

1. Give the details of the derivation of Equations 10.18 and 10.21 on page 361 of the textbook (Auletta, Fortunato & Parisi, Cambridge University Press, 2009.)
2. Consider a particle of mass m that moves on the real line and interacts with the potential

$$v(x) := \begin{cases} \zeta e^{-x} & \text{for } x \in [0, a], \\ \infty & \text{for } x \notin [0, a], \end{cases}$$

where a and ζ are real numbers and a is positive. Use perturbation theory to obtain the energy eigenvalues of the particle up to and including second order terms in ζ .

3. Use first-order perturbation theory to find eigenvalues and a set of orthonormal eigenvectors of the following matrices

$$\mathbf{M} := \begin{bmatrix} 1 + \zeta & \zeta & 0 & -\zeta \\ \zeta & 2 & 2\zeta & -\zeta \\ 0 & 2\zeta & 3 & -i\zeta \\ -\zeta & -\zeta & i\zeta & 4 \end{bmatrix}, \quad \mathbf{N} := \begin{bmatrix} 2 + \zeta & \zeta & 0 & -\zeta \\ \zeta & 2 & 2\zeta & -\zeta \\ 0 & 2\zeta & 3 & -i\zeta \\ -\zeta & -\zeta & i\zeta & 4 \end{bmatrix}.$$