PROBLEM 1 (10 points) Let \((s_n)\) be a sequence defined by \(s_0 = 1\) and \(s_{n+1} = \sqrt{1 + s_n}\) for \(n \geq 0\). Prove that \((s_n)\) is convergent and find the \(\lim_{n \to \infty} s_n\).

PROBLEM 2 (10 points) Let \((x_p)_{p \in \mathbb{N}}\) and \((y_p)_{p \in \mathbb{N}}\) be two sequences in \(\mathbb{R}^3\). If \((x_p) \to x \in \mathbb{R}^3\) and \((y_p) \to y \in \mathbb{R}^3\) then show that \((x_p \cdot y_p) \to x \cdot y \in \mathbb{R}\) (Here \(\cdot\) represents the dot product of vectors).

PROBLEM 3 (10 points) Let \(f : \mathbb{R}^2 \to \mathbb{R}\) be a continuous function. Show that if \(f(a) > 12.7\) for some \(a \in \mathbb{R}^2\) then there exists a real number \(\delta > 0\) such that \(f(x) > 12.7\) for all \(x \in B_\delta(a)\).

PROBLEM 4 (10 points) Let \(f : \mathbb{R} \to \mathbb{R}\) be a function defined by
\[
f(x) = \begin{cases} \sqrt{2} & \text{if } x \in \mathbb{Q} \setminus \{2\} \\ \sqrt{3} & \text{if } x \notin \mathbb{Q} \\ 2 & \text{if } x = 2 \end{cases}
\]
Decide whether \(f\) is continuous at \(x = \sqrt{2}\). Prove your answer.

PROBLEM 5 (10 points) Find the closure of the set \(A\) in \(\mathbb{R}^3\) defined as follows:
\[A = \{(x, y, z) \mid x^2 + y^2 + z^2 = 4\} \]
Prove your answer.

PROBLEM 6 (10 points) Prove the following statement if you think it is true, or find a counterexample if you think that it is false: If \(V\) and \(W\) are nonempty compact subsets of \(\mathbb{R}\), then \(V \times W\) is a compact subset of \(\mathbb{R} \times \mathbb{R} = \mathbb{R}^2\).

PROBLEM 7 (10 points) Give an example of a family of closed subsets \(A_\alpha\) of \(\mathbb{R}^2\) (where \(\alpha\) belongs to some indexing set) whose union is not closed in \(\mathbb{R}^2\). Prove your answer.

PROBLEM 8 (10 points) Let \(f : \mathbb{R}^3 \to \mathbb{R}\) be function such that \(\frac{\partial f}{\partial x}(x, y, z) = \frac{\partial f}{\partial y}(x, y, z) = \frac{\partial f}{\partial z}(x, y, z) = 0\) for every \((x, y, z) \in \mathbb{R}^3\). Show that \(f\) is a constant function.

PROBLEM 9 (10 points) Let \(f : \mathbb{R}^n \to \mathbb{R}^n\) be differentiable and has a differentiable inverse \(f^{-1} : \mathbb{R}^n \to \mathbb{R}^n\). Show that for every \(a \in \mathbb{R}^n\) we have
\[
(f^{-1})'(a) = [f'(f^{-1}(a))]^{-1}.
\]