



International Workshop on Distribution Logistics IWDL 2006

Brescia, ITALY

October 2nd–5th, 2006

Collection Center Location Problem with Incentive & Distance Dependent Returns

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Agenda

- Literature Review
- Model Description
- Solution Procedure
- Experimental Results
- Conclusions

Introduction

Traditional focus of supply chains:

- fine-tuning the logistics of products from raw material to the end customer

Modern supply chain view:

- increasing flow of products in the reverse direction

Definition of Reverse Logistics

*“The process of planning, implementing, and controlling backward flows of raw material, in-process inventory, packaging, and finished goods from **a manufacturing, distribution or reuse point, to a point of origin for the purpose of recapturing value or proper disposal.**”*

(REVLOG, 2002)

Reverse Logistics Process (Fleischmann et al., 2000)

- **Collection:** process of rendering used products available and physically moving them to some point for further treatment. It may include activities purchasing, transportation, and storage
- **Collection costs are 30-50% of overall EOL product management costs (Mulder et al., 1999)**
- **Reprocessing:** transformation of a used product into a usable product again
- **Disposal:** done for products that cannot be re-used for technical or economical reasons
- **Redistribution:** distributing reusable products to a potential market

Types of Collection Strategies

■ Curbside collection:

- ❑ consumers sort recyclable material from other solid wastes and place in bins at curbside
- ❑ convenient, high return rate

■ Drop-off centers:

- ❑ requires more consumer effort, so lower return rates
- ❑ consumers are not offered reimbursement for materials delivered

■ Buy-back centers:

- ❑ consumers provide the initial transportation and drop-off recyclables
- ❑ they receive reimbursement

Common practices for Collection

(Mulder et al., 1999)

- Netherlands, Belgium, and Italy
 - municipalities: ensure separate collection of EOL electronics from households
 - producers: in charge of recycling
- Switzerland and Sweden
 - producers are fully responsible for organizing separate collection structures for EOL electronics
- Denmark
 - municipalities are responsible for separate collection of EOL electronic products and also their proper treatment
 - funded through local waste taxes

Multi Echelon Reverse Logistics Network

(Min et al., 2006)

- *customers* drop products to *initial collection points (ICP)*
- products are transferred from ICPs to *Central Return Centers (CRC)*
- cost components: *renting, inventory carrying, material handling, setup, shipping*
- CRCs have a limited capacity
- service level requirement for ICPs
- **Objective:** finding optimum ***number and location of ICP and CRCs***, and ***frequency of shipments*** from ICP to CRCs

Model Description

- Problem is faced by a company that collects used products from consumers
- Sites at which collection centers can be located
- Consumers who have end-of-life products
- Drop-off model
- Return decision based on the:
 - financial incentive offered by the company (R)
 - travel distance of the consumers (d_{ij})
- Objective: to maximize profit by determining
 - sites at which collection centers must be set up
 - financial incentive (unit acquisition price) offered to customers

Model Description

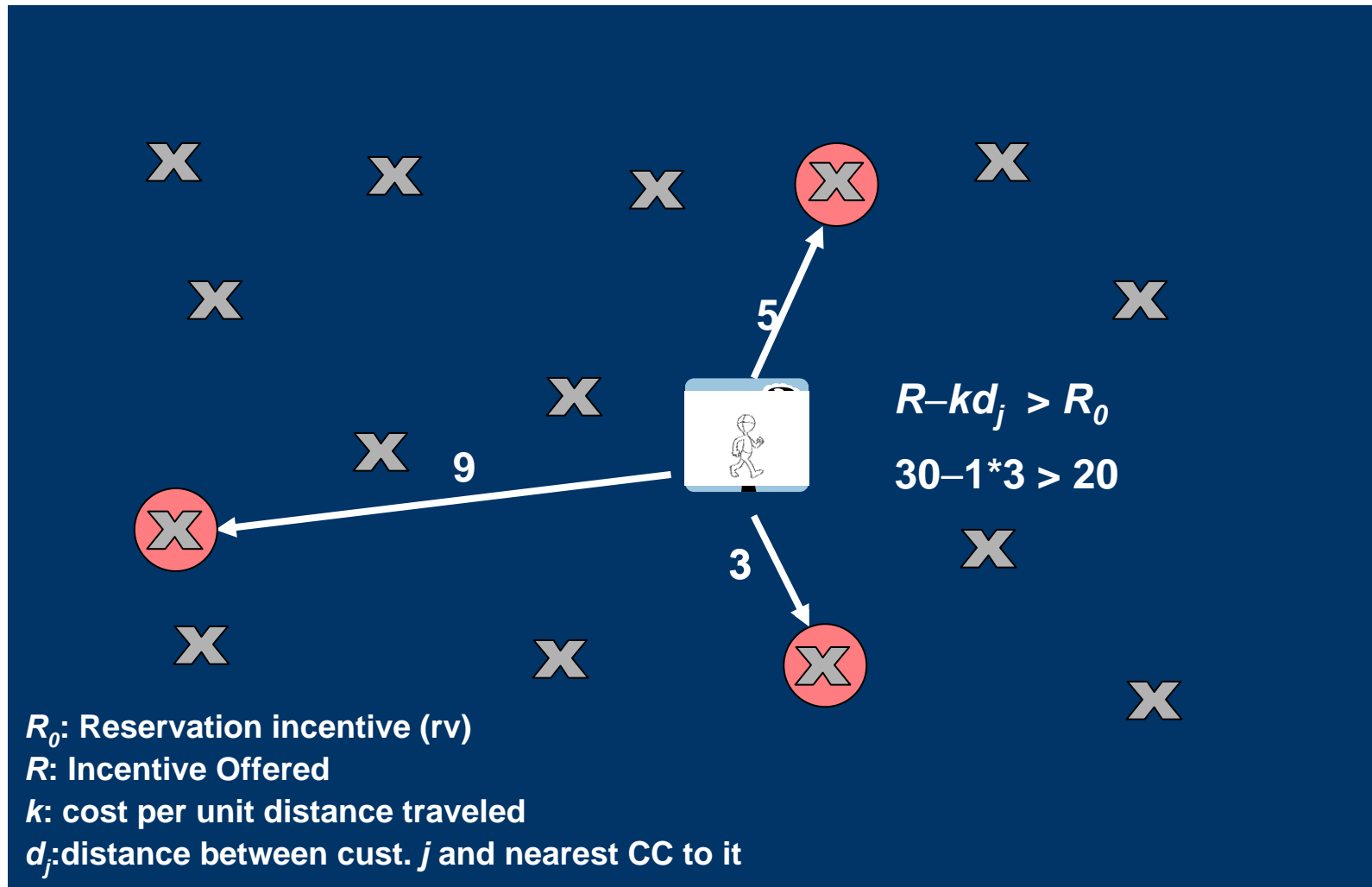
Model-1: UCCLP

- the number and locations of CCs & incentive to be offered are decided
- tradeoff :
 - high fixed cost and high return rate
 - low fixed cost and less return rate

Model-2: p -CCLP

- UCCLP in which the number of collection facilities to be opened is predetermined similar to the famous p -median problem
- locations of CCs and incentive to be suggested are decided

Model Description



Model Description

R_0 : Reservation incentive (rv)
 R : Incentive Offered
 k : cost per unit distance traveled
 d_j : distance between cust. j and nearest CC to it

$R - kd_j > R_0$
 $20 - 1 * 3 < 20$

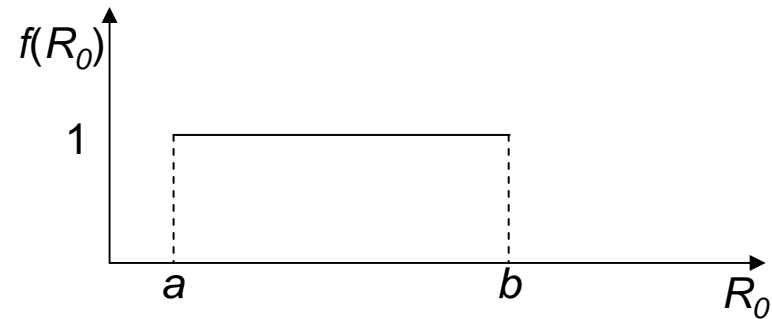
Model Description

$R_0 \rightarrow R_0 \sim U(a,b)$: measure of consumer willingness

Ray et al., 2005; Wojanowski et al., 2003

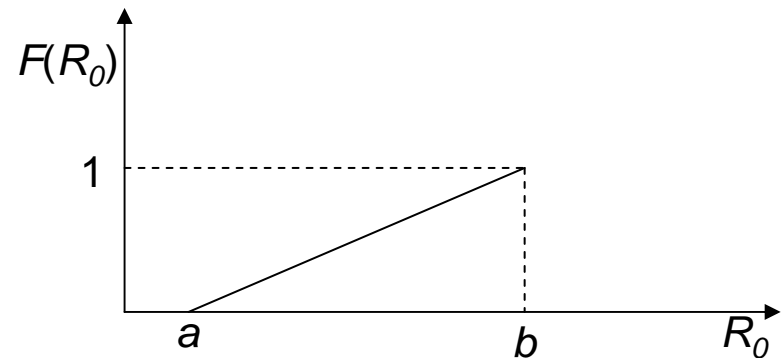
Probability density function of R_0

$$f(R_0) = \frac{1}{b-a}$$



Probability distribution function of R_0

$$F(R_0) = \frac{R_0 - a}{b - a}$$



Parameters

e_{ij}	: Euclidean distance between customer zone j and candidate site i
f_i	: fixed cost of opening and operating a collection center at site i
h_j	: number of product holders located at zone j
s	: unit revenue from a return
M	: a large number

Variables

$$Y_i = \begin{cases} 1 & \text{if a collection center is located at site } i \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ij} = \begin{cases} 1 & \text{if product holders at zone } j \text{ are assigned to the collection center at site } i \\ 0 & \text{otherwise} \end{cases}$$

R : amount of incentive offered by the company

P_j : proportion of product holders at zone j who drop off their product

$$\delta_{1j} = \begin{cases} 1 & \text{if product holders at zone } j \text{ do not drop off their product } (P_j = 0) \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{2j} = \begin{cases} 1 & \text{if all product holders at zone } j \text{ drop off their product } (P_j = 1) \\ 0 & \text{otherwise} \end{cases}$$

δ_{1j} & $\delta_{2j} \rightarrow$ auxiliary binary variables used for formulating P_j

