

## MARKET RISK

**DEFINITION:** Market risk is the risk that a firm will lose money due to changes in market prices including interest rates, foreign exchange rates, equity prices, and commodity prices.

### TWO ISSUES:

- 1) Identify the risk factors that affect the value of your portfolio and the correlation between the risk factors.
- 2) Decide how to manage your market risk exposure.

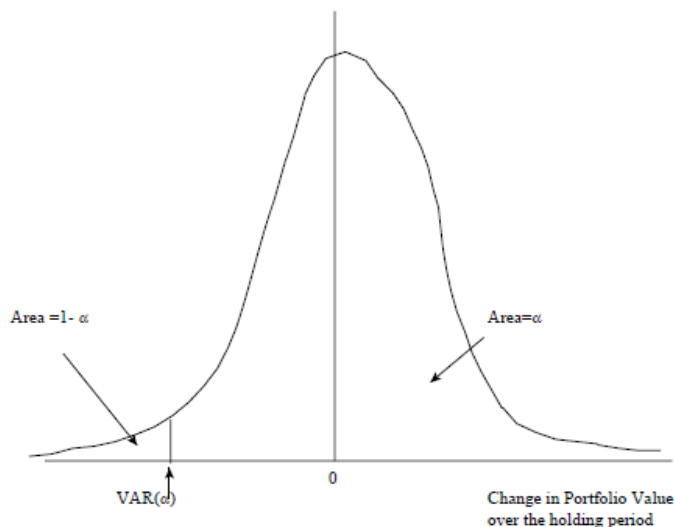
### MEASURING MARKET RISK:

- 1) Identify the set of risk factors ( $F_1, F_2, \dots, F_N$ ) that affect the value of your portfolio.
- 2) Come up with a valuation function: ( $F_1, F_2, \dots, F_N$ )
- 3) Construct the p.d.f. of  $V$  over the holding period.
- 4) Calculate value-at-risk (VAR)

VAR is defined as VAR (holding period;  $1 - \alpha$ ) where  $1 - \alpha$  is your confidence interval.

Example: If VAR (30-day; 95%) = \$10,000,000 then 95% of the time the change in the value of your portfolio in one month will be less than \$10,000,000. Note that this tells you nothing about the magnitude of potential losses in extreme cases.

$$1 - \alpha = \int_{-\infty}^{\text{VAR}(\alpha)} P(x) dx$$



## THREE METHODS OF COMPUTING VAR:

### 1) Historical simulation:

Suppose you have a zero-coupon one year CD with 3.4% interest in your portfolio and the only risk factor is the market interest rate. So you can model the change in the value of your portfolio as a function of changes in interest rates:

$$\Delta V = \frac{100}{1034 + \Delta r} - \frac{100}{1034}$$

With historical simulation, here is what you do:

- i) Collect historical data on changes in market interest rates for a given holding period
- ii) Use the 95<sup>th</sup> percentile of the historical interest rate change to compute VAR for 95% confidence level.

### 2) Monte Carlo simulation:

- i) Collect historical data on changes in market interest rates for a given holding period and compute the mean and standard deviation of interest rate changes.
- (ii) Assume a distribution for the risk factor and estimate its p.d.f. using historical mean and standard deviation.
- (iii) Compute VAR based on estimated p.d.f.

The advantage of Monte Carlo over historical simulation it lets you explore the “tails” of the distribution.

The disadvantage of Monte Carlo simulation is you make an assumption about the distribution of the risk factor. If your assumption is wrong, you will have a misleading VAR.

### 3) Delta-Normal method:

Based on two assumptions:

- (1) The portfolio is a linear function of the risk factors,
- (2) % changes in risk factors are normally distributed.

**Let's start with a portfolio with one risk factor F:**  $V(F_1) - V(F_0) \approx (dV/dF)(F_1 - F_0)$

First-order Taylor series approximation gives:  $dV = (dV/dF)(dF/F) F$

Since  $dV/dF$  is a constant and  $dF/F$  is normally distributed  $dV$  is also normally distributed.

Using  $\sigma$ , the standard deviation of the risk factor we can write:

$$\text{VAR (99\%)} = 2.33 (dV/dF) F \sigma$$

$$\text{VAR (95\%)} = 1.65 (dV/dF) F \sigma$$

2.33 and 1.65 are t-stats associated with 2- and 1-standard deviation (respectively) increases associated with the underlying risk factor (using the standard normal distribution).

If expected change in the value of the portfolio is not zero, subtract it from VAR figures above.

**If there are two risk factors  $F_1$  and  $F_2$ :**

$$\text{VAR}_1 (95\%) = 1.65 (dV/dF_1) F_1 \sigma_1$$

$$\text{VAR}_2 (95\%) = 1.65 (dV/dF_2) F_2 \sigma_2$$

Portfolio VAR =  $[(\text{VAR}_1)^2 + (\text{VAR}_2)^2 + 2 \rho \text{VAR}_1 \text{VAR}_2]$  where  $\rho$  is the correlation coefficient between the two risk factors.

## **VAR PITFALLS**

- 1) Confidence level
- 2) Correlations between risk factors may increase during a crisis
- 3) Holding period
- 4) Estimation period
- 5) Non-normality
- 6) Validation