SPARSE BOUNDED COMPONENT ANALYSIS FOR CONVOLUTIVE MIXTURES

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ABSTRACT

In this article, we propose a Bounded Component Analysis (BCA) approach for the separation of the convolutive mixtures of sparse sources. The corresponding algorithm is derived from a geometric objective function defined over a completely deterministic setting. Therefore, it is applicable to sources which can be independent or dependent in both space and time dimensions. We show that all global optima of the proposed objective are perfect separators. We also provide numerical examples to illustrate the performance of the algorithm.

***Index Terms*—** Convolutive Blind Source Separation, Bounded Component Analysis, Sparse Bounded Component Analysis.

1. INTRODUCTION

Blind Source Separation (BSS) is a central problem in signal processing, with diverse set of applications [1]. It can be described as extracting sources from their linear mixtures, where the mixing can be either only in source dimension (instantaneous BSS) or in both source and sample dimensions (convolutive BSS).

Bounded Component Analysis (BCA) is a recently introduced source separation framework that exploits the boundedness of sources to relax the independence assumption in Independent Component Analysis (ICA), with a less stringent domain separability assumption [2]. In [3], a geometric BCA approach, based on the volume ratios of objects defined related to separator output samples was proposed. The same approach was later extended to convolutive mixtures [4].

Aforementioned algorithmic BCA frameworks are mostly applicable to source vectors whose samples lie in ℓ_∞-norm ball in high dimensional space, which is illustrated to be a nice fit for digital communication constellations, natural images and harmonic signals. More recently, [5] adapted the geometric approach in [3] for unmixing bounded and sparse natured sources from their memoryless mixtures by replacing the ℓ_∞-norm ball in [3] with the ℓ_1-norm ball.

In this article, we extend the instantaneous Sparse BCA (SBCA) approach in [5] to more general convolutive mixtures. We note that the exploitation of sparsity for solving BSS problem has been referred in literature as Sparse Component Analysis (SCA), and several different approaches has been proposed especially for the instantaneous mixing problem (see for example [1, 6] and the references therein). The algorithm proposed in this paper is derived from a novel optimization setting where the global maxima are proven to correspond to perfect separators.

The article is organized as follows: In Section 2, we introduce the Convolutive Sparse Bounded Component Analysis (CSBCA) setup. The proposed CSBCA approach is introduced in Section 3. Finally, numerical examples illustrating the use of the proposed approach is given in Section 4.

2. CONVOLUTIVE SPARSE BOUNDED COMPONENT ANALYSIS SETUP

For the convolutive BSS setup:

- We assume that there are p sources represented by the set $\mathcal{S} = \{\mathbf{s}(n) \in \Re^p\}$. Furthermore, it is assumed that source vectors are bounded in magnitude and lie in ℓ_1 norm ball, i.e., $\mathbf{s}(n) \in \mathcal{B}_s$ where

$$\mathcal{B}_s = \{\mathbf{s} \in \Re^p \,|\, \|\mathbf{s}\|_1 < 1\}. \quad (1)$$

  This assumption implies that the sources have identical unity range for the purpose of simplifying future expressions, without any loss of generality. In the more general case, $\mathcal{B}_s$ can be replaced with the weighted ℓ_1-norm-ball. Note that, we do not make any stochastic assumption about the source vector such as statistical independence of its components.

- The source signals are mixed by a convolutive MIMO channel which is represented by

$$\mathbf{y}(n) = \sum_{k=0}^{K-1} \mathbf{H}(k)\mathbf{s}(n-k), \quad (2)$$

  where $\{\mathbf{H}(k), k \in \{0, \ldots, K-1\}\}$ are the channel impulse response coefficients of dimension $q \times p$. We assume that the mixing system is equalizable [7] having order of $K-1$ and $q \geq p$. Defining $\tilde{\mathbf{H}} = [\mathbf{H}(0), \mathbf{H}(1), \ldots, \mathbf{H}(K-1)]$ and $\tilde{\mathbf{s}}_K(n) = [\mathbf{s}^T(n), \mathbf{s}^T(n-1), \ldots, \mathbf{s}^T(n-K+1)]^T$, the equation for the mixing sequence in (2) is turned into

$$\mathbf{y}(n) = \tilde{\mathbf{H}}\tilde{\mathbf{s}}_K(n), \quad n = 1, \ldots L. \quad (3)$$

