I) Solve Problems: **3.6, 3.7, 3.10, 3.11** on page **139** tof the textbook (Auletta, Fortunato & Parisi, Cambridge University Press, 2009).

II) Solve the following problems.

1. Let $\mathbb{E}^n$ denote $\mathbb{C}^2$ endowed with the Euclidean inner product and $\hat{H}$ and $\hat{O}$ be the linear operators acting in $\mathbb{E}$ that are respectively represented by the following matrices in the standard basis of $\mathbb{C}^2$.

$$\mathbf{H} = E\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{O} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$

where $E \in \mathbb{R}^+$. Consider the quantum system described by $(\mathbb{E}^2, \hat{H})$. Suppose that the system starts its evolution at time $t_0 = 0$ in the state $\lambda(0)$ given by the state vector $\psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

1.a) Find the explicit expression for the eigenvectors and eigenvalues of $\hat{H}$.

1.b) Compute the matrix representation of the time-evolution operator $\hat{U}(t, 0)$ in the standard basis of $\mathbb{C}^2$, i.e., find the explicit expression for the entries of this matrix.

1.c) Find the expectation value of $\hat{O}$ as the system evolves in time, i.e., give the formula for $\langle \hat{O} \rangle_{\lambda(t)}$.

1.d) Find the probability of finding 1, if we measure $\hat{O}$ at time $t_1 := 2\hbar/E$.

1.e) Suppose that we do actually find 1 after we measure $\hat{O}$ at t_1 and that at some time $t_2 > t_1$ we measure $\hat{H}$. Find the smallest value of t_2 such that the result of the second measurement is 0 with a 100% probability.

2) Derive the explicit formula for the propagator $K(x, t; x', t')$ of a free particle moving on the real line $\mathbb{R}$.

3) Use your response to the preceding problem to compute the position wave function $\psi(x, t)$ if $\psi(x, 0) := e^{-ax^2}$, where a is a positive real number. Determine the rate of change of $|\psi(0, t)|$ as a function of time, i.e., compute its time-derivative.

4) Let $\psi(0) \in L^2(\mathbb{R})$ be the state vector with position wave function $\psi(x, 0) := e^{-ax^2}$.

4.a) Find the momentum wave function $\tilde{\psi}(p, 0) := \langle p|\psi(0)\rangle$.

4.b) Suppose that $\psi(0)$ describes the initial state of a free particle moving on $\mathbb{R}$. Find the explicit formula for the momentum wave function $\tilde{\psi}(p, t)$ for the evolving state vector $\psi(t)$.