1) Show that adding total derivative function to the Lagrangian does not change the Lagrange equations.

2) Derive Lagrange equations of motion in case $L = L(q, \ddot{q}, t)$.

3) Consider two dimensional anisotropic harmonic oscillator. Let $\mathbf{m}$ be the mass of the oscillating body and k_x and k_y be the spring constants in the x- and y-directions respectively.
(a) Construct a Hamiltonian for this system
(b) Find a solution using Hamilton-Jacobi method in terms of initial quantities.

4) Consider the Hamiltonian explicitly time-dependent Hamiltonian:

$$H = H_0(q, p) - \varepsilon sin\omega t$$

where $H_0(q, p)$ is a time-independent Hamiltonian, and ε and ω are given constants.
(a) How are the canonical equations modified by the presence of the term $\varepsilon sin\omega t$?
(b) Find a canonical transformation that restores the canonical form of the equations of motion and determine the new Hamiltonian.

5) Consider a classical system determined by the Lagrangian:

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y}^2 - k\sqrt{x^2 + y^2}$$

where a, b, c, f, g and k are constants. Find a Hamiltonian for this system and determine the conserved quantities.

6) Consider the Lagrangian of a charged particle in an electomagnetic field, described by a scaler potential $\phi(\mathbf{r}, t)$ and a vector potential $\mathbf{A}(\mathbf{r}, t)$

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 - e\phi(\mathbf{r}, t) + e\dot{\mathbf{r}}.\mathbf{A}(\mathbf{r}, t)$$

(a) Show that the corresponding Euler-Lagrange equation takes the form

$$m\ddot{\mathbf{r}} = e\mathbf{E} + e\dot{\mathbf{r}} \times \mathbf{B}$$

in terms of the electric field $\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$ and the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$.
(b) Show that the result formulated in problem 1 implies that the equations of motion are invariant under the gauge transformations

$$\phi(\mathbf{r}, t) \to \phi(\mathbf{r}, t) - \frac{\partial \Lambda(\mathbf{r}, t)}{\partial t}$$
$$\mathbf{A}(\mathbf{r}, t) \to \mathbf{A}(\mathbf{r}, t) + \nabla\Lambda(\mathbf{r}, t)$$

In addition check explicitly that this leaves the fields $\mathbf{E}$ and $\mathbf{B}$ unchanged.

(c) Derive the corresponding Hamiltonian $H(\mathbf{p}, \mathbf{r})$.

(d) Is the canonical momentum $\mathbf{p}$ gauge invariant? How about the kinematic momentum $\pi = m\mathbf{r}$?

7) Show by the use of of Poisson brackets that for a one-dimensional harmonic oscillator there is a constant of motion u defined as

$$u(q, p, t) = \ln(p + im\omega q) - i\omega t, \qquad \omega = \sqrt{\frac{k}{m}}$$