- The separator is a convolutive system with the input-output relationship given by

$$\mathbf{z}(n) = \sum_{m=0}^{M-1} \mathbf{W}(m)\mathbf{y}(n-m), \tag{4}$$

where $\{\mathbf{W}(m), m \in \{0,\dots,M-1\}\}$ are the impulse response coefficients of dimension $p \times q$ and $M-1$ is the order of the separator system. Defining $\tilde{\mathbf{W}} = [\mathbf{W}(0), \mathbf{W}(1), \dots, \mathbf{W}(M-1)]$ and $\tilde{\mathbf{y}}_M(n) = [\mathbf{y}^T(n), \mathbf{y}^T(n-1), \dots, \mathbf{y}^T(n-M+1)]^T$, (4) can be written as

$$\mathbf{z}(n) = \tilde{\mathbf{W}}\tilde{\mathbf{y}}_M(n), \quad n = 1, \dots, L+M-1. \tag{5}$$

- By defining the cascade of the mixing and separator systems as

$$\mathbf{G}(k) = \sum_{l=0}^{P-1} \mathbf{W}(l)\mathbf{H}(k-l), \quad k = 0, \dots, P-1. \tag{6}$$

where $P-1 = K+M-2$ is the order of the overall system, we can write the separator outputs in terms of sources as

$$\mathbf{z}(n) = \sum_{l=0}^{P-1} \mathbf{G}(l)\mathbf{s}(n-l). \tag{7}$$

Defining $\tilde{\mathbf{G}} = [\mathbf{G}(0), \mathbf{G}(1), \dots, \mathbf{G}(P-1)]$ and $\tilde{\mathbf{s}}_P(n) = [\mathbf{s}^T(n), \mathbf{s}^T(n-1), \dots, \mathbf{s}^T(n-P+1)]^T$, (7) turns to be

$$\mathbf{z}(n) = \tilde{\mathbf{G}}\tilde{\mathbf{s}}_P(n), \quad n = 1, \dots, L+M-1. \tag{8}$$

- We also define the extended separator output vector $\tilde{\mathbf{z}}_N(n) = [\mathbf{z}^T(n), \mathbf{z}^T(n-1), \dots, \mathbf{z}^T(n-N+1)]^T$ which will be used in the objective function definition in the next section, and $Np \times (N+P-1)p$ size block-Toeplitz matrix

$$\Gamma_N(\tilde{\mathbf{G}}) = \begin{bmatrix} \mathbf{G}(0), \mathbf{G}(1), \dots, \mathbf{G}(P-1), \dots, \mathbf{0} \\ \vdots \qquad \ddots \qquad \ddots \qquad \vdots \\ \mathbf{0}, \dots, \mathbf{G}(0), \mathbf{G}(1), \dots, \mathbf{G}(P-1) \end{bmatrix}, \tag{9}$$

where $N \geq P$. This yields,

$$\tilde{\mathbf{z}}_N(n) = \Gamma_N(\tilde{\mathbf{G}})\tilde{\mathbf{s}}_{N+P-1}(n). \tag{10}$$

As a result, the convolutive channel generates $L+M-1$ output samples. Now after defining the BCA setup, we extend the deterministic instantaneous sparse BCA approach introduced in [5] to the convolutive case.

3. CONVOLUTIVE SBCA APPROACH

In this section we propose an objective function for the convolutive SBCA. We then prove that all global maxima of this objective are perfect separators, and derive an iterative algorithm for obtaining the separator matrix. As the basis, we introduce the following ***local dominance*** assumption:

Assumption (A1): The source sample set $S_{N+P-1} = \{\tilde{\mathbf{s}}_{N+P-1}(N+M-1), \tilde{\mathbf{s}}_{N+P-1}(N+M), \dots, \tilde{\mathbf{s}}_{N+P-1}(L)\}$ contains the vertices of its bounding l_1-norm-ball $\mathcal{B}_s$.

