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PROFIT MAXIMIZATION IN VEHICLE ROUTING PROBLEMS

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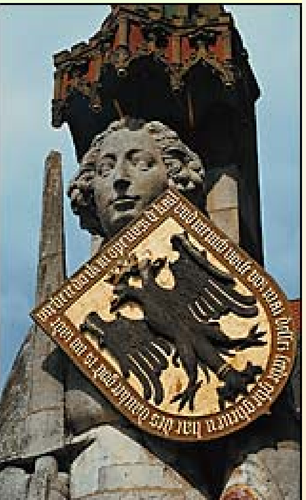


Intro: Capacitated Vehicle Routing

- ***𝒩𝒫*-hard** problem as a total distance / traveling cost minimization problem.
- Natural extension of the notorious traveling salesman problem (**TSP**) with...
  - Delivery vehicles (*homogeneous or heterogeneous fleet*) with limited capacity.
  - One or more central nodes (*single depot vs. multi-depots*) as origins of tours.
  - Each tour terminates at its own origin (*closed route*), or either at a different origin *(different depot*) or at a customer node *(open route)*.

Intro: Capacitated Vehicle Routing (cont.)

- Natural extension of the notorious (TSP) with... (*cont.*)
  - Customers with a known demand to be delivered (*linehaul customers*) or customers with a known supply to be picked up (*backhaul customers as in VRP with Backhauls*.)
    - VRP with Simultaneous or Exclusive Pickups and Deliveries **(VRPPD)**
  - A QoS (*quality of service*) guarantee in the form of two-sided time windows (TW), time deadlines (TD), maximum route duration or maximum route length.

Intro: Capacitated Vehicle Routing (cont.)

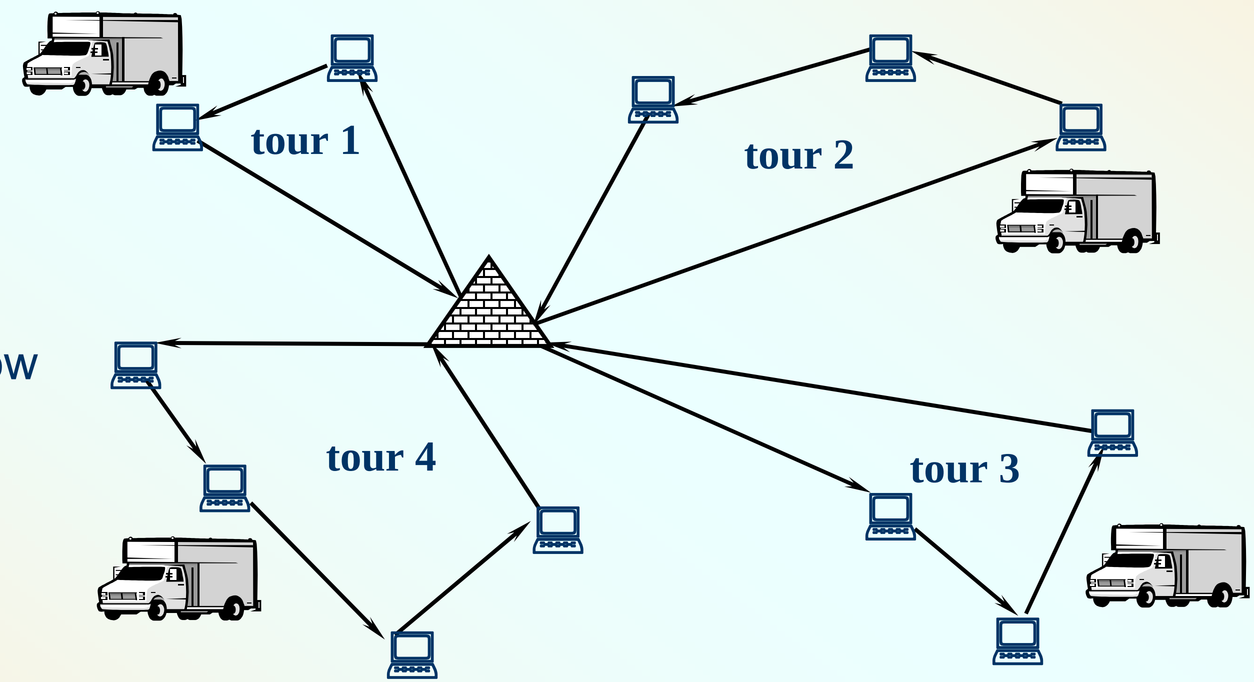
- Natural extension of the notorious (TSP) with… (*cont.*)
  - The vehicle fleet size can be fixed (no costs associated with vehicles) or …
  - A vehicle acquisition cost (*daily rental cost* or *discounted vehicle purchasing price*) can be incurred.

