Manami Yamagishi (Department of Physics, The University of Tokyo, Japan)

Title: Dynamics of a quantum active particle based on non-Hermitian quantum walks

Abstract: We present a model of a quantum active particle using non-Hermitian quantum walks in one and two dimensions (1D and 2D) and analyze its dynamics. Although there are a number of works on active matter, most of them are conducted in classical systems. Adachi *et al.* [1] used a non-Hermitian quantum spin system to simulate a "stoquastic" active matter. In contrast to their work with many-body systems, we start with simpler one-particle systems to allow systems real-time evolution in a fully quantum range. In particular, we utilize a model of quantum walks proposed by Yamagishi *et al.* [2] to model dynamics of quantum active matter in two dimensions. We observed not only activeness but also quantumness at the same time.

We believe that the following two points are essential properties for a system to be an active matter: (i) energy nor momentum are not conserved and (ii) kinetic motion depends on particles' internal states. A system without energy conservation is realized with our non-Hermitian Hamiltonian

$$H_{\mathrm{NH}} = \sigma^0 \otimes \tau^0 \otimes \begin{pmatrix} -\varepsilon & -w\mathrm{e}^{-g} \\ -w\mathrm{e}^{+g} & +\varepsilon \end{pmatrix},$$

where σ^0 and τ^0 are 2×2 identity matrices for the space spanned by the leftward and rightward states ($|\mathrm{L}\rangle$ and $|\mathrm{R}\rangle$) and the space spanned by the downward and upward states ($|\mathrm{D}\rangle$ and $|\mathrm{U}\rangle$), respectively. We introduce new internal states, the ground state $|\mathrm{G}\rangle$ and the excited state $|\mathrm{E}\rangle$. The non-Hermiticity parameter g promotes transition from $|\mathrm{G}\rangle$ to $|\mathrm{E}\rangle$, and hence the particle takes up energy from the environment. We use different parameter values for $|\mathrm{G}\rangle$ and $|\mathrm{E}\rangle$, which means that the kinetic motion depends on particle's internal state, to realize a system without momentum conservation.

We aim to reproduce similar phenomena that Schweitzer *et al.* [3] numerically found, that is, the dynamics of their (active) Brownian particle changes depending on energy-take-up term. To simulate "Brownian motion" under a harmonic potential in a 2D quantum system, we use quantum walks. The quantum walk (QW) is a quantum analogue of random walk. Instead of stochastic fluctuations of a classical random walker, a quantum walker moves under interference of quantum fluctuations at each site, which deterministically governs the walker's dynamics. We start with proposing a 2D Dirac Hamiltonian [2]

$$H_{\mathrm{D}}^{(2)} := (\epsilon\sigma^z p_x + m_x(x)\sigma^y) \otimes \tau^0 \otimes \upsilon^0 + \sigma^x \otimes (\epsilon\tau^z p_y + m_y(y)\tau^y) \otimes \upsilon^0, \tag{1}$$

which can be mapped to a 2D QW as well as to a Schrödinger Hamiltonian as we will show in the talk. Here, $\{\sigma^x, \sigma^y, \sigma^z\}$ and $\{\tau^x, \tau^y, \tau^z\}$ are the Pauli matrices for the spaces spanned by $\{|\mathrm{L}\rangle, |\mathrm{R}\rangle\}$ and $\{|\mathrm{D}\rangle, |\mathrm{U}\rangle\}$, respectively, and υ^0 is a 2×2 identity matrix for the space spanned by $|\mathrm{G}\rangle$ and $|\mathrm{E}\rangle$. We let $m_x(x)$ and $m_y(y)$ denote the mass terms, which are proportional to the parameters $\theta_x(x)$ and $\theta_y(y)$ for the coin operators for QW [4]. By setting $\theta_x(x)$ and $\theta_y(y)$ linear to x and y respectively, we realize the harmonic potential in 2D. The momenta p_x and p_y can be rewritten in the forms of $-\mathrm{i}\partial/\partial x$ and $-\mathrm{i}\partial/\partial y$, respectively. Figure 1 shows probability distributions after 50 time steps with different values of g in the harmonic potential in 1D. We observe that less amount of wave functions are bound around the origin of the potential

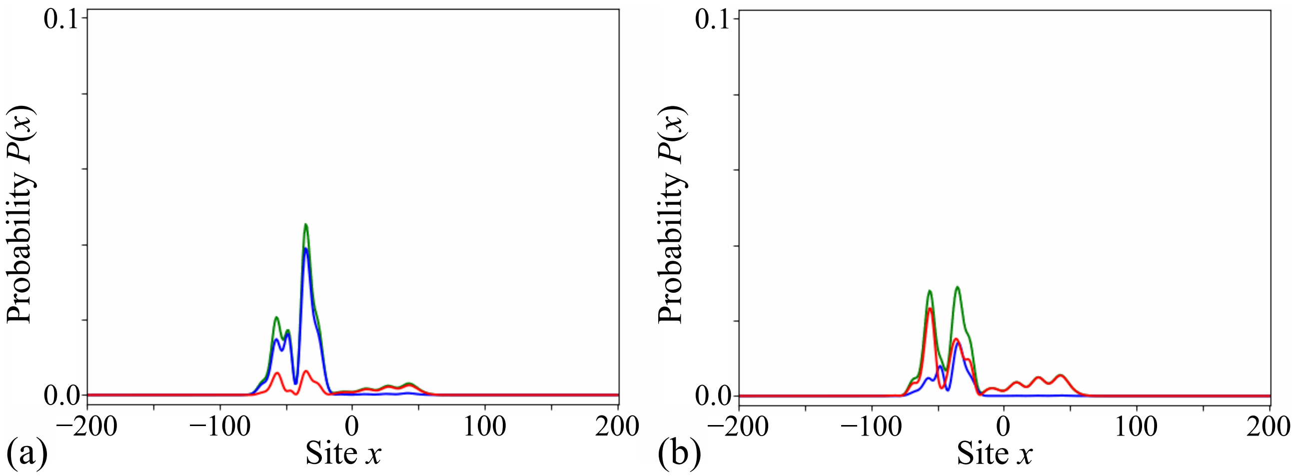

Figure 1: Probability distribution after 50 time steps of evolution in 1D. (a) $g = 0$ and (b) $g = 1$. The blue and red curves show the probability distributions of $|\mathrm{G}\rangle$ and $|\mathrm{E}\rangle$, respectively, and the green curve shows the summation of both states. $\varepsilon = w = 0.25$.

($x = -40$) in the $g = 1$ case than in the $g = 0$ case. This means that more wave functions have gone out, climbing up the potential wall in the $g = 1$ case.

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References:

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