3.1. Objective Function

Similar to [5], the objective function is defined as the volume ratio of the following two objects defined for the samples of the separator output vector $\mathbf{z}(n)$ and the extended separator output vector $\tilde{\mathbf{z}}_N(n)$:

- *The bounding ℓ_1-norm ball* for the extended separator vector samples $\tilde{\mathbf{z}}_N(n)$, which is defined as $\mathcal{B}_{\mathbf{Z}} = \{\mathbf{q} | \|\mathbf{q}\|_1 \leq \max_{n \in \{N,\dots,L_1\}} \|\tilde{\mathbf{z}}_N(n)\|_1\}$ where $L_1 = L+M-1$.
- *Principal Hyperellipsoid* for the extended separator vector samples $\tilde{\mathbf{z}}_N(n)$, which is defined as $\mathcal{E}_{\mathbf{Z}} = \{\mathbf{q} | (\mathbf{q} - \hat{\mu}_{\tilde{\mathbf{z}}_N})^T \hat{\mathbf{R}}_{\tilde{\mathbf{z}}_N}^{-1} (\mathbf{q} - \hat{\mu}_{\tilde{\mathbf{z}}_N}) \leq 1\}$ where $\hat{\mu}_{\tilde{\mathbf{z}}_N} = \frac{1}{L_2} \sum_{n=N}^{L_1} \tilde{\mathbf{z}}_N(n)$, $\hat{\mathbf{R}}_{\tilde{\mathbf{z}}_N} = \frac{1}{L_2} \sum_{n=N}^{L_1} (\tilde{\mathbf{z}}_N(n) - \hat{\mu}_{\tilde{\mathbf{z}}_N})(\tilde{\mathbf{z}}_N(n) - \hat{\mu}_{\tilde{\mathbf{z}}_N})^T$ where $L_2 = L_1 - N + 1$,

Based on these definitions, we define the convolutive SBCA objective as the volume ratio

$$J(\tilde{\mathbf{W}}) = \frac{\sqrt{\det(\hat{\mathbf{R}}_{\tilde{\mathbf{z}}_N})}}{(\max_{n \in \{N,\dots,L_1\}} \|\tilde{\mathbf{z}}_N(n)\|_1)^{Np}} \tag{11}$$

3.2. Global Optimality of Perfect Separators

The following theorem shows that all global optima of (11) are perfect separators.

Theorem: Given the BCA setup in section 2 and assuming $\tilde{\mathbf{H}}$ is equalizable by an FIR extractor matrix of order $M-1$, then all global maxima of (11) are perfect separators if the assumption ($\mathbf{A}$1) is correct.

Proof: We start by rewriting the objective function, in terms of the argument $\mathbf{G}(k) = \sum_{l=0}^{P-1} \mathbf{W}(l)\mathbf{H}(k-l)$ for $k = 0, \dots, P-1$, and $\tilde{\mathbf{G}} = [\mathbf{G}(0), \mathbf{G}(1), \dots, \mathbf{G}(P-1)]$. We define the operator Γ_N such that $\Gamma_N(\tilde{\mathbf{G}})$ is a block Toeplitz matrix of dimension $(Np) \times (N+P-1)p$ whose first block row is $[\mathbf{G}(0), \mathbf{G}(1), \dots, \mathbf{G}(P-1), \mathbf{0}, \dots, \mathbf{0}]$ and first block column is $[\mathbf{G}(0), \mathbf{0}, \dots, \mathbf{0}]^T$ where the zero matrices are of dimension $p \times p$. The resulting output sample vector and output sample covariance matrix are $\tilde{\mathbf{z}}_N(n) = \Gamma_N(\tilde{\mathbf{G}})\tilde{\mathbf{s}}_{N+P-1}(n)$ for $n = N,\dots,L_1$, and $\hat{\mathbf{R}}_{\tilde{\mathbf{z}}_\mathbf{N}} = \Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{\mathbf{N+P-1}}}\Gamma_N(\tilde{\mathbf{G}})^T$ where $\tilde{\mathbf{s}}_{N+P-1}$ is of dimension $(N+P-1)p \times 1$. We redefine the objective function as,