Model Description

- R : amount of incentive company offers
- k : cost per unit distance traveled
- d_j : distance between cust. zone j and nearest facility
- kd_j : cost of carrying product from zone j to nearest facility
- P_j : proportion of consumers at zone j who return their product

$$P_j = \Pr(R - kd_j - R_0 > 0) = \frac{(R - kd_j - a)^+}{b - a}$$



$$P_j = \begin{cases} 0 & R < kd_j + a \\ \frac{R - kd_j - a}{b - a} & kd_j + a \leq R < kd_j + b \\ 1 & R \geq kd_j + b \end{cases}$$

$$R < kd_j + a$$

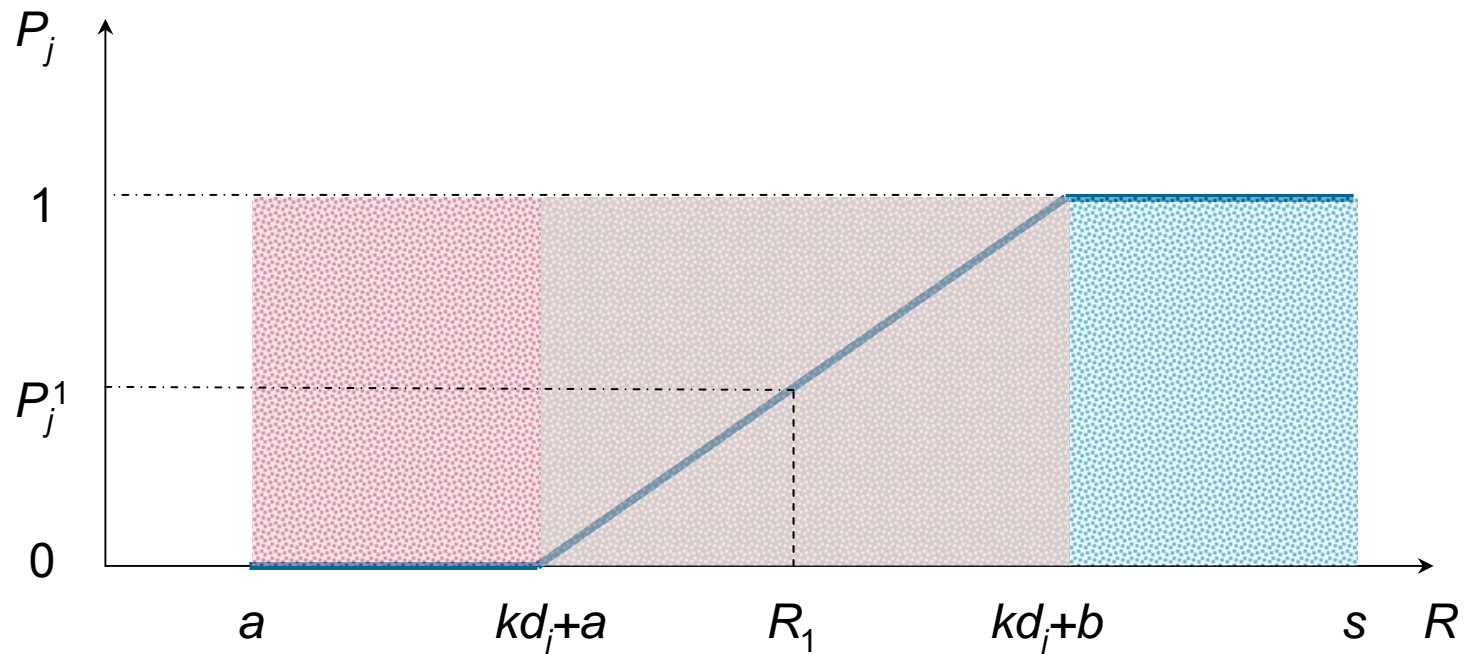
$$kd_j + a \leq R < kd_j + b$$

$$R \geq kd_j + b$$

Return Rate Function

$$R_0 \rightarrow R_0 \sim U(a,b)$$

$$P_j = \begin{cases} 0 & R < kd_j + a \\ \frac{R - kd_j - a}{b - a} & kd_j + a \leq R < kd_j + b \\ 1 & R \geq kd_j + b \end{cases}$$



Model Description

- h_j : number of consumers that own the product of the company
- $h_j P_j$: total number of returns from zone j
- s : value of a returned product
- $(s-R)$: profit gained from each return
- $\Pi_j = h_j P_j (s-R)$: profit from customer zone j

$$\Pi_j = \begin{cases} 0 & R < kd_j + a \\ \frac{h_j}{b-a} (-R^2 + \alpha_j R - \beta_j) & kd_j + a \leq R < s \\ 0 & R \geq s \end{cases}$$

$$R < kd_j + a$$

$$kd_j + a \leq R < s$$

$$R \geq s$$

Objective Function - UCCLP

$$\max \quad \Pi = \sum_{j=1}^n h_j P_j (s - R) - \sum_{i=1}^m f_i Y_i$$

Total expected profit

Total fixed cost of opening CCs

Mixed Integer Nonlinear Programming Problem

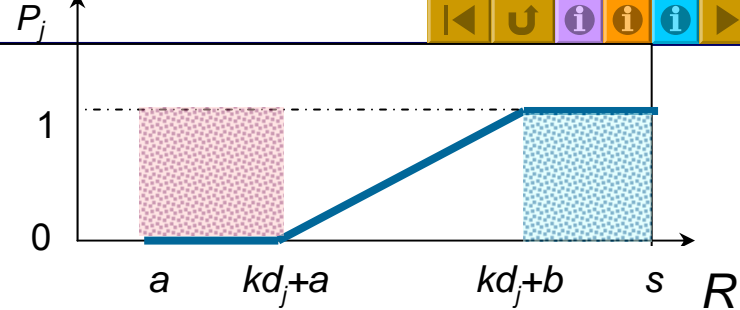
Constraints (UCCLP)

subject to

- (1) $\sum_{i=1}^m X_{ij} = 1$ $j = 1, \dots, n$
- (2) $X_{ij} \leq Y_i$ $i = 1, \dots, m, j = 1, \dots, n$
- (11) $R \geq a$
- (12) $R \leq s$
- (13) $X_{ij} \in \{0,1\}$ $i = 1, \dots, m, j = 1, \dots, n$
- (14) $Y_i \in \{0,1\}$ $i = 1, \dots, m$

$$\delta_{1j} = 1 \quad \text{and} \quad \delta_{2j} = 1$$

→ **Infeasible**



$$(3) \quad R \leq \left(\sum_{i=1}^m ke_{ij} X_{ij} \right) + a + M(1 - \delta_{1j}) \quad j = 1, \dots, n$$

$$(4) \quad R \geq \left(\sum_{i=1}^m ke_{ij} X_{ij} \right) + a - M\delta_{1j} \quad j = 1, \dots, n$$

$$(5) \quad R \leq \left(\sum_{i=1}^m ke_{ij} X_{ij} \right) + b + M\delta_{2j} \quad j = 1, \dots, n$$

$$(6) \quad R \geq \left(\sum_{i=1}^m ke_{ij} X_{ij} \right) + b - M(1 - \delta_{2j}) \quad j = 1, \dots, n$$

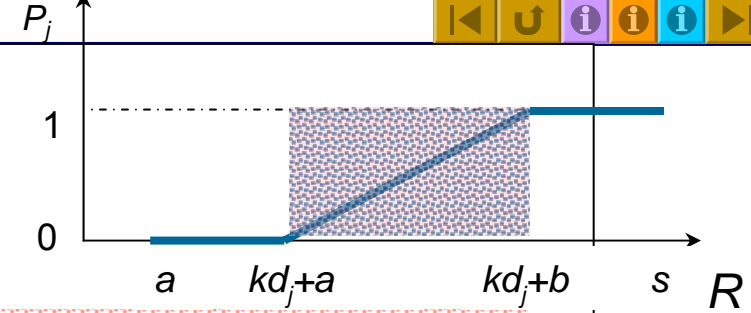
$$(7) \quad P_j \leq 1 - \delta_{1j} \quad j = 1, \dots, n$$

$$(8) \quad P_j \geq \delta_{2j} \quad j = 1, \dots, n$$

$$(9) \quad P_j \leq \frac{R - \left(\sum_{i=1}^m ke_{ij} X_{ij} \right) - a}{b - a} + M(\delta_{1j} + \delta_{2j}) \quad j = 1, \dots, n$$

$$(10) \quad P_j \geq \frac{R - \left(\sum_{i=1}^m ke_{ij} X_{ij} \right) - a}{b - a} - M(\delta_{1j} + \delta_{2j}) \quad j = 1, \dots, n$$

$$\delta_{1j} = 0 \quad \text{and} \quad \delta_{2j} = 0$$



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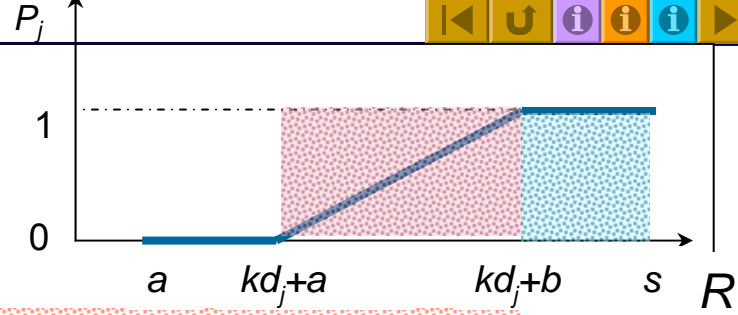
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$$\delta_{1j} = 0 \quad \text{and} \quad \delta_{2j} = 1$$



$$(3) \quad R \leq \left(\sum_{i=1}^m ke_{ij} X_{ij} \right) + a + M(1 - \delta_{1j}) \quad j = 1, \dots, n$$

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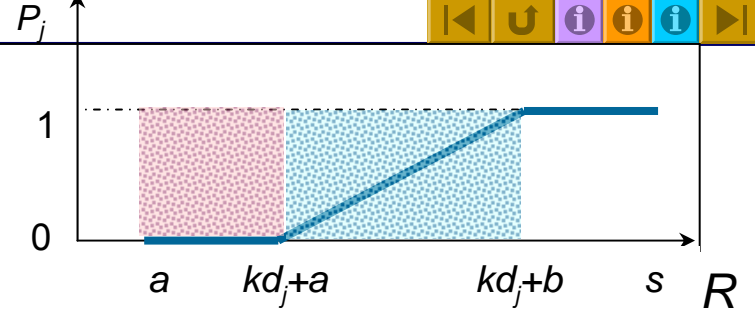
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$$\delta_{1j} = 1 \text{ and } \delta_{2j} = 0$$



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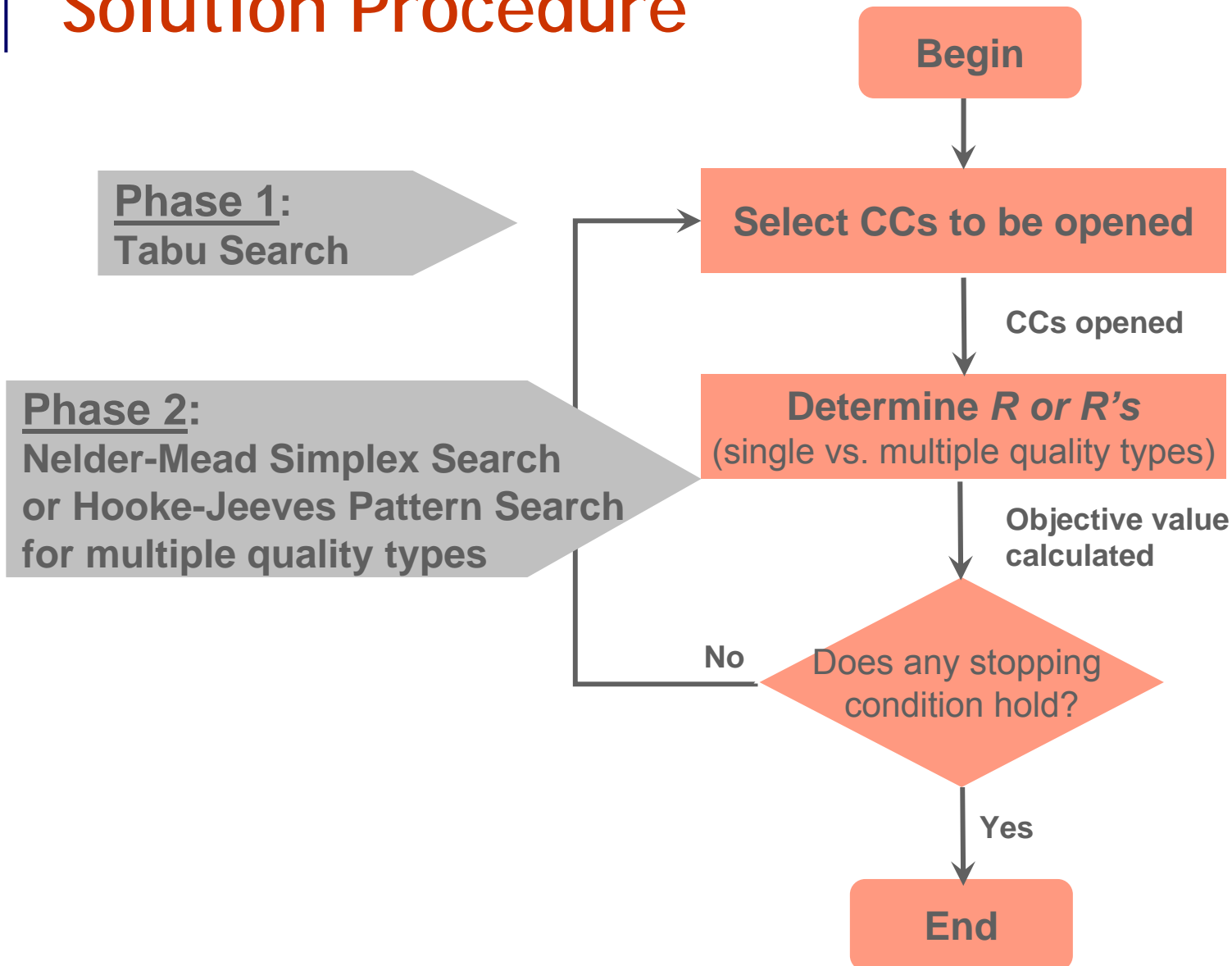
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Extension: p -CCLP

