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Title: $\mathcal{PT}$-symmetric potentials beyond the Natanzon class

Abstract: The exact solutions of quantum mechanical potentials are typically obtained in terms of some special functions of mathematical physics. The most widely studied class is Natanzon potentials, in which case the solutions are expressed using (confluent) hypergeometric functions. For bound states they usually reduce to orthogonal polynomials (Jacobi, generalized Laguerre, Hermite). The Heun type differential equations offer a natural way to expand the range of solvable potentials. They contain more parameters, so, in principle, potentials with more flexible shapes can be generated from them. However, the solutions of the Heun type differential equations are much less known than the (confluent) hypergeometric functions.

The $\mathcal{PT}$-symmetrization of the Natanzon (1) class potentials has been discussed in detail previously (1, 2). The direct generalization of these results can be worked out for the confluent Heun equation, which reduces to the confluent hypergeometric differential equation for the special choice of the parameters. The $\mathcal{PT}$-symmetrization of potentials obtained from the confluent Heun equation has been discussed in Ref. (3). Due to the symmetries of this equation and its solutions, this procedure can be implemented on the formal level, without the use of explicit solutions and potentials. Similarly to the Natanzon potential class, the polynomial solutions play a central role in this case too. A systematic search for the polynomial solutions of the confluent Heun equation has been carried out recently (4). Besides the generalized Laguerre and the rationally extended X_1 type Laguerre polynomias, two further polynomial solutions have been identified, which exhibit characteristic of semi-classical orthogonal polynomials. Eventually, all known potentials with solutions expressed in terms of the generalized Laguerre polynomials within, oroutside the Natanzon confluent potential class, were recovered. Another member of the Heun family, the bi-confluent Heun differential equation has also been investigated. Polynomial solutions have been found to play an important role in this case too: the sextic oscillator, which has been identified previously as a quasi-exactly solvable potential emerged from this procedure in a natural way (5).

References:

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