OBJECTIVES *subject to* system constraints and QoS guarantees

- Minimization of total route length.
- Minimization of total operational costs.

CVRP Applications (cont.)

- A **must** in e-commerce
  - the e-fulfillment of online orders for perishable goods
  - the realization of the last mile of delivery



A **quality of service** guarantee is time window restricted deliveries to customers' dwellings.

: Physical store serving also online customers

: Online (delivery) customer

CVRP Literature

To TSP with Profits

- A wide variety of real-world applications since its first introduction in **1959** (*G.B. Dantzig* and *J.H. Ramser* in *Mgmt Sci*) and since its most common solution method proposed in **1964** (*G. Clarke* and *J.V. Wright* in *Oper Res*)...

- A few surveys and manuals of the VRP

1. **Computers & Operations Research**
   The entire Volume 10, No. 2, 1983 special issue:
   Routing and Scheduling of Vehicles and Crews – The State of the Art by *Lawrence Bodin, Bruce L. Golden, Arjang Assad and Michael Ball*

2. **American Journal of Mathematical and Management Sciences,** the entire Volume 13, No. 3 and No. 4, 1993
   Ulrich Derigs, Gregor Grabenbauer, Julien Bramel, Chung-Lun Li, David Simchi-Levi, Sam R. Thangiah, Ibrahim H. Osman, Rajini Vinayagamoorthy, Tong Sun, I-Ming Chao, Bruce L. Golden and Edward Wasil among many.

CVRP Literature (cont.)

3. **"Algorithms for the vehicle routing and scheduling problems with time window constraints"**
   by *Marius M. Solomon,*
   Operations Research, Vol. 35, No. 2, March-April 1987

4. **"Fleet Management and Logistics"**
   by *Teodor Gabriel Crainic and Gilbert Laporte (eds),*
   Centre for Research on Transportation,
   Copyright © 1998 by Kluwer Academic Publishers.

5. **"A Computational Study of Vehicle Routing Applications"**
   doctoral thesis by *Jennifer L. Rich,* **RICE UNIVERSITY**, Houston, May 1999, USA (Advisor: William J. Cook)

CVRP Literature (cont.)

6. **"The Dynamic Vehicle Routing Problem" IMM-PHD-2000-73**, doctoral thesis by *Allan Larsen*, Department of Mathematical Modelling (IMM) at the **TECHNICAL UNIVERSITY OF DENMARK** (DTU), Lyngby June 2000, Denmark (Advisor: Oli B.G. Madsen)
7. **"A heuristic method for the open vehicle routing problem"** by *Sariklis D and S Powell* in **Journal of the Operational Research Society**, Vol. 51, No. 5, May 2000
8. **"The Vehicle Routing Problem"** by *Paolo Toth and Daniele Vigo (eds),* **SIAM Monographs on Discrete Mathematics and Applications**, Copyright © 2002, SIAM Publishing.
9. **Featured Issue: "New technologies in transportation systems,"** *Y. Siskos and E. Sambracos (eds),* **European Journal of Operational Research**, Vol. 152, No. 2, January 16th 2004, ELSEVIER B.V. Copyright © 2004.

CVRP Literature (cont.)

10. **"A guide to vehicle routing heuristics"** by *Jean-Françoise Cordeau, Michel Gendreau, Gilbert Laporte, Jean-Yves Potvin, Frédéric Semet* in **Journal of the Operational Research Society**, Vol. 53, No. 5, May 2002

11. **"Real-time vehicle routing: Solution concepts, algorithms and parallel computing strategies"** by *Ghiani G, Guerriero F, Laporte G, Musmanno R* in **European Journal of Operational Research**, Vol. 151, No. 1, November 2003

12. **The Ultimate List of Vehicle Routing References** at the **Center for Traffic and Transport Research (CTT)** of the **Technical University of Denmark** **http://www.imm.dtu.dk/or/vrp_ref/vrp.html**

CVRP Literature (cont.)