Objective function \rightarrow $\max \quad \Pi = \sum_{j=1}^n h_j P_j (s - R)$

Additional constraint \rightarrow $\sum_{i=1}^m Y_i = p$

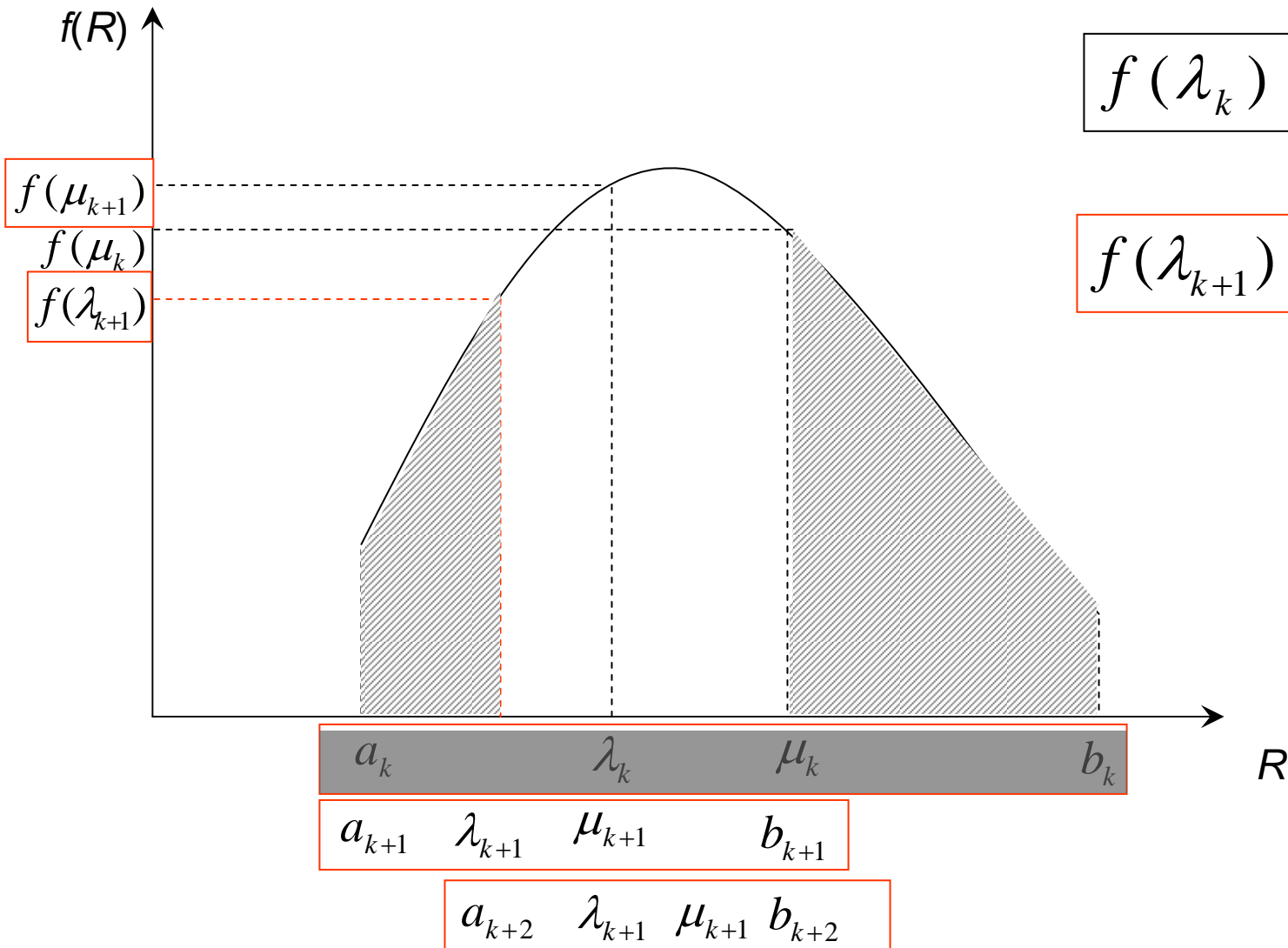
Solution Procedure



p -CCLP: 1st phase - Tabu Search

- Initially p facilities opened with the smallest total distance from customer zones
- SWAP move operators used (*symmetrical 1-Swap, 2-Swap, and 3-Swap*) to generate a neighborhood of the current solution
- For each move operator: $num_neigh = p(m-p)/3$
- Tabu status is tracked for every facility:
 $tabu_tenure \in [1, 2, \dots, Max_Tabu_Tenure = 25]$
- Search terminates IF EITHER max_iter iterations have been executed, OR IF the best solution found so far does not improve for max_nonimp_iter consecutive iterations

p -CCLP & UCCLP: 2nd phase - Fibonacci Search



UCCLP: 1st phase - Tabu Search

- initially 1 random facility opened
- 1-ADD, 1-DROP, 1-SWAP move operators to generate a neighborhood of the current solution
- For 1-ADD: $num_neigh = (m - \rho)$
For 1-DROP: $num_neigh = \rho$
For 1-SWAP: $num_neigh = \min\{3m, \rho(m - \rho)\}$
- Tabu status is tracked for every facility:
 $tabu_tenure \in [1, 2, \dots, Max_Tabu_Tenure = 25]$
- Search terminates IF EITHER max_iter iterations have been executed,
OR IF the best solution found so far does not improve for max_nonimp_iter consecutive iterations

TS Heuristic vs. Exhaustive Search on p -CCLP

Instance (n, p, s)	Exhaustive Enumeration		TS heuristic			Percent Error (%)
	Incentive	Profit	Incentive	Profit	CPU (s)	
(20,1,50)	45.34	123.7	45.34	123.7	0.02	0.00
(20,1,75)	60.00	2906.4	60.00	2906.4	0.02	0.00
(20,1,100)	66.40	8736.2	66.40	8736.2	0.02	0.00
(20,2,50)	45.20	245.9	45.20	245.9	0.02	0.00
(20,2,75)	60.00	4774.5	60.00	4774.5	0.03	0.00
(20,2,100)	66.40	14245.6	66.40	14245.6	0.03	0.00
(20,3,50)	45.15	367.5	45.15	367.5	0.05	0.00
(20,3,75)	60.00	6473.0	60.00	6473.0	0.06	0.00
(20,3,100)	74.85	18713.5	74.87	18703.3	0.06	0.05
(20,4,50)	45.10	488.7	45.10	488.7	0.06	0.00
(20,4,75)	59.95	7943.1	59.95	7943.1	0.08	0.00
(20,4,100)	68.05	23013.9	68.05	23013.9	0.09	0.00
(20,5,50)	-	-	45.08	599.9	0.08	-
(20,5,75)	-	-	59.65	9415.6	0.11	-
(20,5,100)	-	-	65.44	25922.6	0.13	-
(50,1,50)	45.63	149.9	45.63	149.9	0.03	0.00
(50,1,75)	62.82	4779.6	62.82	4779.6	0.05	0.00
(50,1,100)	77.12	20864.7	77.12	20864.7	0.05	0.00
(50,2,50)	45.65	282.1	45.65	282.1	0.11	0.00
(50,2,75)	62.62	7959.0	62.62	7959.0	0.16	0.00
(50,2,100)	76.15	33675.0	76.15	33675.0	0.17	0.00

Table 2: Results of some p -CCLP instances by TS heuristic and exhaustive enumeration.

SBB Solver of the GAMS Suite v22.0

No. of customer zones	20	50	100	200	400
No. of solns. returned	15	15	2	0	0
No. of solns. as good as TS	6	1	0	0	0
No. of inferior solns.	9	14	2	0	0

Table 4: Results of the SBB solver on p -CCLP instances.

- ❖ SBB calls CPLEX 9.0 for the IP subproblems, and either MINOS or CONOPT as subsolver for NLP
- ❖ OQNLP and DICOPT as MINLP solvers perform even worse.

