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Title: Poisson wave trace formula for Dirac resonances at thresholds and its applications

Abstract: Resonances, which are metastable states and are associated with the complex poles of the scattering matrix, play a central role in diverse processes in chemistry and physics [7,8]. We study resonances of the Dirac operator $D = D_0 + V(x)$, where D_0 is the free three-dimensional Dirac operator and $V(x)$ is a smooth compactly supported Hermitian matrix potential [1, 2, 3, 4, 5, 6, 9, 10]. We define resonances of D as poles of the meromorphic continuation of its cut-off resolvent. By analyzing the resolvent behaviour at the spectrum edges $\pm m$, we establish a generalized Birman-Krein formula, taking into account possible resonances at $\pm m$. As an application of the new Birman-Krein formula we establish the Poisson wave trace formula in its full generality. The Poisson wave trace formula links the resonances with the trace of the difference of the wave groups. The Poisson wave trace formula, in conjunction with asymptotics of the scattering phase, allows us to prove that, under certain natural assumptions on V, the perturbed Dirac operator has infinitely many resonances; a result similar in nature to Melrose's classic 1995 result for Schrödinger operators.

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