13. **The VRP Web and the Networking and Emerging Optimization Group** at the **University of Málaga in Spain** **http://neo.lcc.uma.es/radi-aeb/WebVRP/**

14. **Branch Cut and Price Applications : Vehicle Routing Links** maintained by *Ted Ralphs* (ted@branchandcut.org) at **http://branchandcut.org/VRP/**

15. **Canada Research Chair in Distribution Management** with a collection of standard benchmark problems **http://www.hec.ca/chairedistributique/**

When $$$ enters the scene: The case of TSP

16. **“A tabu search heuristic for the undirected selective travelling salesman problem”** by *Michel Gendreau, Gilbert Laporte, Frédéric Semet* in **European Journal of Operational Research**, Vol. 106, No. 2-3, April 1998

17. **“A branch-and-cut algorithm for the undirected selective traveling salesman problem”** by *Michel Gendreau, Gilbert Laporte, Frédéric Semet* in **NETWORKS**, Vol. 32, No. 4, December 1998

18. **“An Exact Algorithm for the Elementary SPP with Resource Constraints: Application to Some VRPs”** by *Dominique Feillet, Pierre Dejax, Michel Gendreau and Cyrille Gueguen* in **NETWORKS**, Vol. 44, No. 3, October 2004

19. **“Traveling salesman problems with profits”** by *Dominique Feillet, Pierre Dejax and Michel Gendreau*, **Transportation Science,** Vol. 39, No. 2, May 2005

TSP: Profit Maximization Case (cont.)

Traveling Salesman Problems with Profits (TSPs with Profits) are a generalization of the Traveling Salesman Problem (TSP) where it is not necessary to visit all vertices. With each vertex is associated a profit. The overall goal pursued is the simultaneous optimization of the collected profit and the travel costs. These two optimization criteria appear either in the objective function or as a constraint.

...

Conclusions emphasize the interest of this class of problems, with respect to applications as well as theoretical results.

Dominique Feillet, Pierre Dejax and Michel Gendreau

TSP: Profit Maximization Case (cont.)

Alternative Objectives

- **Minimize total vehicle distance** *whilst collecting a specified minimum amount of total profit from visited customers.*

- **Minimize total vehicle distance** *whilst serving a specified minimum number of customers.*

- **Maximize total profit collected from customers** *whilst a specified maximum total distance is traveled.*

- **JOINT OBJECTIVE FUNCTION**

MAXIMIZE Net Profit = Total Profit Collected – Total Cost of Traveling

OP and then TOP

- Unit demand
- No node-specific temporal constraints
- No vehicle capacity

20. **"Heuristic Methods Applied to Orienteering"** by *Theodore A. Tsiligiridis* in **Journal of Operational Research Society**, Vol. 35, 1984.
21. **"The team orienteering problem"** by *I-Ming Chao, Bruce L. Golden and Edward Wasil* in **European Journal of Operational Research**, Vol. 88, 1996.
22. **"A Genetic Algorithm with an Adaptive Penalty Function for the Orienteering Problem"** by *M. Fatih Taşgetiren* in **Journal of Economic and Social Research**, Vol. 4, 2002.
23. **"A tabu search heuristic for the team orienteering problem"** by *Huang Tang and Elise Miller-Hooks* in **Computers & Operations Research**, Vol. 32, 2005.

OP → *Maximum Tour Duration Limit* → TOP

MCP, and then MTMCP

MULTIPLE TOUR MAXIMUM COLLECTION PROBLEM (MTMCP)

24. **"An optimal solution procedure for the multiple tour maximum collection problem using column generation"** by *Butt and Ryan* in **Computers & Operations Research**, Vol. 26, 1999.
25. **"A heuristic for the multiple tour maximum collection problem"** by *Butt and Cavalier* in **Computers & Operations Research**, Vol. 21, 1994.

MCP → *Maximum Tour Duration Limit* → MTMCP

Single-VRP with Unrestricted Backhauls

26. "The Single-Vehicle Routing Problem with Unrestricted Backhauls" by *Haldun Süral and James H. Bookbinder* in **Networks**, Vol. 41, 2003.