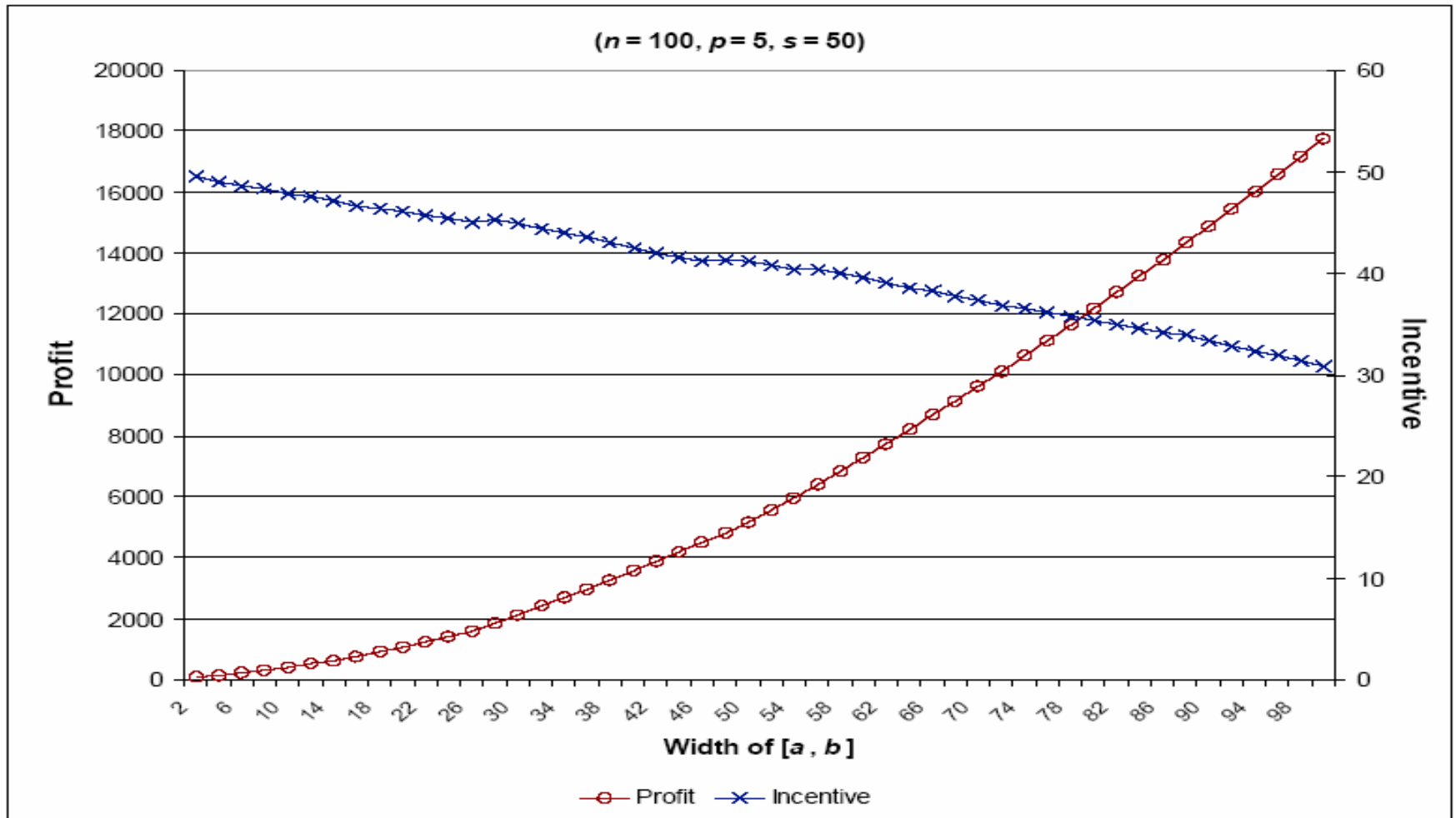
SBB Solver vs. TS Heuristic on UCCLP instances

Instance	TS heuristic				SBB			
	q	Incentive	Profit	CPU (s)	q	Incentive	Profit	CPU (s)
(20, l_0 , 50)	15	45.07	909.1	0.14	15	45.07	908.7	7200
(20, h_1 , 50)	6	45.12	110.7	0.08	8	45.06	-31.1	7200
(20, l_0 , 75)	11	58.71	8929.8	0.14	9	58.54	8791.6	7200
(20, h_1 , 75)	1	60.00	259.4	0.05	3	59.85	-1679.1	7200
(20, l_0 , 100)	9	60.00	21901.1	0.14	9	60.00	21295.1	7200
(20, h_1 , 100)	1	66.40	1282.2	0.05	2	69.22	-884.9	7200
(50, l_0 , 50)	38	45.10	2320.5	1.53	41	45.10	2278.9	7200
(50, h_1 , 50)	4	45.49	95.3	0.41	28	49.06	-2781.7	7200
(50, l_0 , 75)	22	59.10	24770.8	1.08	24	59.07	24754.4	7200
(50, h_1 , 75)	1	62.83	849.7	0.27	5	68.02	-8437.6	7200
(50, l_0 , 100)	14	66.17	61365.8	1.05	16	65.09	59392.7	7200
(50, h_1 , 100)	1	77.12	5330.3	0.25	4	85.00	-37502.4	7200
(100, l_0 , 50)	60	45.21	3711.9	11.81	-	-	-	7200
(100, h_1 , 50)	3	46.03	95.8	2.59	-	-	-	7200
(100, l_0 , 75)	28	59.57	48803.9	7.05	-	-	-	7200
(100, h_1 , 75)	3	61.69	3395.6	3.58	-	-	-	7200
(100, l_0 , 100)	12	68.06	120192	6.05	-	-	-	7200
(100, h_1 , 100)	2	77.95	24306.2	3.17	-	-	-	7200
(200, l_0 , 50)	97	45.42	5906.6	98.09	-	-	-	7200
(200, h_1 , 50)	1	46.70	24.9	9.05	-	-	-	7200
(200, l_0 , 75)	33	60.00	87480.7	49.44	-	-	-	7200
(200, h_1 , 75)	2	64.93	3804.5	21.74	-	-	-	7200
(200, l_0 , 100)	17	67.48	247157.5	51.80	-	-	-	7200
(200, h_1 , 100)	1	79.00	18374.6	15.28	-	-	-	7200
(400, l_0 , 50)	151	45.66	12820.3	1025.38	-	-	-	7200
(400, h_1 , 50)	2	46.75	95.8	132.00	-	-	-	7200
(400, l_0 , 75)	26	60.92	138359.0	414.50	-	-	-	7200
(400, h_1 , 75)	3	64.97	10147.7	279.89	-	-	-	7200
(400, l_0 , 100)	29	66.40	529076.6	479.77	-	-	-	7200
(400, h_1 , 100)	2	77.72	34062.0	217.70	-	-	-	7200

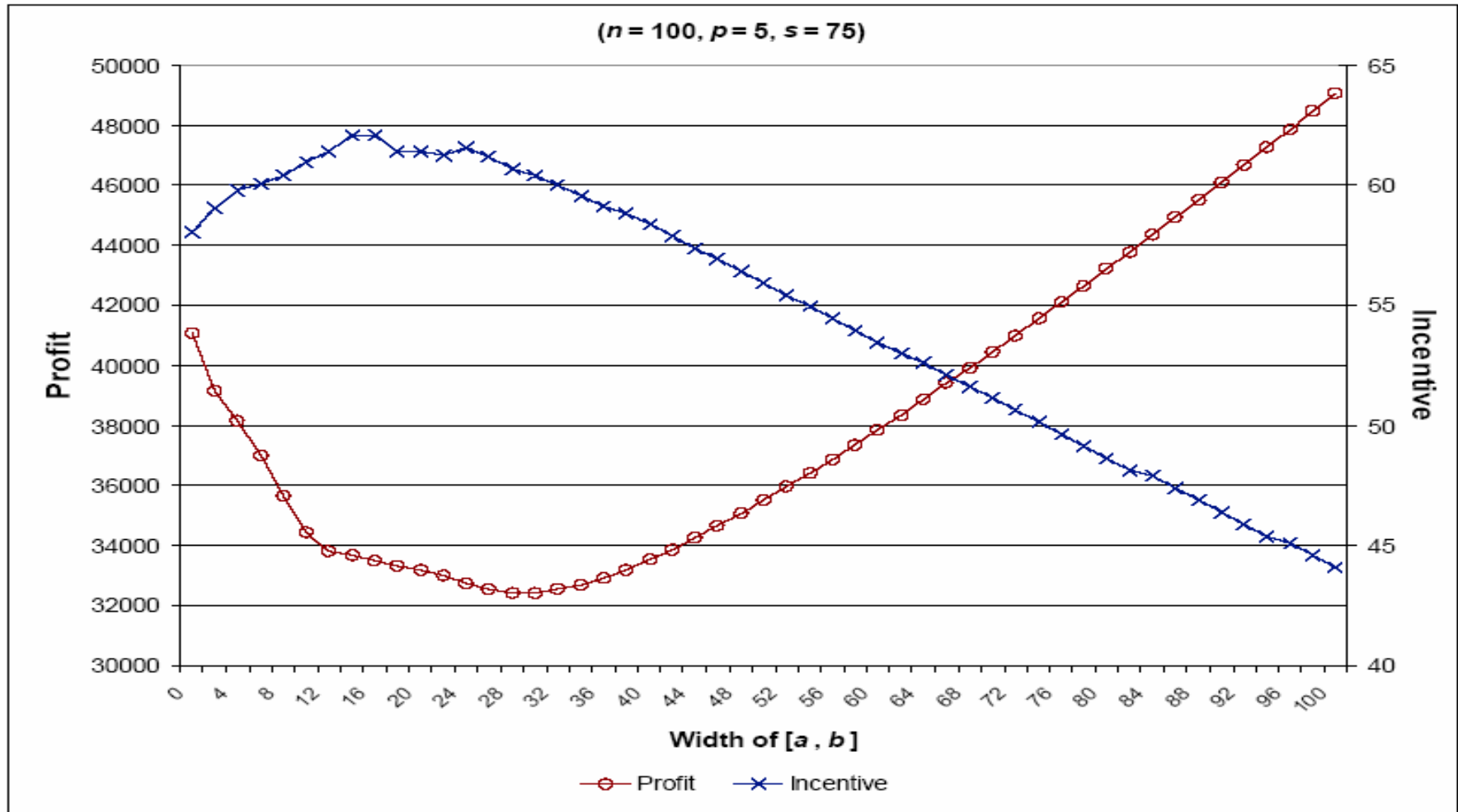
Table 5: Results of the CCLP instances by TS heuristic.



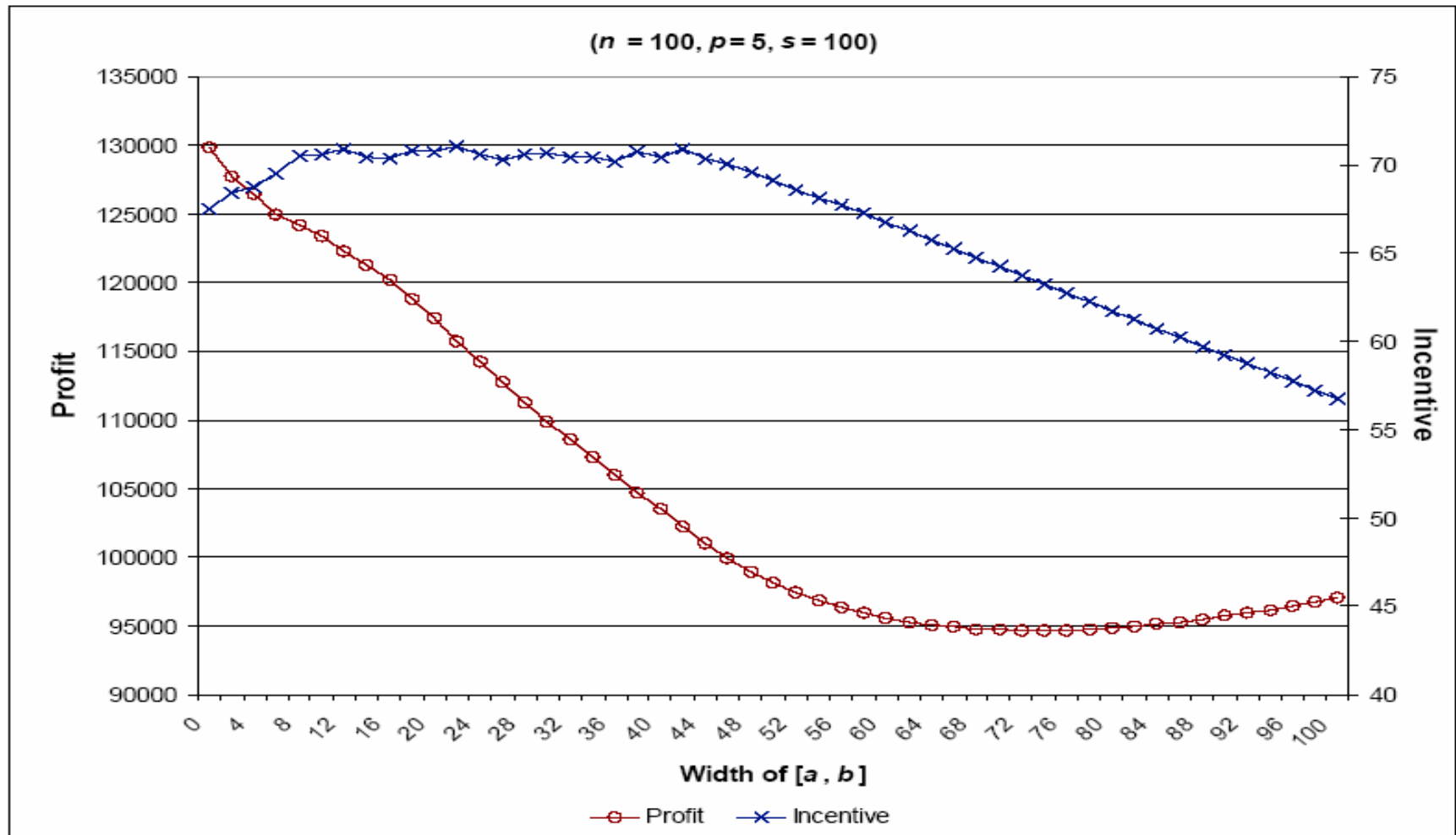
Effect of the variability of R_0 - Width of $[a, b]$



