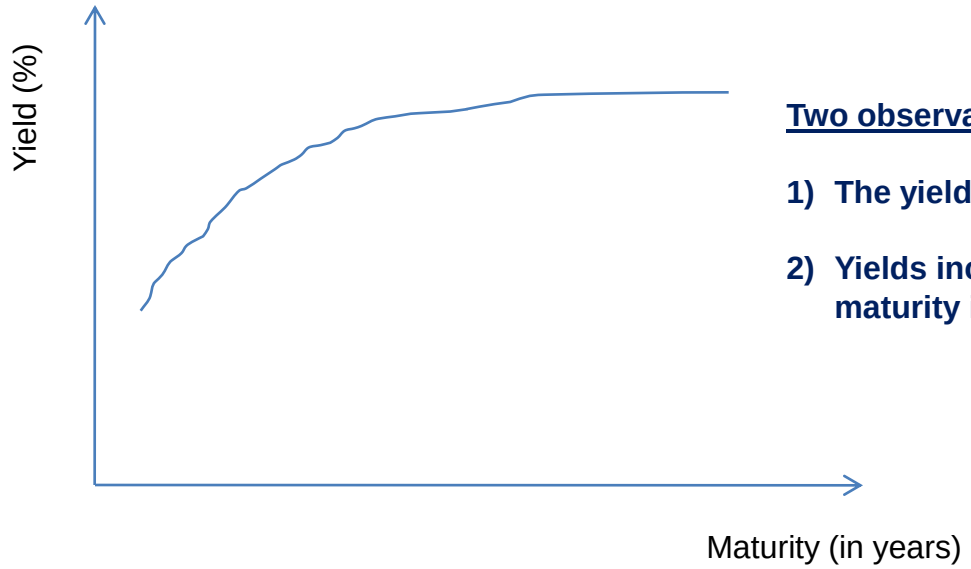
The term structure of interest

Definition

- The relationship between yield to maturity on securities differing only in length of time to maturity is known as the **term structure of interest rates**.



Two observations:

1) **The yield curve is typically upward sloping.**
2) **Yields increases at a decreasing rate as maturity increases.**

Theories that explain the shape of the yield curve

1. The pure expectations theory
2. Liquidity
3. Market segmentation

The pure expectations theory

- The idea: The expected one-period rate of return on an investment is the same, regardless of the maturity of security in which one invests.

- Specifically: If an individual's investment horizon were one year, it would make no difference whether he or she
  a) invested in a 1-year security,
  b) Invested in a 2-year security and sold it at the end of the year,
  b) invested in a 5-year security and sold it at the end of the year,

Forward rates of interest

- The rate of interest, or yield, on an actual bond of a particular yield is known as its **spot rate of interest**.

- When considering the expectations theory, it is useful to transform spot rates of interest into **forward rates**, the rates at which two parties agree to borrow and lend money for a specified period of time in the future.

- Example: A forward contract might be for a two year loan beginning one year in the future.

Forward rates and term structure

- Implied in the term structure at any moment is a set of forward rates.

- Examine the following equation where R stands for the spot rate of interest at time t (you get R from the yield curve), and r stands for forward interest rates for one year loans implied by the yield curve.

$$(1+R_{t,n})^{n} = (1+R_{t,1})(1+r_{t+1,1})(1+r_{t+2,1})\ldots(1+r_{t+n-1,1})$$

- Thus a loan for n years is equivalent to a one year loan plus a series of forward contracts, each renewing the loan for a successive year.

Calculating forward rates

- What is the one-period forward rate beginning at time t + n, implied by the term structure at time t?

$$r_{t+n,1} = \frac{(1+R_{t,n+1})^{n+1}}{(1+R_{t,n})^{n}} - 1$$

- What is the J-period forward rate beginning at time t + n, implied by the term structure at time t?

$$r_{t+n,j} = \sqrt[j]{\frac{(1+R_{t,n+j})^{n+j}}{(1+R_{t,n})^{n}}} - 1$$

Now, back to the PET

- The pure expectations theory argues that securities of different maturities are perfect substitutes for one another. Investors have to choose one of the three effectively equivalent strategies:

1. Invest in a security having a maturity corresponding exactly to your anticipated holding period;

2. Invest in short-term securities, reinvesting in short term at each maturity over the holding period (*rollover*);

3. Invest in a security having a maturity longer than the anticipated holding period and sell at the end of the holding period.

Example

- Suppose the yields presented in column 2 of the following table prevailed in the market for default-free Treasury securities.

Maturity	Yield	1-year forward rates
1 year	5%	
2 year	6%	1.06^2 / 1.05 – 1 = 7.01%
3 year	7%	1.07^3 / 1.06^2 – 1 = 9.03%
4 year	7.5%	1.075^4 / 1.07^3 – 1 = 9.01%

- If the investor has a three-year investment horizon, he can invest in a 3-year loan and earn 7% or he can rollover one-year loans and earn

$$\sqrt[3]{(1.05)(1.0701)(1.0903)} - 1 = 7\%$$

So, what does the PET say about the shape of the yield curve?

- PET says, the shape of the yield curve is purely determined by the investors' expectations of future interest rates.
- A flat yield curve, according to PET, implies that market participants expect future short rates to be the same as the current short rate.
- A downward sloping yield curve signifies that future short rates are expected to fall.
- An upward sloping yield curve implies that future short rates are expected to rise.

Liquidity

- The longer the maturity of the security, the greater the risk of fluctuation in the value of principal to the investor.
- Because of this greater risk, investors prefer to lend short. Borrowers, on the other hand, prefer to borrow long-term in order to reduce the risk of inability to meet principal payments.
- Because of the weak supply of funds for long-term borrowing, borrowers have to pay a premium to induce investors to purchase long-term securities.
- Liquidity premium, in general, monotonically increases with maturity and might be time-varying.

Market segmentation

- Because of behavioral and legal restrictions, institutional lenders are said to have preferred maturity ranges in which they operate.

- For example, commercial banks typically prefer short- to medium-term maturities because of the nature of their deposit liability and their traditional emphasis on liquidity. Insurance companies and pension funds, on the other hand, prefer longer maturities.

- The market segmentation theory implies that the rate of interest for a particular maturity is determined solely by demand and supply conditions for that maturity, with no reference to conditions for other maturities. In other words, borrowers and lenders have rigid maturity preferences and do not deviate from these preferences no matter how attractive the yield of other maturities.

Empirical evidence

1. Interest rate expectations affect the shape of the yield curve

2. Forward rates of interest are not unbiased estimates of expected future spot rates of interest

3. Term premiums vary directly with the volatility of interest rates

4. Market segmentation appears to have a moderate effect on term structure. However, monetary policies aimed at affecting the yield of certain maturities seem to influence the yields of all maturities (yields of securities with varying maturities tend to move together, contrary to the market segmentation theory)