1) Let $\mathbb{E}^2$ denote $\mathbb{C}^2$ endowed with the Euclidian inner product and $\psi := \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ be a nonzero element of $\mathbb{E}^2$.

1.a) Compute the matrix representation $\mathbf{P}_\lambda$ of the projection operator $P_\lambda = \frac{|\psi\rangle\langle\psi|}{\langle\psi|\psi\rangle}$ onto the state λ defined by ψ.

1.b) Find $(x, y, z) \in \mathbb{R}^3$ such that

$$\mathbf{P}_\lambda = \begin{pmatrix} z & x - iy \\ x + iy & 1 - z \end{pmatrix},$$

i.e., express x, y, z as function of w_1 and w_2.

1.c) Show that (x, y, z) determines a point on the two-dimensional sphere S^2 defined by

$$x^2 + y^2 + (z - \frac{1}{2})^2 = \frac{1}{4}.$$

1.d) Determine the projection operators corresponding to the north and south poles of S^2.

1.e) Show that there is a one-to-one correspondence between the points of S^2 and the rays λ in $\mathbb{E}^2$ (i.e., points of the projective Hilbert space $\mathscr{P}(\mathbb{E}^2)$.)

2) Consider a quantum system that is represented using the Hilbert space obtained by endowing $\mathbb{C}^3$ with the Euclidean inner product as the Hilbert space and a Hamiltonian operator. Let $\hat{A}$ be the observable represented in the standard basis of $\mathbb{C}^2$ by the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

2.a) Determine the eigenstates of $\hat{A}$ and the corresponding projection operators.

2.b) Compute the probability of measuring each of the eigenvalues of $\hat{A}$ in the state λ determined by the state vector $\psi_\lambda = \begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix}$.

2.c) Compute the expectation value of measuring $\hat{A}$ in the state λ_ψ first using $\langle\hat{A}\rangle_\lambda = \frac{\langle\psi_\lambda|\hat{A}\psi_\lambda\rangle}{\langle\psi_\lambda|\psi_\lambda\rangle}$ and then using $\langle\hat{A}\rangle_\lambda = \sum_{n=1}^{3} a_n \mathrm{Prob}_{a_n}(\lambda)$, where a_1, a_2, a_3 are the eigenvalues of $\hat{A}$ and $\mathrm{Prob}_{a_n}(\lambda)$ stands for the probability of finding a_n as a result of measuring $\hat{A}$ in the state λ.

2.d) Compute the expectation value of measuring $\hat{A}^2$ in the state λ.

2.e) For all $s \in \mathbb{R}$ compute the matrix representation of $e^{s\hat{A}}$ in the standard basis of $\mathbb{C}^3$.