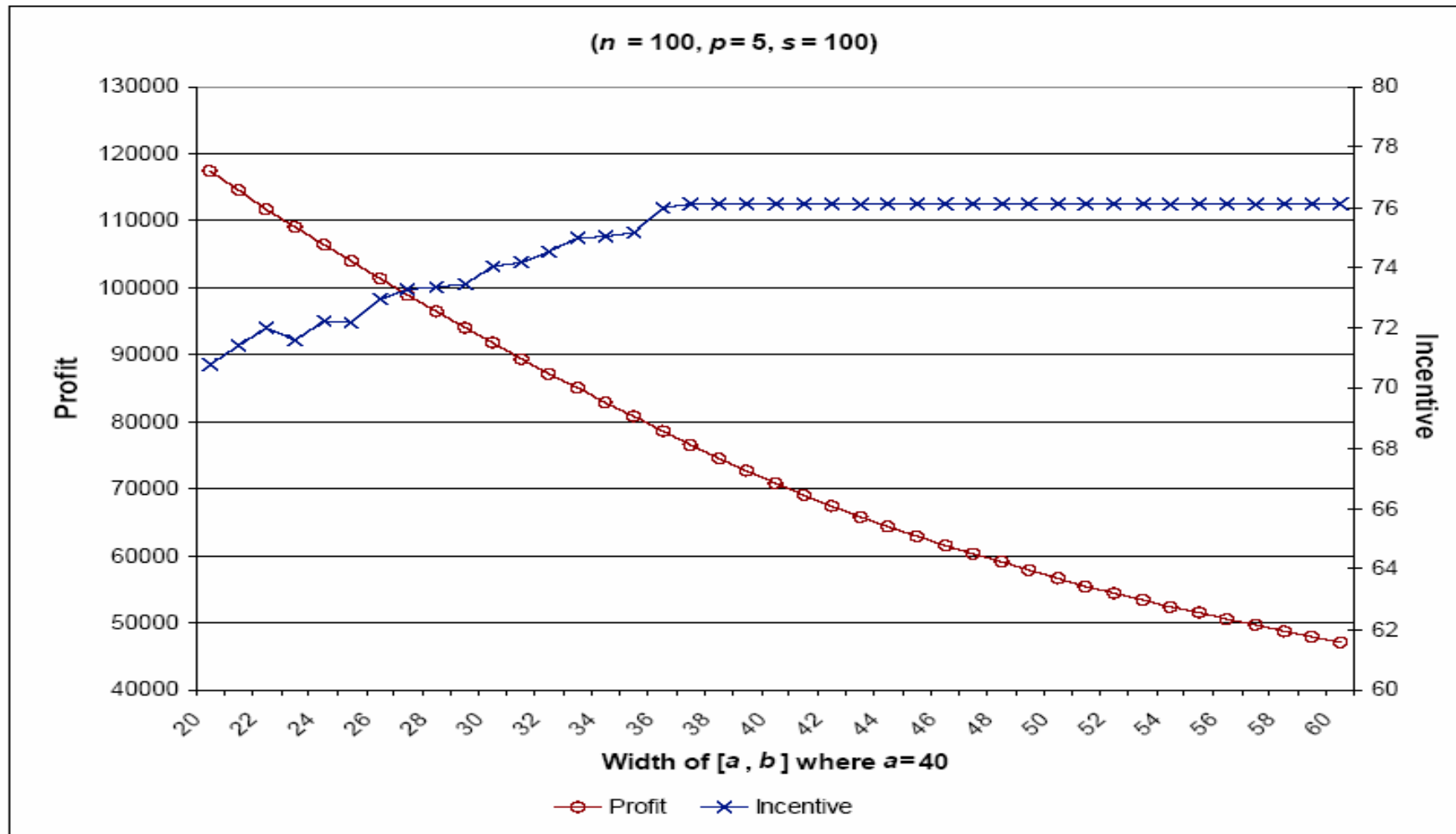
Effect of the variability of R_0 - Width of $[a, b]$



Effect of the variability of R_0 - Width of $[a, b]$



Effect of the variability of R_0 - Width of $[a, b]$ when $a = \text{const.}$



p -CCLP Results - Summary

- as p increases
 - profit increases, but the marginal profit decreases with each additional facility
 - return rate increases
 - financial incentive offered decreases
- when s is higher
 - incentive is higher
 - collection rate is higher

p-CCLP - Effect of the Variability of R_0

- *As mean and standard deviation increases*
 - profit decreases
 - return rate decreases
 - incentive increases to some extent

p-CCLP -Effect of the Variability of R_0

- *As standard deviation increases, mean being constant*
 - Profit
 - increases, for $s=50$ and $s=75$
 - first decreases then increases for $s=100$
 - Return rate
 - increases for $s=50$
 - decreases for $s=75$ and $s=100$
 - Incentive decreases

UCCLP Results - Summary

- when fixed cost is higher
 - profit is lower
 - # of opened CCs is lower
 - return rate is lower
- when s is higher
 - profit & incentive is higher
 - # of CCs lower

Conclusions

- a CC location model is proposed for *collection of EOL products from customers*
- *number and locations of CCs* that should be opened and *financial incentive* are determined
- customers are assumed to aggregate at some points
- a *reservation price* is assumed for each customer
- two factors affect customers' decision on returning the product:
 - *financial incentive*
 - *proximity to the nearest* collection center
- *returned product still has a value* for the company which can be extracted by subsequent reprocessing activities

Conclusions

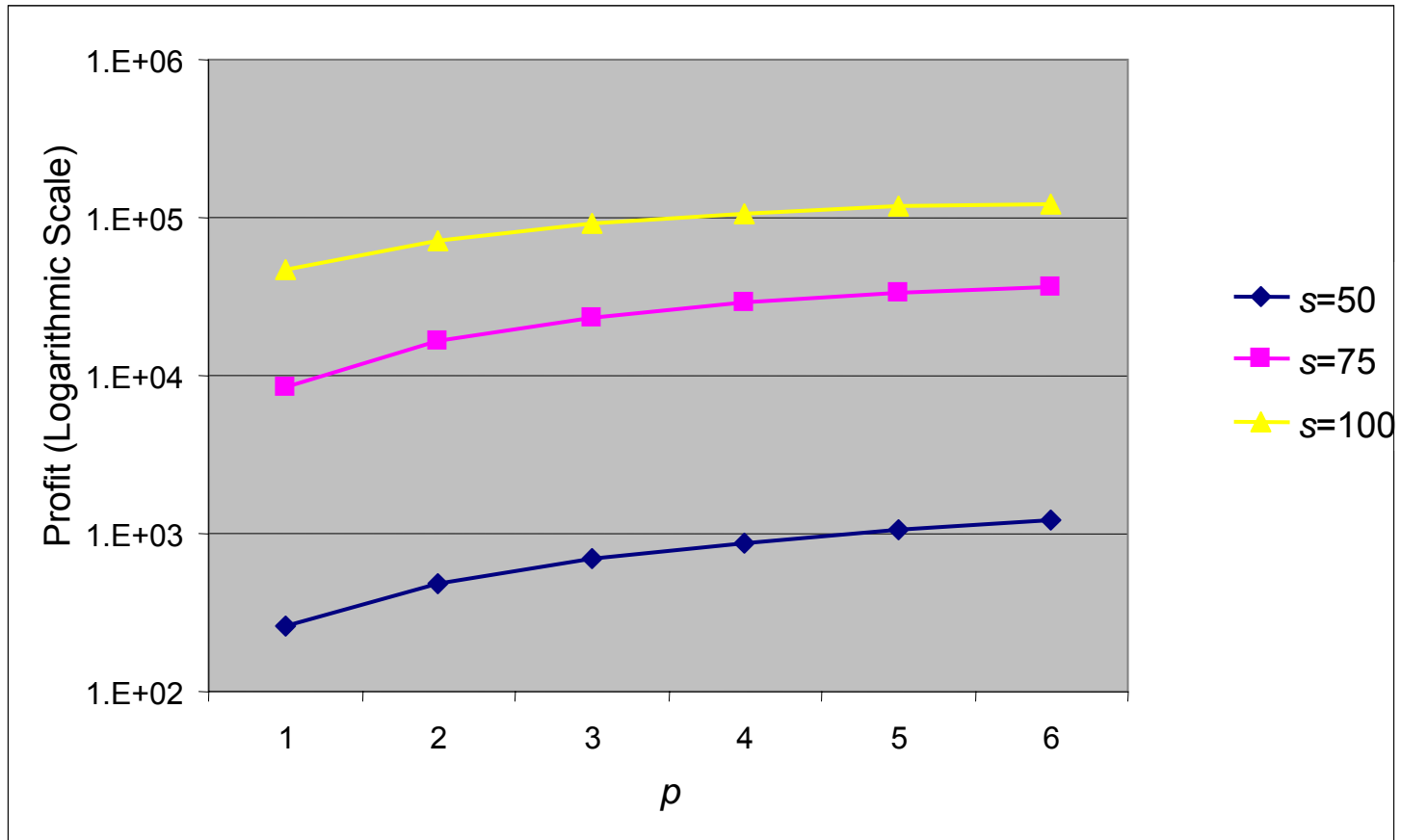
- problem is formulated as a *Mixed-Integer Nonlinear Facility Location (MINLP)* model to determine the collection center locations and the financial incentive to be offered
- a *2-phase solution methodology* is used
 - *Tabu search & Fibonacci search*
- 3870 experiments are made in total
- results are presented for two models: p-CCLP and UCCLP

Questions & Comments?