$$J(\tilde{\mathbf{G}}) = \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{(\max_{n\in\{N,\dots,L_1\}}\|\Gamma_N(\tilde{\mathbf{G}})\tilde{\mathbf{s}}_{L_3}\|_1)^{Np}}. \quad (12)$$

where $L_3 = N+P-1$. Following similar steps as in [5] under the assumption $(\mathbf{A}1)$, we can write for the denominator of (12),

$$(\max_{n\in\{N,\dots,L_1\}}\|\Gamma_N(\tilde{\mathbf{G}})\tilde{\mathbf{s}}_{L_3}\|_1)^{Np} \le \|\Gamma_N(\tilde{\mathbf{G}})\|_{1,1}^{Np} \quad (13)$$

where $\|\Gamma_N(\tilde{\mathbf{G}})\|_{1,1}$ can be explicitly written as $\|\Gamma_N(\tilde{\mathbf{G}})\|_{1,1} = \left\|\left[\ \|\Gamma_N(\tilde{\mathbf{G}})_{:,1}\|_1,\dots,\|\Gamma_N(\tilde{\mathbf{G}})_{:,(L_3)p}\|_1\ \right]\right\|_\infty$ [8]. If Assumption $(\mathbf{A}1)$ holds, then (13) is an equality, and we can rewrite (12):

$$\begin{aligned}
J(\tilde{\mathbf{G}}) &= \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{\left\|\left[\ \|\Gamma_N(\tilde{\mathbf{G}})_{:,1}\|_1\cdots\|\Gamma_N(\tilde{\mathbf{G}})_{:,L_3p}\|_1\ \right]\right\|_\infty^{Np}} && (14)\\
&= \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{\left\|\left[\ \|\Gamma_N(\tilde{\mathbf{G}})_{1,:}\|_1\cdots\|\Gamma_N(\tilde{\mathbf{G}})_{Np,:}\|_1\ \right]\right\|_\infty^{Np}} && (15)\\
&\le \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{\left(\left\|\left[\ \|\Gamma_N(\tilde{\mathbf{G}})_{1,:}\|_1\cdots\|\Gamma_N(\tilde{\mathbf{G}})_{Np,:}\|_1\ \right]\right\|_1/Np\right)^{Np}} && (16)\\
&\le \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{\|\Gamma_N(\tilde{\mathbf{G}})_{1,:}\|_1\|\Gamma_N(\tilde{\mathbf{G}})_{2,:}\|_1\cdots\|\Gamma_N(\tilde{\mathbf{G}})_{Np,:}\|_1} && (17)\\
&\le \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{\Gamma_N(\tilde{\mathbf{G}})_{1,:}\|_2\|\Gamma_N(\tilde{\mathbf{G}})_{2,:}\|_2\cdots\|\Gamma_N(\tilde{\mathbf{G}})_{Np,:}\|_2} && (18)
\end{aligned}$$

We note that for any $\tilde{\mathbf{G}}$ whose rows are not linearly independent we have $\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T) = 0$; therefore, the corresponding $\tilde{\mathbf{G}}$ can not be global maxima of (11). Hence for any $\tilde{\mathbf{G}}$ whose rows are linearly independent, in order to complete $\Gamma_N(\tilde{\mathbf{G}})$ into a full rank square matrix we introduce a $(P-1)p \times (L_3)p$ matrix $\mathbf{Y} = \mathbf{DP}$ where $\mathbf{D} = diag(\mathbf{a}_1;\mathbf{a}_2;\dots;\mathbf{a}_{(P-1)p})$ is a full rank diagonal matrix and $\mathbf{P}$ is a permutation matrix such that $\det(\mathbf{YBY}^T) = 1$ where we define $\mathbf{B} = \hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}} - \hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T$ $(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)^{-1}\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}$. Using the matrix $\mathbf{Y}$, we accomplish the following equality:

$$\begin{aligned}
&\det\left(\begin{bmatrix}\Gamma_N(\tilde{\mathbf{G}})\\ \mathbf{Y}\end{bmatrix}\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}[\ \Gamma_N(\tilde{\mathbf{G}})^T \quad \mathbf{Y}^T\ ]\right) = \\
&\det\left(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T\right) * \\
&\det(\mathbf{Y}(\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}} - \hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T * \\
&(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)^{-1}\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}})\mathbf{Y}^T) = \\
&\det\left(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T\right)\det(\mathbf{YBY}^T) = \\
&\det\left(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T\right) \qquad (19)
\end{aligned}$$

$\mathbf{YBY}^T$ is called the Schur complement of the expression $\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T$. We can choose an appropriate $\mathbf{Y}$ matrix satisfying (19). When choosing $\mathbf{Y}$ appropriately and using Hadamard's Inequality, it yields

$$\begin{aligned}
&\det\left(\begin{bmatrix}\Gamma_N(\tilde{\mathbf{G}})\\ \mathbf{Y}\end{bmatrix}\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\left[\ \Gamma_N(\tilde{\mathbf{G}})^T \quad \mathbf{Y}^T\ \right]\right) \le \\
&\prod_{m=1}^{Np}\|\ \Gamma_N(\tilde{\mathbf{G}})_{m,:}\ \|_2^2\prod_{n=1}^{(P-1)p}\|\ \mathbf{Y}_{n,:}\ \|_2^2\det(\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}) \qquad (20)
\end{aligned}$$

If we replace the equality for $\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)$ in (20) into (18), the expression for the upper bound of the objective becomes

$$J(\tilde{\mathbf{G}}) \le \prod_{n=1}^{(P-1)p}\|\ \mathbf{Y}_{n,:}\ \|_2\det(\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}})^{1/2} \qquad (21)$$

The inequality (16) is due to norm inequality (between l_1 and l_∞ norms) and (17) is due to arithmetic-geometric mean inequality, with equality if and only if all the rows of $\tilde{\mathbf{G}}$ have the same l_1 norm. (18) is due to norm inequality (between l_1 and l_2 norms), with equality if each row of $\tilde{\mathbf{G}}$ has only one non-zero entry. (20) is due to Hadamard's Inequality that is achieved if and only if the rows of $\Gamma_N(\tilde{\mathbf{G}})$ are orthogonal to each other and to the rows of $\mathbf{Y}$. Since we choose $P \le N$, there is at least one block column inside $\Gamma_N(\tilde{\mathbf{G}})$ that contains $\tilde{\mathbf{G}}^T$ in upside down form. This fact have two results: Firstly, (15) is achieved. And secondly, the non-zero entries of $\tilde{\mathbf{G}}$ must not be in the same position inside $\Gamma_N(\tilde{\mathbf{G}})$ with respect to mod p to achieve (20). As a result, the upper bound for the objective $J(\tilde{\mathbf{G}})$ on the right hand-side of (21) is achieved if and only if $\mathbb{G}(z) = diag\left(\alpha_1 z^{-d_1}, \alpha_2 z^{-d_2},\dots,\alpha_p z^{-d_p}\right)\mathbf{P}$.

