Problem 1 (15 pts) Solve the following system by using Gauss-Jordan elimination.

$$\begin{aligned} 3x + 8y - z &= -18 \\ 2x + y + 5z &= 8 \\ 2x + 4y + 2z &= -4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & 8 & -1 & -18 \\ 2 & 1 & 5 & 8 \\ 2 & 4 & 2 & -4 \end{array}\right]\begin{array}{l} \\ \\ R_3/2 \end{array} \rightarrow \left[\begin{array}{ccc|c} 3 & 8 & -1 & -18 \\ 2 & 1 & 5 & 8 \\ 1 & 2 & 1 & -2 \end{array}\right]\begin{array}{l} R_1 \leftrightarrow R_3 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ 2 & 1 & 5 & 8 \\ 3 & 8 & -1 & -18 \end{array}\right]\begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ 0 & -3 & 3 & 12 \\ 0 & 2 & -4 & -12 \end{array}\right]\begin{array}{l} \\ R_2/-3 \\ R_3/2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ 0 & 1 & -1 & -4 \\ 0 & 1 & -2 & -6 \end{array}\right]\begin{array}{l} R_1-2R_2 \\ \\ R_3-R_2 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & -1 & -2 \end{array}\right]\begin{array}{l} \\ \\ R_3/-1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 2 \end{array}\right]\begin{array}{l} R_1-3R_3 \\ R_2+R_3 \\ \\ \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