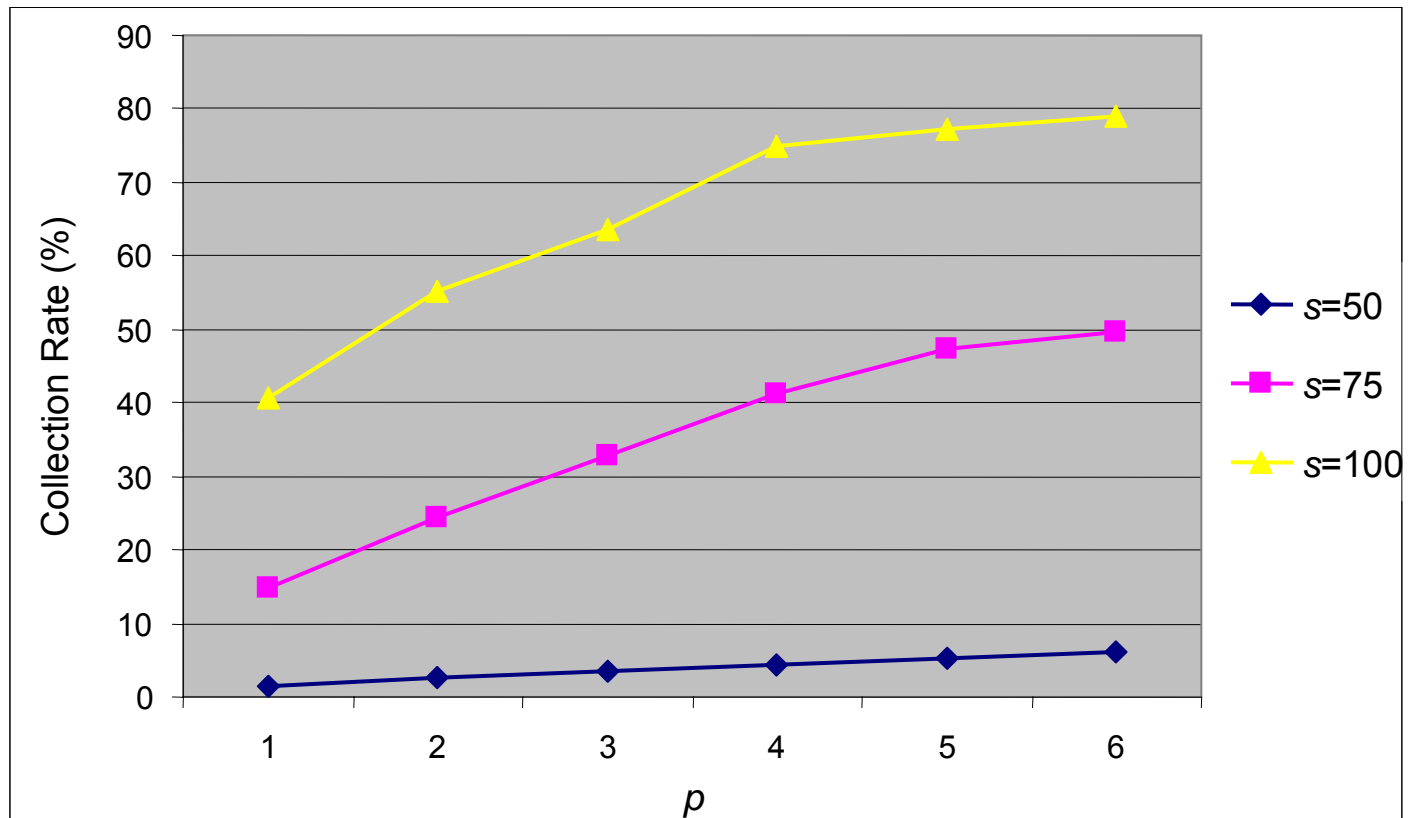
- ▶ Computational Results
- ▶ p -CCLP Sensitivity Results with respect to p and s
- ▶ p -CCLP Sensitivity with respect to Variance and Mean
- ▶ Effect of the FC value in UCCLP
- ▶ Effect of R_0 variability in p -CCLP