3.3. Iterative Algorithm for Convolutive SBCA

Taking the logarithm of the SBCA objective in (11) converts the ratio form into a difference form which can be written as a modified objective

$$\begin{aligned}
\mathcal{J}(\tilde{\mathbf{W}}) &= \log(J(\tilde{\mathbf{W}})) \qquad (22)\\
&= \frac{1}{2}\log\left(\det(\Gamma_N(\tilde{\mathbf{W}})\hat{\mathbf{R}}_{\tilde{\mathbf{y}}_{\mathbf{N+M-1}}}\Gamma_N(\tilde{\mathbf{W}})^T)\right)\\
&\quad -Np\log(\max_{n\in\{N,\dots,L_1\}}\|\tilde{\mathbf{z}}_N(n)\|_1)
\end{aligned}$$

The modified SBCA objective $\mathcal{J}(\tilde{\mathbf{W}})$ in (22) is convenient for the iterative algorithm derivation, due to its additive form. Although $\mathcal{J}(\tilde{\mathbf{W}})$ is non-convex and not differentiable everywhere, we can still utilize Clarke subdifferential [9] for deriving iterative algorithms. More explicitly, we can write the subdifferential set corresponding to $\mathcal{J}(\tilde{\mathbf{W}})$ as

$$\partial\mathcal{J}(\tilde{\mathbf{W}}) = \sum_{k=0}^{N-1}\mathbf{X}_{kp+1:(k+1)p,kq+1:(k+M)q} \quad (23)$$

$$-\frac{Np\sum_{l^{(t)}\in\mathcal{I}_{\tilde{\mathbf{W}}}}\sum_{k=0}^{N-1}\lambda_l\mathbf{o}_{kp+1:(k+1)p,kq+1:(k+M)q}}{\max_{n\in\{N,\ldots,L_1\}}\|\tilde{\mathbf{z}}_N(n)\|_1}$$

$$|\lambda_l \geq 0, \sum_{l\in\{N,\ldots,L_1\}}\lambda_l = 1\}$$

where $\mathbf{o} = \text{sign}(\tilde{\mathbf{z}}_N(l^{(t)}))\tilde{\mathbf{y}}_{N+M-1}(l^{(t)})^T$,
$\mathbf{X} = \left(\Gamma_N(\tilde{\mathbf{W}})\hat{\mathbf{R}}_{\tilde{\mathbf{y}}_{N+M-1}}\Gamma_N(\tilde{\mathbf{W}})^T\right)^{-1}\Gamma_N(\tilde{\mathbf{W}})\hat{\mathbf{R}}_{\tilde{\mathbf{y}}_{N+M-1}}$,
$\hat{\mathbf{R}}_{\tilde{\mathbf{y}}_{N+M-1}} = \frac{1}{L_2}\sum_{n=N}^{L_1}(\tilde{\mathbf{y}}_{N+M-1}(n) - \hat{\mu}_{\tilde{\mathbf{y}}_{N+M-1}})$
$(\tilde{\mathbf{y}}_{N+M-1}(n) - \hat{\mu}_{\tilde{\mathbf{y}}_{N+M-1}})^T$ and
$\hat{\mu}_{\tilde{\mathbf{y}}_{N+M-1}} = \frac{1}{L_2}\sum_{n=N}^{L_1}\tilde{\mathbf{y}}_{N+M-1}(n)$. $\mathcal{I}_{\tilde{\mathbf{W}}}$ represents the subset of $\{N,\ldots,L_1\}$ and it corresponds to indices for which maximum ℓ_1-norm at the separator output is achieved. We can generate an iterative update of simple form, by selecting a special subgradient from the subddifferential set in (24), where only one λ_l term is non-zero:

$$\tilde{\mathbf{W}}^{(t+1)} = \tilde{\mathbf{W}}^{(t)} + \sigma^{(t)}(\sum_{k=0}^{N-1}\mathbf{X}^{(t)}_{kp+1:(k+1)p,kq+1:(k+M)q}$$

$$-Np\frac{\sum_{k=0}^{N-1}\mathbf{o}_{kp+1:(k+1)p,kq+1:(k+M)q}}{\max_{n\in\{N,\ldots,L_1\}}\|\tilde{\mathbf{z}}_N(n)\|_1}) \quad (24)$$