Notation Scheme for VRP with Backhaul Opportunities

$$(\boxed{\alpha} / \boxed{\beta} / \boxed{\gamma})$$

- α: Number of vehicles
- β: Backhaul Service Options [**must** *or* **free**]
- γ: Precedence order between deliveries (*linehaul)* and pickups (*backhaul*) [**prec** *or* **any**]

VRPPTD: Profit Maximization Case

Our Problem of Interest

- *S*ingle-depot capacitated vehicle routing problem with a flexible size fleet of homogeneous vehicles.

- Demand of all customers + Profit from all customers + coordinates of all customers and the depot known in advance with certainty.

- Time deadline + maximum route duration/length constraints are to be satisfied.

- JOINT OBJECTIVE FUNCTION

MAXIMIZE Net Profit = Total Profit Collected — Total Cost of Deliveries

VRPPTD: Profit Maximization Case (cont.)

Mathematical Formulation (No TDs) – Alternative I

max. $$\Pi = \sum_{i \in N \setminus \{0\}} p_i y_i - \mathit{UNITCOST} \times \sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} c_{ij} x_{ij}$$

s.t. $$\sum_{\substack{j \in N \\ j \neq i}} x_{ij} = \sum_{\substack{j \in N \\ j \neq i}} x_{ji} \qquad \forall i \in N$$

$$\sum_{\substack{j \in N \\ j \neq i}} x_{ij} = y_i \qquad \forall i \in N \setminus \{0\}$$

$$\sum_{i \in N \setminus \{0\}} x_{0i} \geq \sum_{i \in N \setminus \{0\}} \frac{1}{Q} d_i y_i$$

Exponential number of Subtour Elimination Constraints

$$\sum_{i \in S} \sum_{\substack{j \in S \\ j \neq i}} x_{ij} \leq |S| - L_S \qquad \forall S \subseteq N \setminus \{0\} \;\wedge |S| \geq 2$$

$$L_S = \left\lceil \frac{1}{Q} \sum_{i \in S} d_i \right\rceil$$

$$y_i, x_{ij} \in \{0,1\}$$

VRPPTD: Profit Maximization Case (cont.)

Mathematical Formulation (No TDs) – Alternative II

max. $\Pi = \sum_{i \in N \setminus \{0\}} p_i y_i - \text{UNITCOST} \times \sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} c_{ij} x_{ij}$

s.t. $\sum_{\substack{j \in N \\ j \neq i}} x_{ij} = \sum_{\substack{j \in N \\ j \neq i}} x_{ji} \qquad \forall i \in N$

$\sum_{\substack{j \in N \\ j \neq i}} x_{ij} = y_i \qquad \forall i \in N \setminus \{0\}$

$\sum_{i \in N \setminus \{0\}} x_{0i} \geq \sum_{i \in N \setminus \{0\}} \frac{1}{Q} d_i y_i$

$u_i - u_j + Q \times x_{ij} \leq Q - d_j \qquad \forall (i,j) \in N \setminus \{0\} \quad \wedge i \neq j$

$OR: \quad u_i - u_j + Q \times x_{ij} + (Q - d_i - d_j) \times x_{ji} \leq Q - d_j \qquad \forall (i,j) \in N \setminus \{0\} \quad \wedge i \neq j$

$d_i \leq u_i \leq Q \qquad \forall i \in N \setminus \{0\}$

$y_i, x_{ij} \in \{0,1\}$

Miller-Tucker-Zemlin valid inequalities (***ACM***, Vol. **7**, 1960) extended to the VRP by Kulkarni and Bhave (***EJOR***, Vol. **20**, 1985), ***Lifted Version*** by Kara, Laporte and Bektaş ***EJOR***, Vol. **158** (2004)

VRPPTD: Maximize the NET PROFIT

Solving VRPP and its time constrained variants

- **Classical Heuristics preferred when solution time is more critical than solution quality.**

- **(Parallel) Savings Algorithm** by Clarke and Wright (1964) ☑
- **Sweep Algorithm** by *Gillett and Miller* (1974)
- **Push-Forward-Insertion** by *Solomon* (1987), by *Thangiah et al.* (1993)
- **Nearest Neighbourhood Search** by *Rosenkratz, Stearns and Lewis* (1977), *Solomon* (1987), *Fisher* (1994).

VRPPTD: Maximize the NET PROFIT (cont.)