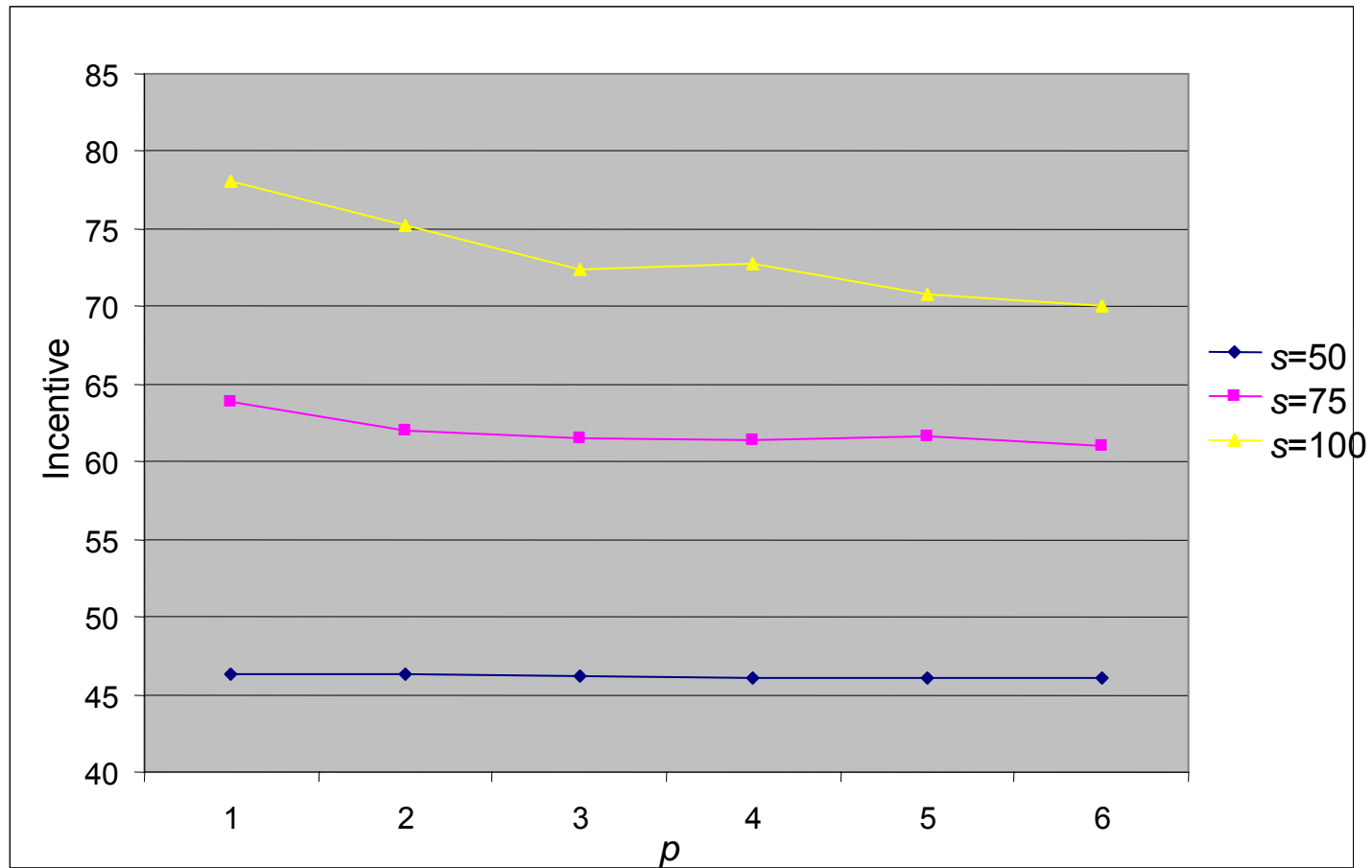
Effect of p on Profit



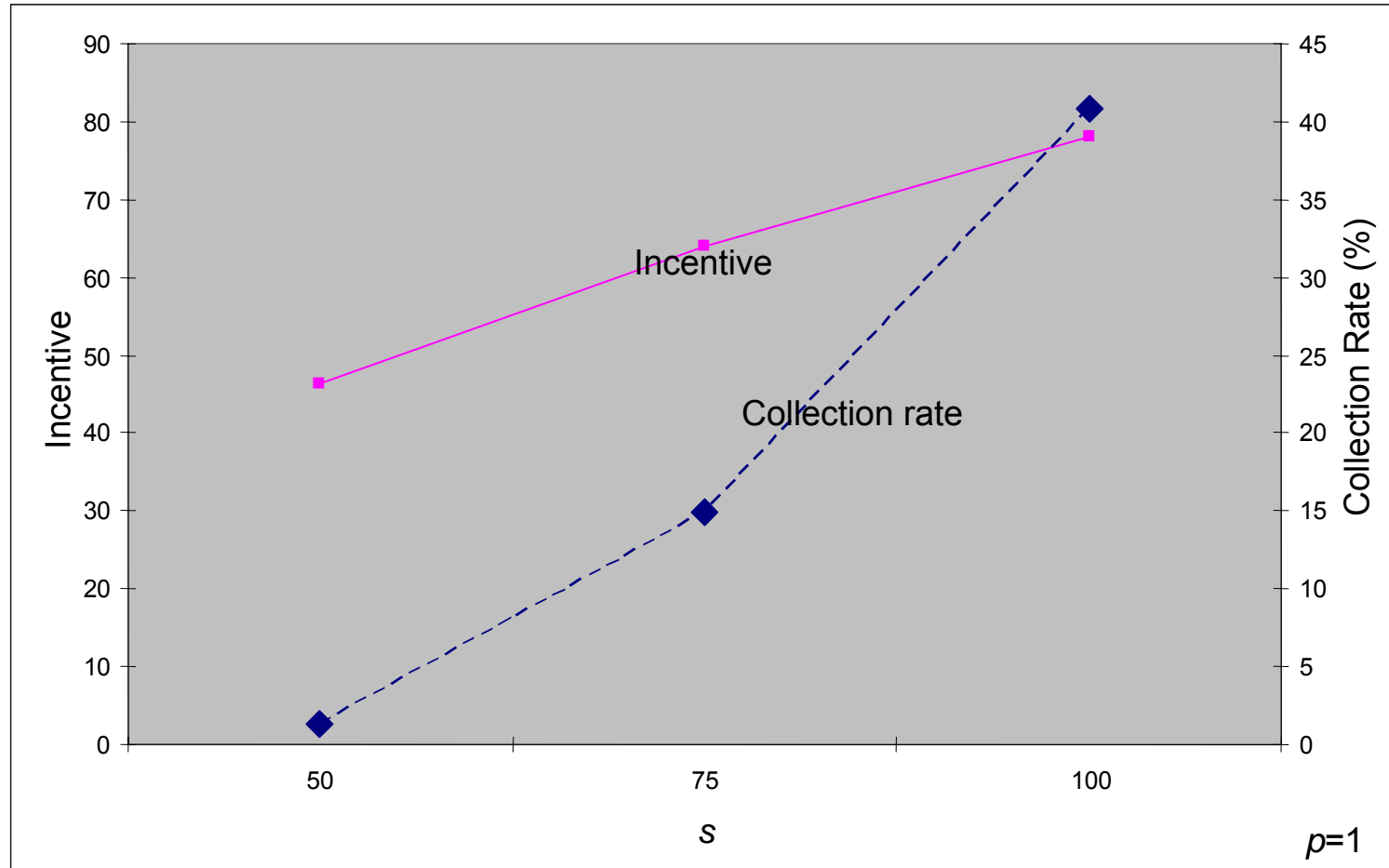
Effect of p on Collection Rate



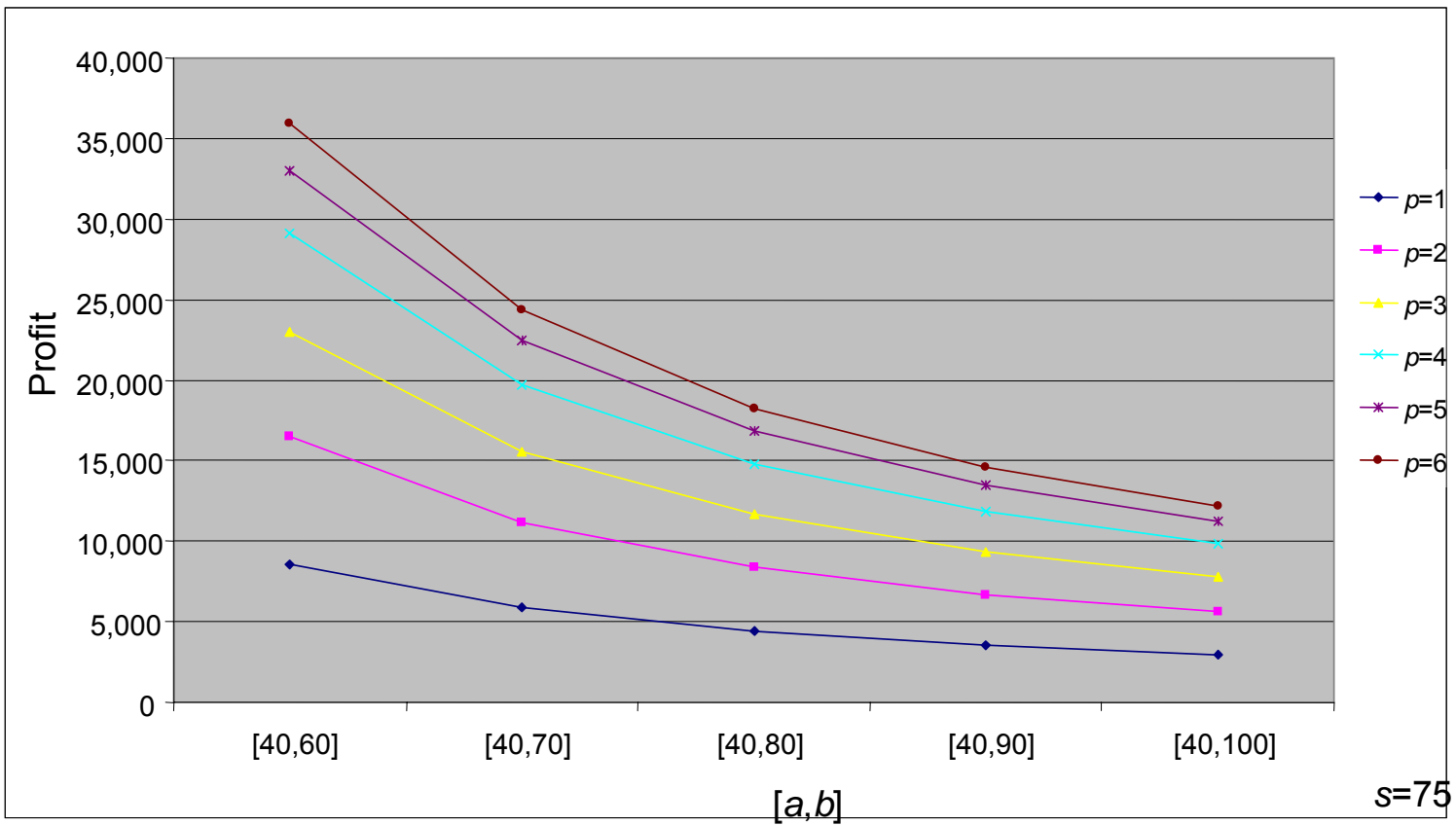
Effect of p on Incentive



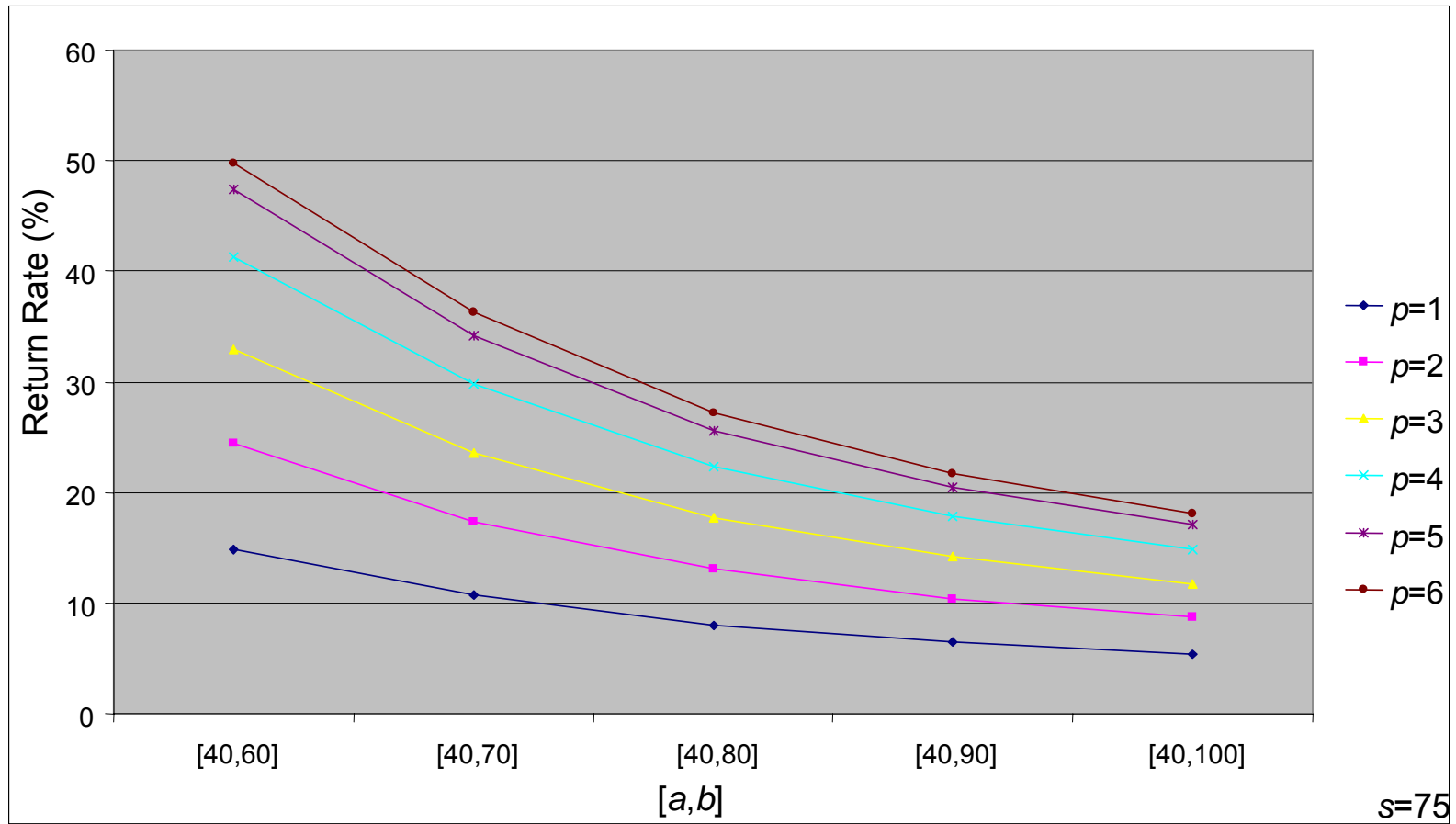
Effect of s on Incentive & Collection Rate



