Interest rate risk

Motivation

- Banks hold fixed income securities on both sides of their balance sheets; mainly loans and mortgages on the asset side and deposits on the liability side.

- The value of fixed income securities change when interest rates change.

- So, in order to understand how bank balance sheets are affected by interest rate changes, we need to first understand how the values of simple fixed income securities change when interest rates change.

Computing bond prices

$$\Pr ice = \sum_{t=1}^{Maturity} \frac{Coupon}{(1+YTM)^t} + \frac{Facevalue}{(1+YTM)^{Maturity}}$$

Let's examine the prices of 6% coupon $1,000 FV bonds with 3- and 10-year maturities

			Year									
	Price	**% Δ Price**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3-year bond @ 8%	$948.5		$55.6	$51.4	$841.5	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0
3-year bond @ 7%	$973.8	2.667%	$56.1	$52.4	$865.3	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0
3-year bond @ 9%	$924.1	-2.572%	$55.0	$50.5	$818.5	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0
10-year bond @ 8%	$865.8		$55.6	$51.4	$47.6	$44.1	$40.8	$37.8	$35.0	$32.4	$30.0	$491.0
10-year bond @ 7%	$929.8	7.388%	$56.1	$52.4	$49.0	$45.8	$42.8	$40.0	$37.4	$34.9	$32.6	$538.9
10-year bond @ 9%	$807.5	-6.737%	$55.0	$50.5	$46.3	$42.5	$39.0	$35.8	$32.8	$30.1	$27.6	$447.8

Three observations

1. Bond prices fall when interest rates increase (and vice versa)
2. The longer term bond falls more than the shorter term bond.
3. A 1% increase in interest rates is associated with a bigger price impact (in absolute value) than a 1% decrease in interest rates ("convexity").

	% Δ Price	
Interest rate change:	3-year bond	10-year bond
1% decrease	2.67%	7.39%
1% increase	-2.57%	-6.74%

Duration: Motivation

- In reality, what matters in terms of the interest rate sensitivity of a bond is its *effective maturity*.

- For example, a 10-year coupon bond has a shorter effective maturity than a 10-year zero-coupon bond, because you get your money back faster with the former.

- The effective maturity of a bond is measured as time-weighted average of all cash flows paid by the bond (or duration).

Duration: Formula and Calculation

$$\text{Duration} = \sum_{t=1}^{\text{Maturity}} t w_t \text{, where } w_t = \frac{PV(\text{Payment})_t}{\text{Price}}$$

INPUTS:	
YTM	8%
Coupon	6%
Maturity	3 years
Face value	$1,000

Year	Payment	PV @ 8%	Weight = PV/Price	Year * Weight
1	$60	$55.6	5.9%	0.06
2	$60	$51.4	5.4%	0.11
3	$1,060	$841.5	88.7%	2.66
	Price=	$948.5	**Duration=**	**2.83**

INPUTS:	
YTM	8%
Coupon	0%
Maturity	3 years
Face value	$1,000

Year	Payment	PV @ 8%	Weight = PV/Price	Year * Weight
1	$0	$0.0	0.0%	0.00
2	$0	$0.0	0.0%	0.00
3	$1,000	$793.8	100.0%	3.00
	Price=	$793.8	**Duration=**	**3.00**

When pricing bonds, people are interested in measuring the sensitivity of the price to changes in bond yield. This is typically calculated by using Taylor series expansion. Below we examine the second-order Taylor series expansion for bond price that is a function of yield to maturity. Since we are not interested in higher order expansion we will assume that epsilon equals 0.

$$P(y) = P\,(y_o) + \frac{\partial P}{\partial y} * (y - y_o) + \frac{1}{2}\frac{\partial^2 P}{\partial y^2}(y - y_o)^2 + \varepsilon$$

$$P(y) - P\,(y_o) = \frac{\partial P}{\partial y} * (y - y_o) + \frac{1}{2}\frac{\partial^2 P}{\partial y^2}(y - y_o)^2 + \varepsilon$$

$$\frac{\Delta P}{P} \cong \frac{1}{P}\frac{\partial P}{\partial y} * \Delta y + \frac{1}{2P}\frac{\partial^2 P}{\partial y^2}(\Delta y)^2$$

$$-\frac{1}{P}\frac{\partial P}{\partial y} = Modified\ Duration = \frac{Duration}{1 + y} \qquad \frac{1}{P}\frac{\partial^2 P}{\partial y^2} = Convexity$$

Duration: Example

- Suppose you have a 3-year bond that makes 6% coupon payments annually. YTM=8% and face value is $1,000. What happens to the price of this bond if YTM increases to 9%?