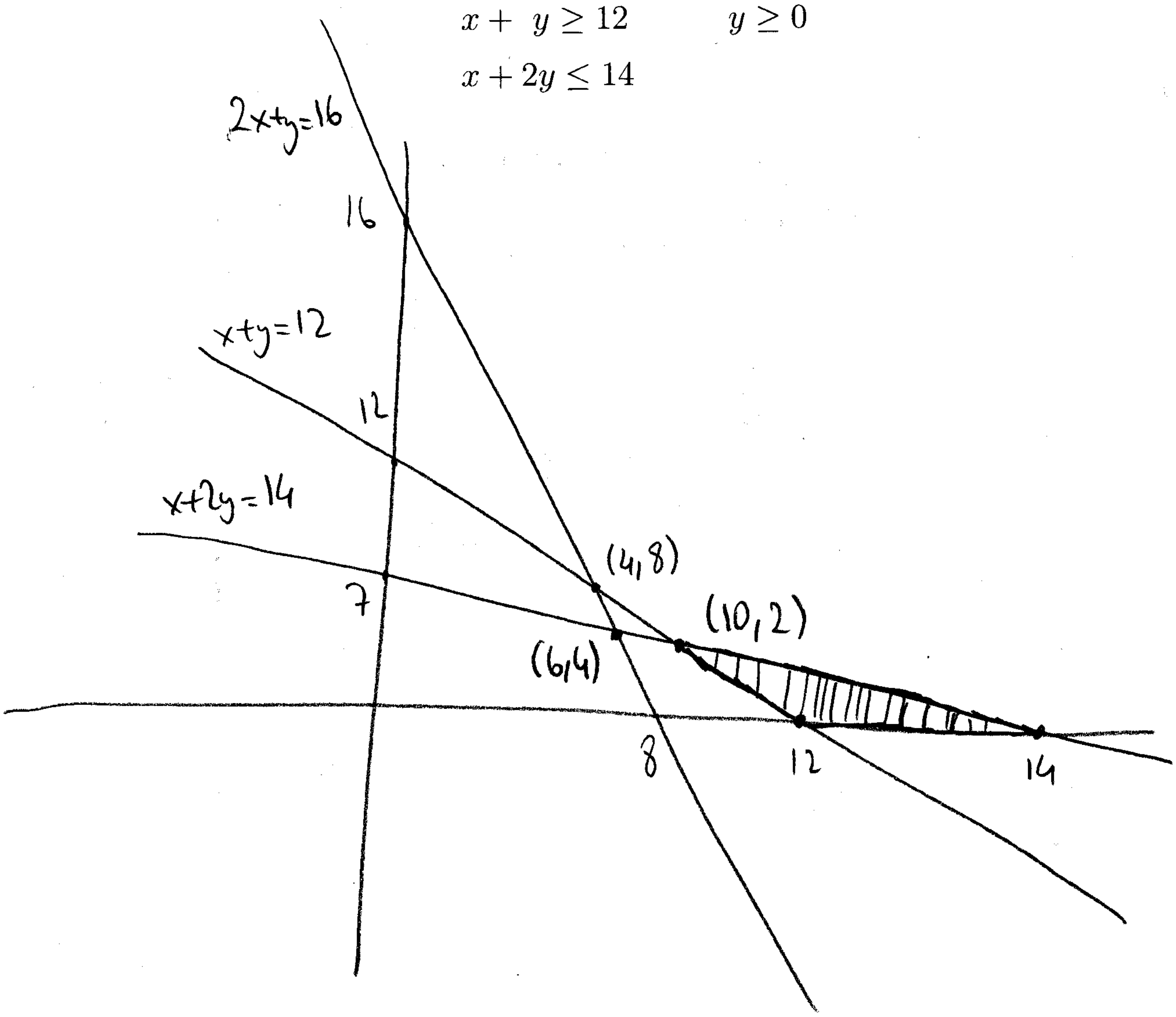
$x = 0$

$y = -2$

$z = 2$

Problem 2 (15 pts) Solve the following system of linear inequalities graphically, and find the corner points.

$$2x + y \geq 16 \qquad x \geq 0$$
$$x + y \geq 12 \qquad y \geq 0$$
$$x + 2y \leq 14$$



$$\begin{aligned} 2x+y &= 16 \\ x+y &= 12 \\ \hline x &= 4 \\ y &= 8 \end{aligned}$$

$$\begin{aligned} 2x+y &= 16 \\ x+2y &= 14 \\ \hline x &= 6 \\ y &= 4 \end{aligned}$$

$$\begin{aligned} x+y &= 12 \\ x+2y &= 14 \\ \hline x &= 10 \\ y &= 2 \end{aligned}$$

Problem 3 a) (10 pts) Construct a truth table for the following compound propositions. Check that if they are equivalent propositions.

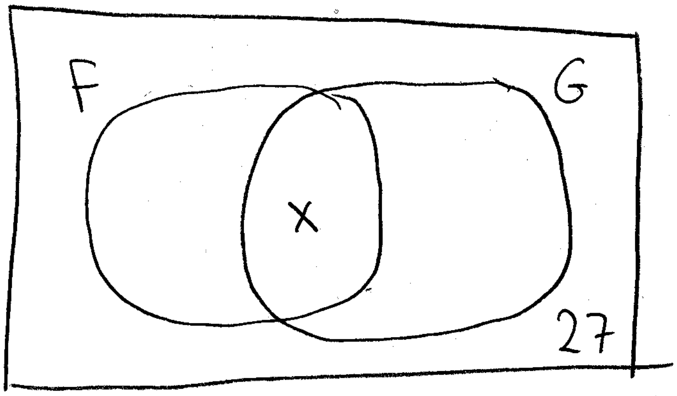
$$p \vee (p \rightarrow q) \qquad\qquad p \rightarrow (p \vee q)$$

p	q	$p \vee q$	$p \rightarrow q$	$p \vee (p \rightarrow q)$	$p \rightarrow (p \vee q)$
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	T	T

$\equiv$

$$p \vee (p \rightarrow q) \equiv p \rightarrow (p \vee q)$$

b) (10 pts) A group of 120 people touring Europe includes 52 people who speak French, 65 who speak German, and 27 who speak neither language. How many people in the group speak both German and French?



$n(F \cup G) = n(F) + n(G) - n(F \cap G)$

$= 52 + 65 - x$

$= 117 - x$

$n(F \cup G) = 120 - 27$

$= 93$

$117 - x = 93$

$\boxed{x = 24}$

Problem 4 a) (7 pts) How many ways can 3 people be seated in a row of 5 chairs?

order is important!

$$P(5,3) = \frac{5!}{(5-3)!} = 5 \cdot 4 \cdot 3 = 60$$

b) (8 pts) How many ways can a 4-person subcommittee consisting of 1 president and 3 members be selected from a committee of 9 people?