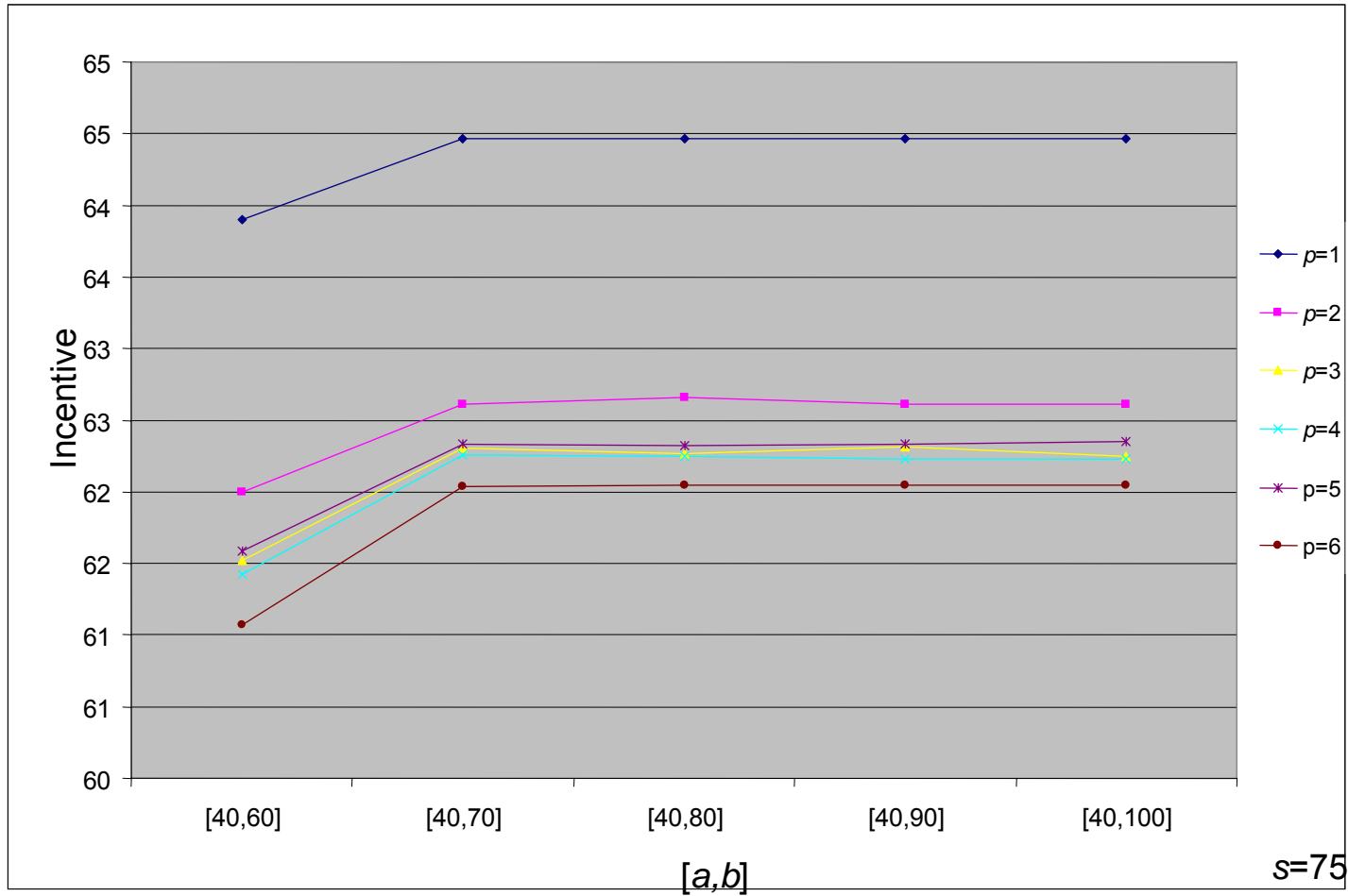
Varying μ and σ : Effect on Profit



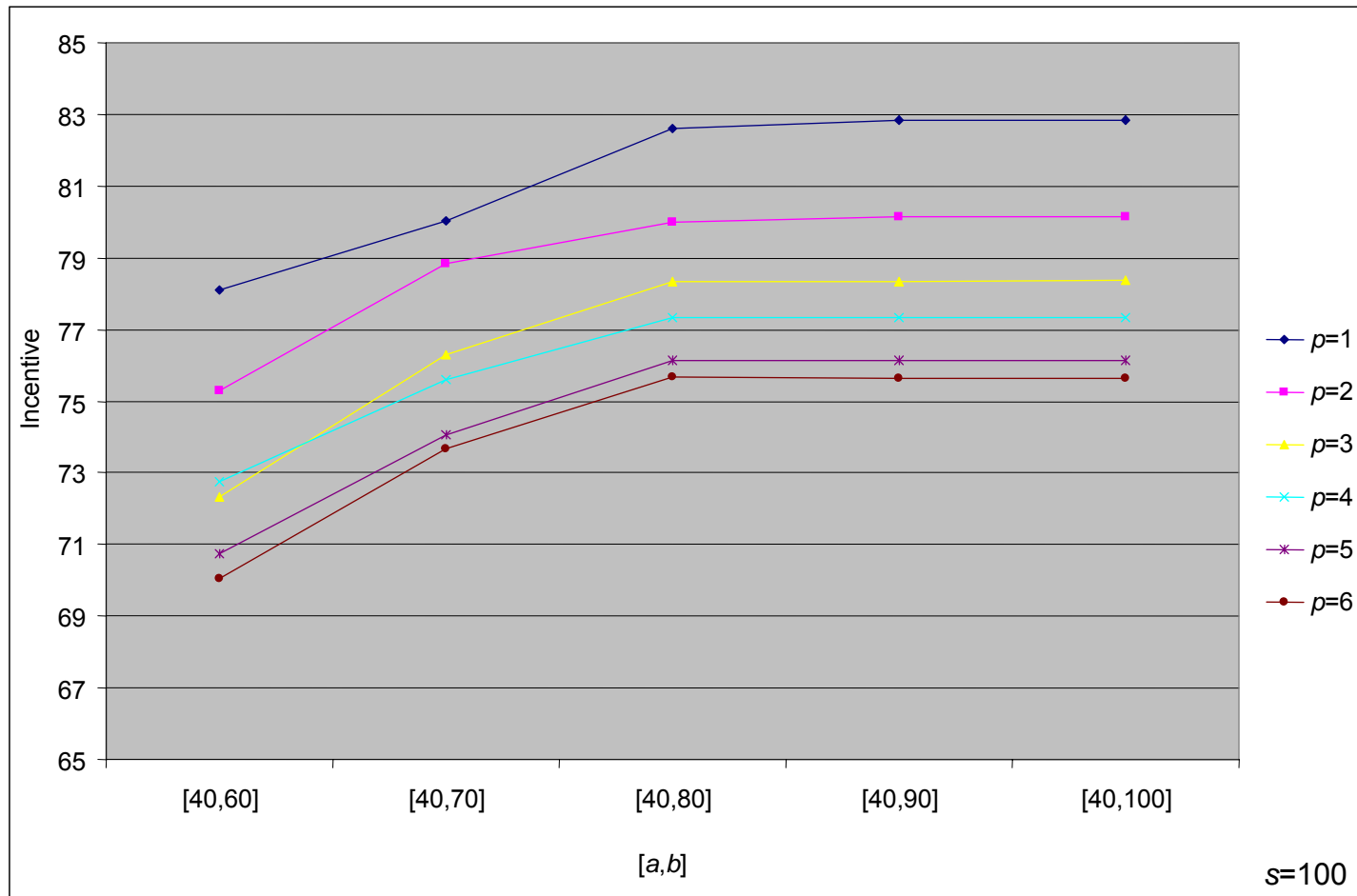
Varying μ and σ : Effect on Return Rate



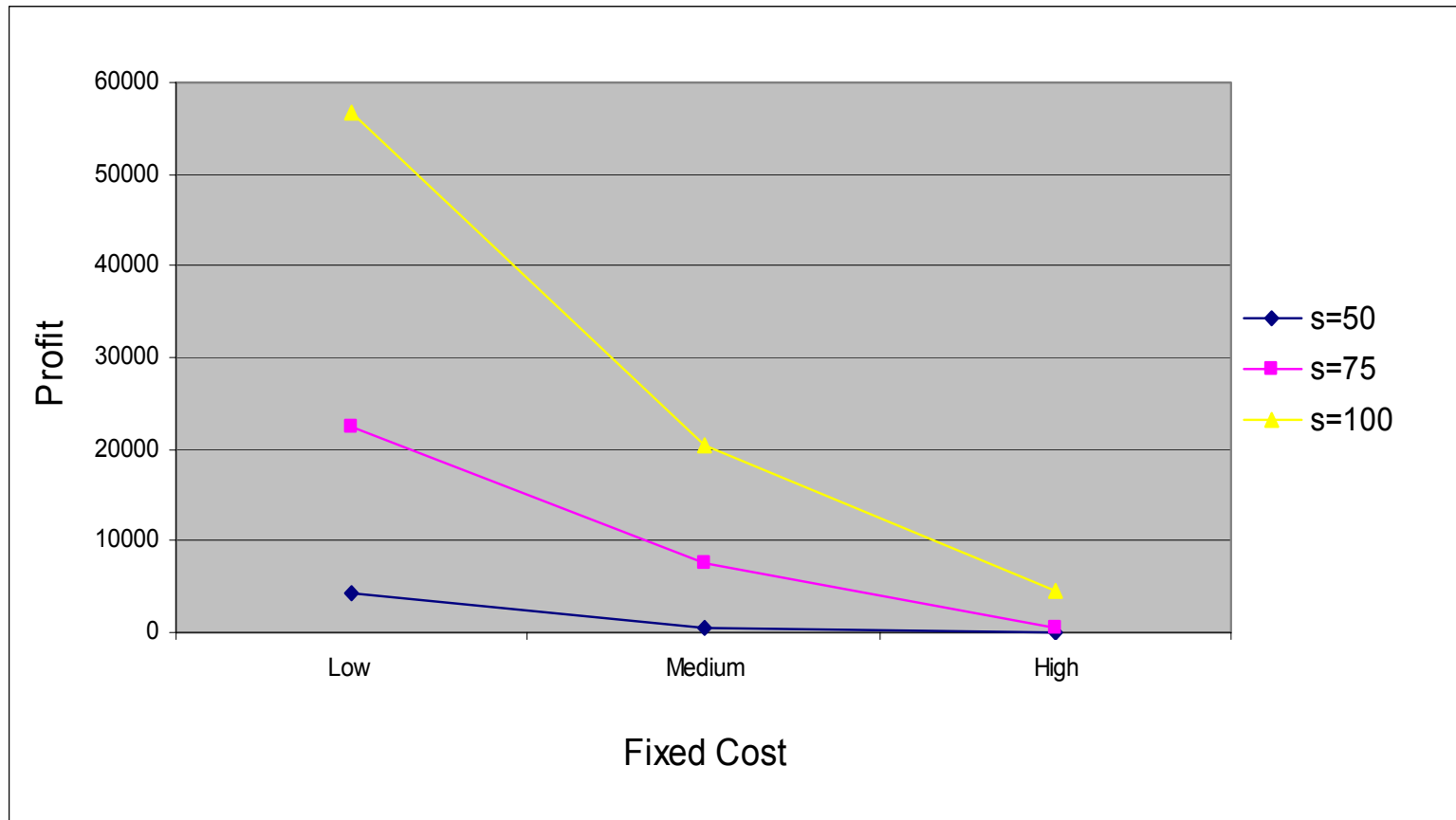
Varying μ and σ : Effect on Incentive, $s=75$



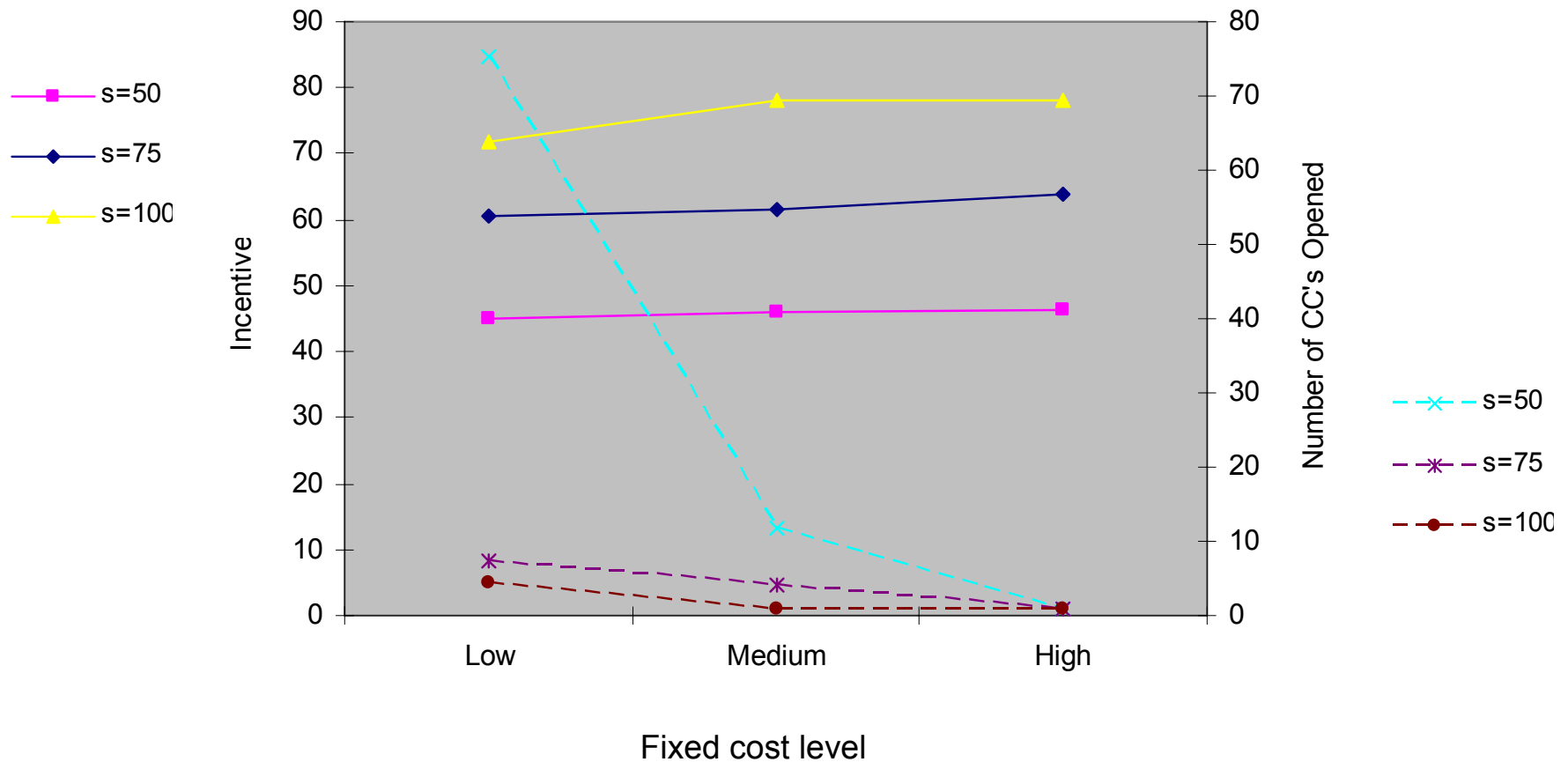
Varying μ and σ : Effect on Incentive, $s=100$



Effect of Fixed Cost on Profit



Effect of Fixed Cost on Incentive and # of CCs



Effect of Fixed Cost on Return Rate

