particle moves on a meridian of the sphere, find the frequency of small oscillations of the particle about its equilibrium configuration.

2) Consider a particle of mass m constrained to move on a circle of radius a in the absence of gravity. Suppose that this circle rotates with a constant angular speed ω about an axis that intersects the circle at a fixed point and is orthogonal to the plane containing it.

a. Find the equations of motion of the particle using appropriate generalized coordinates in a frame rotating with the circle.

b. Show that in this frame the system has the same dynamics as a planar pendulum in a constant gravitation field.

c. Find initial condition for which the motion of the particle can be determined exactly and explicitly and express the position of the particle in these cases as a function of time in an inertial frame.

3) Consider a pendulum obtained by attaching a particle of mass M to a massless rigid rod of length L whose other end of the rod is connected to a point p of a horizontal disc of radius a such that the point p is at distance b of the center of the disc.

a. Suppose that the disc is no longer rotating but accelerating with a constant acceleration in an arbitrary direction in space (while it remains horizontal all the time). Choose a set of convenient generalized coordinates and write down a Lagrangian for this system in a frame that moves with the disc. Derive the equations of motion in this frame.

b. Suppose that the disc is rotating with a constant angular speed ω about the vertical axis that passes through the center of the disc (i.e., the disc's symmetry axis.) Choose a set of convenient generalized coordinates and write down a Lagrangian for this system in a frame that rotates together with the disc. Derive the equations of motion in this frame.