4. NUMERICAL EXAMPLES

In the first numerical example, we illustrate the Signal to Distortion Ratio (SDR) performance of the proposed algorithm for the synthetic sparse signal set given in the website of RIKEN Brain Science Institute [10]. We consider a scenario with 5 sources and 1000 samples. The convolutive mixing system is i.i.d. Gaussian with order 3, and the separator is of order 4. Mixture outputs are also corrupted by Gaussian noise. We compare the performance of the algorithm with 3 different algorithms, i.e., Castella's algorithm optimizing kurtosis based contrast function [11], and Koldovský's algorithm [12] based on EFICA which is an extension of the famous ICA method. We utilized the MATLAB toolbox called BSS Eval [13] to measure the performance of the algorithms. Based on the results in Fig.1-(b) and Fig.2, we can comment that the algorithm almost consistently provides improvement over the other algorithms. In the second example,

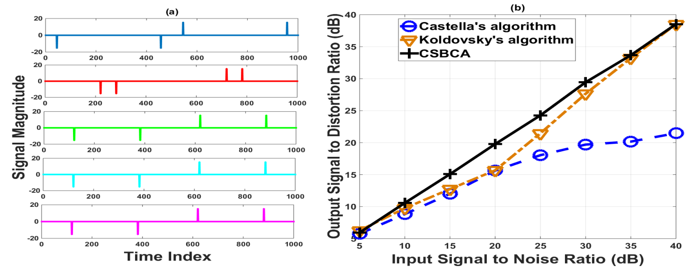

Fig. 1. a)- Synthetic sparse signals. b)- Output SDR vs. input SNR for 5 sources and 10 mixture channels.

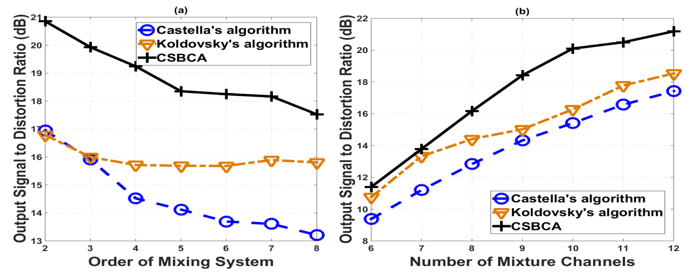

Fig. 2. a)- Output SDR vs. mixing order for 5 sources and 10 mixture channels under SNR=20 dB. b)- Output SDR vs. number of mixture channels for 5 sources under SNR=20 dB.

we synthetically generate sparse dependent sources by using Copula-T distribution with four degrees of freedom, and then transforming i.i.d. uniform random vector $\mathbf{u} \in [-1,1]^p$ through the mapping

$$\mathbf{s} = \begin{cases} \mathbf{u}, & \mathbf{u} \in \mathbf{B}_r \\ \mathbf{0}, & otherwise \end{cases} \quad (25)$$

where $\mathbf{B}_r = \mathbf{x} : \|\mathbf{x}_r\| \leq 1$ with $0 \leq r \leq 1$. In the scenario, there are 2000 samples, an i.i.d. Gaussian convolutive mixing system of order 3 and a separator of order 4. The performance of the proposed algorithm for different correlation degrees is examined. The results are illustrated in Fig.3. We note that dependency used in source generation effects the performances, and still the proposed algorithm yields better performance than the other algorithms for all cases.

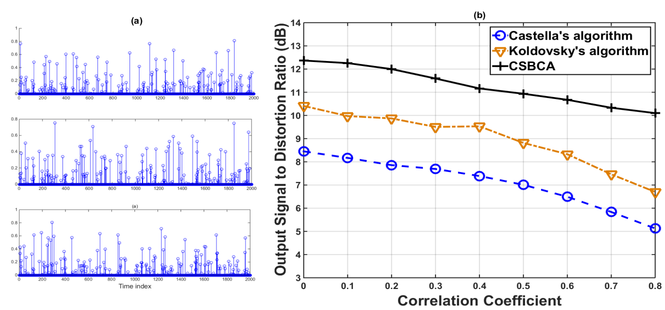

Fig. 3. a)- Copula-T distributed random sparse sequences. b)- Output SINR vs. correlation for 3 sources under SNR=20 dB.

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