Choice of president: $\binom{9}{1}$

choice of othe members: $\binom{8}{3}$

$$\# = \binom{9}{1} \cdot \binom{8}{3} = 9 \cdot \frac{8!}{5!\,3!} = 9 \cdot \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3 \cdot 2 \cdot 1}} = 504$$

Problem 5 Find the following limits, if exists.

a) (6 pts) $\lim\limits_{x\to 2} \frac{x^2+3x-10}{x^2-7x+10}$

$$\lim_{x\to 2} \frac{x^2+3x-10}{x^2-7x+10} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}(x-5)} = \lim_{x\to 2} \frac{x+5}{x-5} = \frac{7}{-3}$$

b) (7 pts) $\lim\limits_{x\to 3} \frac{x^2-9}{|x-3|}$

$$\lim_{x\to 3^+} \frac{x^2-9}{|x-3|} = \lim_{x\to 3^+} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \lim_{x\to 3^+} x+3 = 6$$

$$\lim_{x\to 3^-} \frac{x^2-9}{|x-3|} = \lim_{x\to 3^-} \frac{\overset{-1}{\cancel{(x-3)}}(x+3)}{\cancel{(-x+3)}} = \lim_{x\to 3^-} -(x+3) = -6$$

No limit!

c) (7 pts) $\lim\limits_{x\to -\infty} \frac{x^3+\sqrt{x}-9}{(x^2+1)(2x-1)}$

$$\lim_{x\to -\infty} \frac{x^3+\sqrt{x}-9}{(x^2+1)(2x-1)} = \lim_{x\to -\infty} \frac{x^3+\sqrt{x}-9}{2x^3-x^2+2x-1} =$$

$$\lim_{x\to -\infty} \frac{x^3\left(1+\cancel{\frac{1}{x^{5/2}}}-\cancel{\frac{9}{x^3}}\right)}{x^3\left(2-\cancel{\frac{1}{x}}+\cancel{\frac{2}{x^2}}-\cancel{\frac{1}{x^3}}\right)} = \lim_{x\to -\infty} \frac{x^3}{2x^3} = \frac{1}{2}$$

List of formulas

$(x^n)' = n.x^{n-1}$ $\quad (e^x)' = e^x$ $\quad (\ln x)' = \frac{1}{x}$

$(f(x).g(x))' = f'(x).g(x) + f(x).g'(x)$ $\quad \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x).g(x) - f(x).g'(x)}{g^2(x)}$

Problem 6 a) (7 pts) Find the derivative of $f(x) = \frac{x+e^x}{\sqrt{x}}$

$$f(x) = \frac{x+e^x}{\sqrt{x}} = \sqrt{x} + \frac{e^x}{\sqrt{x}}$$

$$f'(x) = (\sqrt{x})' + \left(\frac{e^x}{\sqrt{x}}\right)' = \frac{1}{2\sqrt{x}} + \frac{e^x.\sqrt{x} - e^x.\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$= \frac{1}{2\sqrt{x}} + \frac{e^x(2x-1)}{2x\sqrt{x}} = \frac{x + 2xe^x - e^x}{2x\sqrt{x}}$$

b) (8 pts) Find the equation of the tangent line of the function $f(x) = x.\ln x$ at $x = 1$.

$$f(x) = x.\ln x \Rightarrow f'(x) = 1.\ln x + x.\frac{1}{x} = \ln x + 1$$

$$f'(1) = \ln 1 + 1 = 0 + 1 = 1$$

$m = 1$ $\quad f(1) = 1.\ln 1 = 0 \Rightarrow$ point: $(1,0)$

$$(y - y_0) = m(x - x_0)$$

$m = 1$

$(x_0, y_0) = (1,0)$ $\Rightarrow$ $y - 0 = 1.(x-1)$

$$\boxed{y = x - 1}$$