Prices and Yields of Bonds and Money Market Instruments

Review of present values

- Fixed income securities involve investing funds today with the promise of cash returns in the future. So, to understand how we calculate prices of fixed income securities, it's first important to understand how we calculate present values.

- Suppose A is an amount of funds we are to receive t years from now. The present value of this amount is

$$PV = \frac{A}{(1+r)^t}$$

Annuities

- Rather than receive a single sum in the future, many financial contracts call for equal periodic payments. Such a stream of payments is known as an **annuity**.

- Suppose that a contract calls for equal payments at the end of each of the next t years and that these payments are designated as A_1, A_2, …. A_t . The present value of this stream is

$$PV = \frac{A_1}{1+r} + \frac{A_2}{(1+r)^2} + \ldots + \frac{A_t}{(1+r)^t} = \sum_{t=1}^{t} \frac{A_t}{(1+r)^t}$$

- If t=3, r=7.5%. A=$10, $PV = \frac{\$10}{1.075} + \frac{\$10}{(1.075)^2} + \frac{\$10}{(1.075)^3} = \$26.01$

PV when r is compounded more than once a year

$$PV = \sum_{t=1}^{n} \frac{A_t}{(1+\frac{r}{m})^{mt}}$$

m is the number of times a year interest is compounded (e.g., m=2 for bonds that make coupon payments semi-annually)

The present value of $100 to be received at the end of year 3, with a discount rate of 10% compounded semi-annually:

$$PV = \sum_{t=1}^{3} \frac{\$100}{(1+\frac{.10}{2})^{(2)(3)}} = \$74.62$$

Continuous compounding

$$PV = \frac{A_n}{e^{rn}}$$

e = 2.71828
n = denotes year n

The present value of $100 to be received at the end of 3 years with a discount rate of 10% compounded continuously is:

$$PV = \frac{\$100}{e^{(.10)(3)}} = \$74.80$$

PV of $100 with various compounding intervals

Compounding	Present value
Annually	$75.13
Semiannually	$74.62
Quarterly	$74.36
Monthly	$74.17
Continuously	$74.08

Conclusion: PV decreases but at a decreasing rate as the compounding interval shortens, the limit being continuous compounding.

The Price of a Bond

- The market price of a fixed income security is the promised value of future cash flows, discounted at the appropriate rate of interest. This rate is known as the **yield-to-maturity** (YTM).

- The return on bonds have two components: **coupon payments** and **final principal payment**.

- Most bonds call for coupon payments to be made semiannually. Hence, we will assume semiannual coupon payments in all that follows.

$$P = \sum_{t=1}^{2n} \frac{C/2}{(1+\frac{r}{2})^{t}} + \frac{FV}{(1+\frac{r}{2})^{2n}}$$

n is number of years to maturity; C is $ coupon payment; FV is face value.

Example

- Suppose that the 8% coupon bond of XYZ have twelve years to maturity and that the interest rate, or YTM, is 7.6%. What is the price of the bond?

$$P = \frac{\$4}{1.038} + \frac{\$4}{(1.038)^2} + \frac{\$4}{(1.038)^3} + \ldots + \frac{\$104}{(1.038)^3} = \$103.11$$

If YTM>c then P < FV
if YTM = c then P=FV
if YTM < c then P > FV

Zero Coupon Bonds

- Suppose ABC Corp issued a zero-coupon bond with a face value of \$100 and a maturity of 10 years and that the yield to maturity is 12%. This implies a market price of

$$P = \frac{\$100}{(1.06)^{20}} = \$31.18$$

Yield Calculation for Bonds

- Example: If an 8% coupon bond has 13 years to maturity and the present market price is $96,

$$\$96 = \frac{\$4}{(1+\frac{r}{2})} + \frac{\$4}{(1+\frac{r}{2})^2} + \frac{\$4}{(1+\frac{r}{2})^3} + \ldots + \frac{\$104}{(1+\frac{r}{2})^{26}}$$

- When we solve for r we obtain r=7.39%.
- Note that YTM calculations are based on the assumption that coupon payments are reinvested at YTM. So, if reinvestment rate is less than YTM, then YTM will be greater than the effective yield.

Current Yield

$$\text{Current yield} = \frac{\text{Annual coupon in dollars}}{\text{Market price}}$$

For our earlier example of an 8% bond with a $96 market price, the current yield is $8/$96 = 8.33%, different from the 8.51% YTM. The difference arises because the discount rate is not amortized in the calculation of the current yield. Only when the market price equals $100 will the current yield and YTM will be the same.

Holding period return (HPR)

- YTM will usually differ from the HPR if the bond is sold prior to the maturity.
- HPR (or realized return) is the rate of discount that equates the present value of coupon payments, plus the present value of the terminal value at the end of the holding period, with the price paid for the bond.
- Example: Suppose a bond with a 10% coupon with 12 years to maturity bought for $105. Two years later the bond has a market price of $94, at which time it is sold. For HPR we solve

$$\$105 = \frac{\$5}{(1+\frac{r}{2})} + \frac{\$5}{(1+\frac{r}{2})^2} + \frac{\$5}{(1+\frac{r}{2})^3} + \frac{\$5}{(1+\frac{r}{2})^4} + \frac{\$94}{(1+\frac{r}{2})^4}$$

r=4.46%

YTM for zero-coupon bonds

- If a zero-coupon bond has 8.5 years to maturity and its market price were $48, we would have

$$\$48 = \frac{\$100}{(1+\frac{r}{2})^{17}}$$

- r = 8.82%

Yield for perpetuities

- With a perpetuity, a fixed coupon payment is expected at equal intervals <u>forever</u>.

$$P_0 = \frac{C^*}{1+r} + \frac{C^*}{(1+r)^2} + \ldots + \frac{C^*}{(1+r)^n}$$

$$P_0(1+r) = C^* + \frac{C^*}{1+r} + \ldots + \frac{C^*}{(1+r)^{n-1}}$$

$$P_0(1+r) - P_0 = C^* - \frac{C^*}{(1+r)^n}$$

since $\frac{C^*}{(1+r)^n}$ is 0 as n approaches infinity

$$r = \frac{C^*}{P_0}$$

Money market instrument returns

- Money market instruments carry no coupon but are sold on a discount basis from a face value of $100. The yields on these instruments are quoted in terms of the **bank discount rate**, which is

$$\text{bank discount rate} = \left(\frac{d}{\$100} x \frac{360}{t} \right)$$

$$\text{d} = \text{the dollar amount of the discount}$$

$$\text{t} = \text{the number of days to maturity}$$

- If the discount on a Treasury bill were $1.90 and it had 88 days to maturity, its yield on a bank discount rate basis will be

$$\left(\frac{\$1.90}{\$100} x \frac{360}{88} \right) = 7.77\%$$

Bank discount rate

- Based on the face value of the instrument as opposed to the money actually invested.
- Assumes a 360-day as opposed to a 365-day year.
- If the instrument has a maturity less than six months, one can convert the bank discount rate to what is known as an **equivalent bond yield** using the formula

$$\text{equivalent bond yield} = \frac{365 \times BDY}{360 - (t \times BDY)}$$

- Equivalent bond yield is higher than BDY owing to the discount effect and the 365-day year effect (see next page).

Bank discount rate (cont'd)

$$EBY = \frac{365 \times .0777}{360 - (88 \times .0777)} = 8.03\%$$

EBY does not take account of compound interest. If compounding interval in our example were every 88 days, the effective return would be

$$\left(1 + \frac{\$1.90}{\$98.10}\right)^{365/88} - 1 = 8.28\%$$

LESSON: You cannot compare money market yields with capital market yields unless you adjust the former.