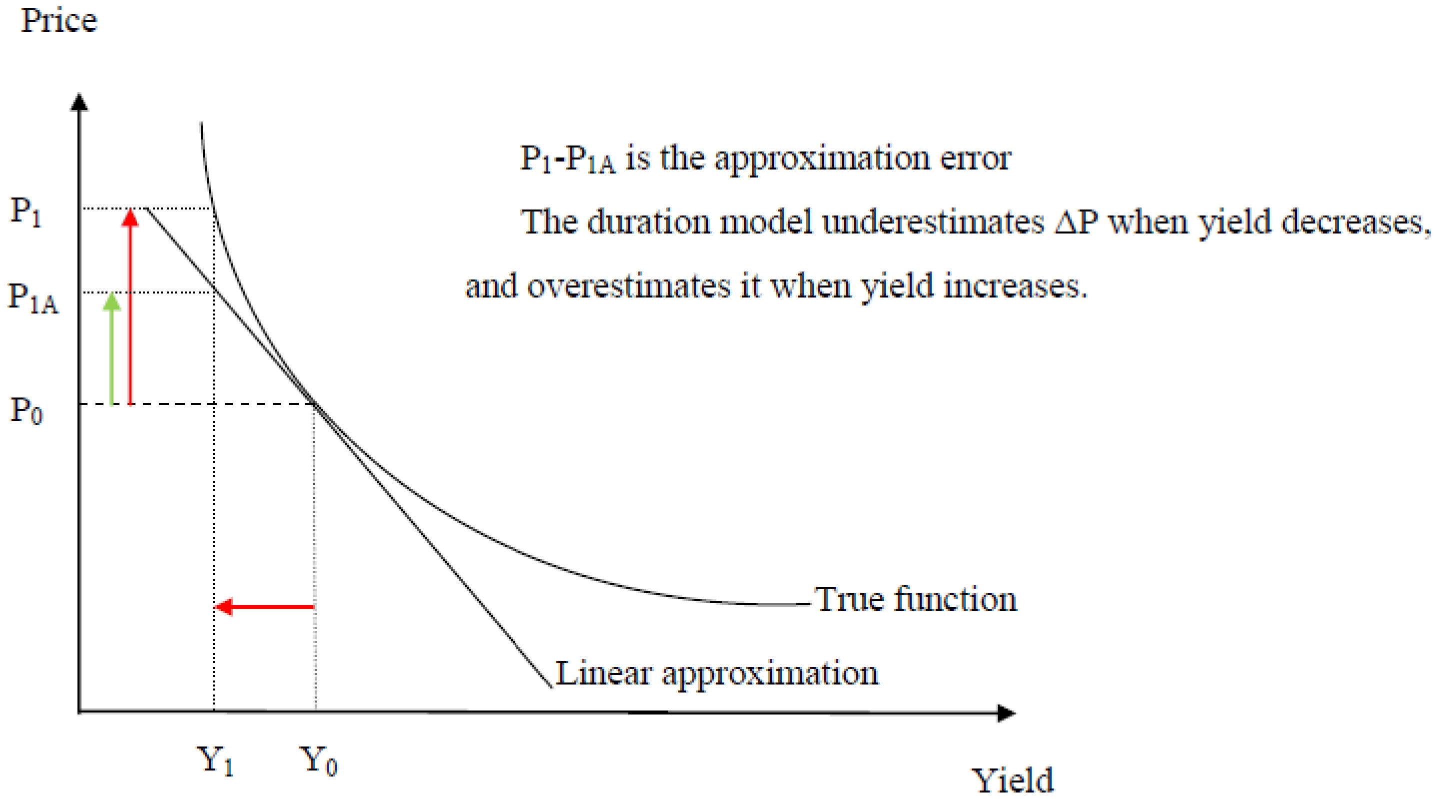
INPUTS:	
YTM	8%
Coupon	6%
Maturity	3 years
Face value	$1,000

Year	Payment	PV @ 8%	Weight = PV/Price	Year * Weight
1	$60	$55.6	5.9%	0.06
2	$60	$51.4	5.4%	0.11
3	$1,060	$841.5	88.7%	2.66
	Price=	$948.5	Duration=	2.83
			Modified duration=	**2.62**

$\Delta P / P = -$ Modified Duration $* \Delta y \quad = -2.62 * 1\% = -2.62\%$

Approximation Error

- The true price change is -2.57%, but the change estimated by the duration formula is -2.62%. Why the difference?



A better approximation with convexity

$$\frac{\Delta P}{P} = -Modified\ duration * \Delta y + \frac{1}{2} Convexity(\Delta y)^2, where\ Convexity = \frac{1}{(1+y)^2} \sum_{t=1}^{T} t(t+1) w_t$$

INPUTS:	
YTM	8%
Coupon	6%
Maturity	3 years
Face value	$1,000

Year	Payment	PV @ 8%	W	t * W	t * (t+1) * W
1	$60	$55.6	5.9%	0.06	0.12
2	$60	$51.4	5.4%	0.11	0.33
3	$1,060	$841.5	88.7%	2.66	10.65
	P=	$948.5	D=	2.83	11.09
			MD=	2.62	**9.51 = Convexity**

ΔP / P = - 2.62 * 1% + 1/2 * 9.51 * (0.01)^2 = -2.572%

Price change is much closer to the actual changes than the approximation based only on duration.

The effect of interest rate changes on bank balance sheets

- Think about a bank that has the following balance sheet.

Assets:	
6% coupon bond w/ maturity 3 years	$1,000
Liabilities:	
1-year CDs zero coupon @ 8%	$900
Net worth	$100

- What happens to the bank's net worth if interest rates increase from 8% to 9%?
- What we need to do is to calculate the change in the value of the bank's assets and liabilities using the duration formula (together with the convexity formula if you want to be precise).

[1] We may use the formula for the duration of net worth:

$$\Delta Net\ worth \approx -(D_A^* - k * D_L^*) * A * \Delta y$$

where A is assets, L is liabilities, D is modified duration, k is leverage, and y is yield.

We know that:

Modified duration of assets	2.62
Modified duration of liabilities	0.93
Leverage (k)	90%

Therefore, the change in net worth associate with 1% increase in interest rates is:

-(2.62-0.9*0.93)*$1,000*1% = -$17.87

[2] An alternative way to attack this problem is:

	1% increase in YTM
Change in asset value	-26.2
- Change in liability value	-8.3
= Change in net worth	-17.87

Change in net worth = -Modified duration of net worth * Net worth * 1%

Therefore, the modified duration of net worth = $17.87 / ($100 * 1%) =17.87