Himanshu Badhani (The Institute of Mathematical Sciences, Chennai, India)

Title: $\mathcal{PT}$-symmetric quantum walk: Exploring the reduced dynamics

Abstract: In this work we study a system undergoing evolution under a $\mathcal{PT}$-symmetric, non-Hermitian, and one-dimensional discrete time quantum walk (NH-DTQW). We explore the properties of the reduced dynamics of such evolutions by exploring their P-divisibility and entanglement growth.

We know that the $\mathcal{PT}$-symmetric evolution can be seen as a unitary dynamics under the appropriate choice the inner product. Furthermore, there is a freedom in the choice of such an inner product, which means that there are an infinite number of Hilbert spaces under which the evolution can be considered to be unitary. One can also map the states and the operators in these non-trivial metric spaces to the states and operators in the Euclidean inner product space, and thereby mapping to the textbook Hermitian evolution. However, a state/operator from different metric spaces is mapped onto different (unitarily equivalent) states/operators in the Euclidean inner product space. This is not a problem for the consistent characterization of the state because as it turns out that the expectation value of any observables with respect to a state in any of the allowable Hilbert spaces is the same.

In this work ,we ask the question: does the choice of the inner product also consistently characterise the reduced system and its dynamics? However, the questions about the reduced system $\mathcal{H}_A$ make sense only if the Hilbert space is of the form $\mathcal{H}_A \otimes \mathcal{H}_B$ with the metric given by $\mathcal{G}_A \otimes \mathcal{G}_B$. We take a DTQW as a toy model of a bipartite system with the position degrees of freedom $\mathcal{H}_p$ and the coin degree of freedom $\mathcal{H}_c$. The unitary quantum walk takes place in the Euclidean inner-product space $\mathcal{H}_p \otimes \mathcal{H}_c$, and here the reduced dynamics of the coin is well defined. We show that for a $\mathcal{PT}$-symmetric NH-DTQW, a metric of the form $\mathcal{G}_p \otimes \mathcal{G}_c$ cannot be constructed, thus not allowing to define the dynamics of a coin state in the non-trivial metric space. In other words, the NH-DTQW does not take place in a Hilbert space of the form $\mathcal{H}_p^G \otimes \mathcal{H}_c^G$ under any choice of the metric. Nevertheless, a such a walk can be mapped to a number of unitary quantum walks onto the $\mathcal{H}_p \otimes \mathcal{H}_c$ space depending on the choice of the metric. We look at the reduced dynamics of these unitary maps by studying their P-divisibility and entanglement growth to investigate if the reduced dynamics in the Euclidean Hilbert space can reveal some non-trivial properties of the dynamics in the non-trivial metric Hilbert space. We use numerics to show that the BLP measure of non-Markovainity of the coin state evolution is independent of the metric choice. We argue that a metric independent characterization of quantum operations allows them to be consistently defined under $\mathcal{PT}$-symmetric QM.