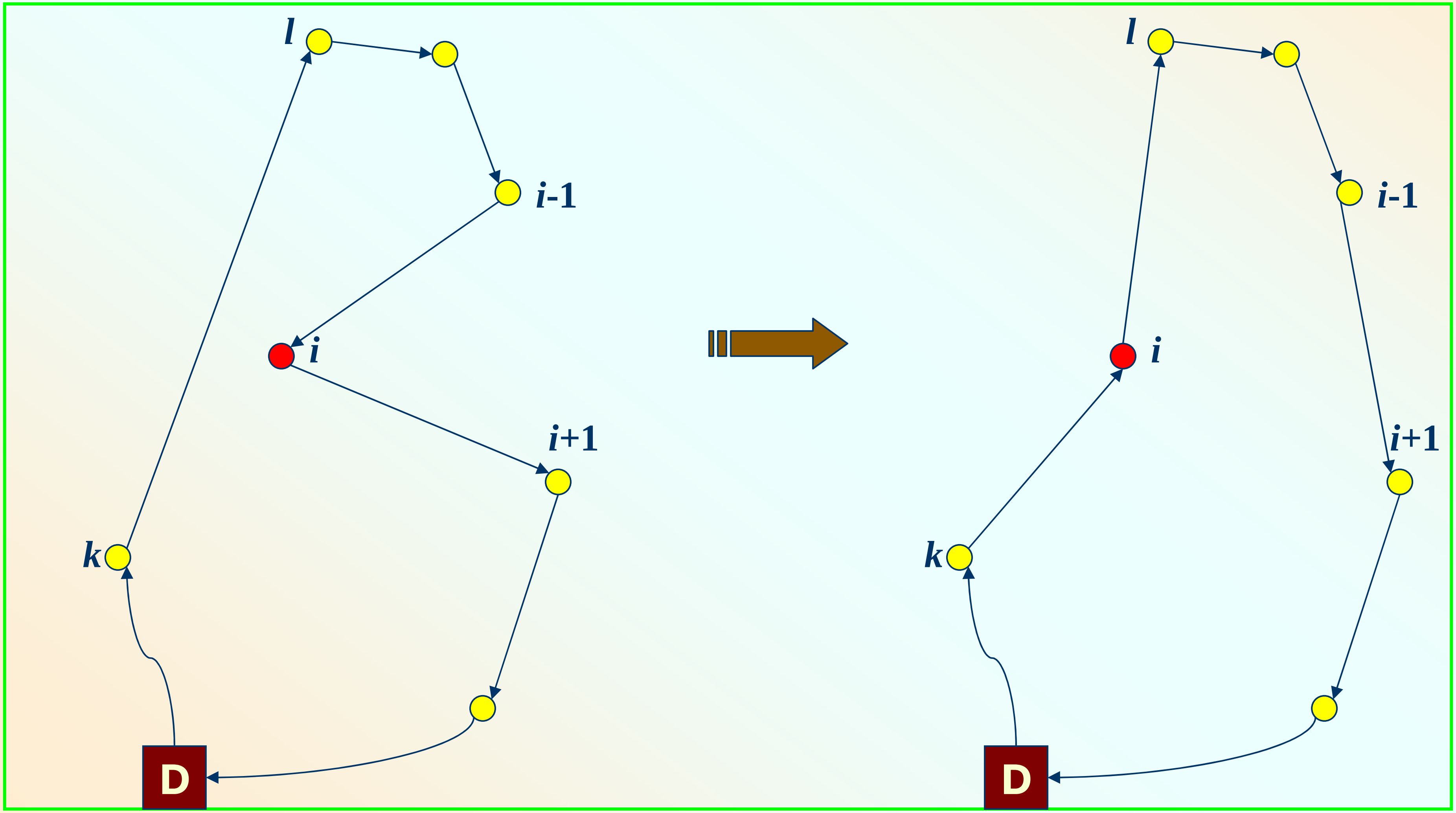
Solving VRPP and its time constrained variants

- **Local Post Optimization (LPO) Procedures**
  a.k.a. *Local Improvement Heuristics (Parallel or First)*

  - **1-0 move (1-Opt) of** *Golden, Magnanti and Nguyen* (1977) ☑
  - **1-1 exchange of** *Waters* (1987) ☑
  - **2-opt of** *Croes* (1958), *Lin* (1965) ☑
  - **3-opt of** *Lin* (1965), **of** *Lin and Kernighan* (1973)
  - **4-opt* of** *Renaud, Laporte and Boctor* (1996)
  - **Or-opt of** *Or* (1976)

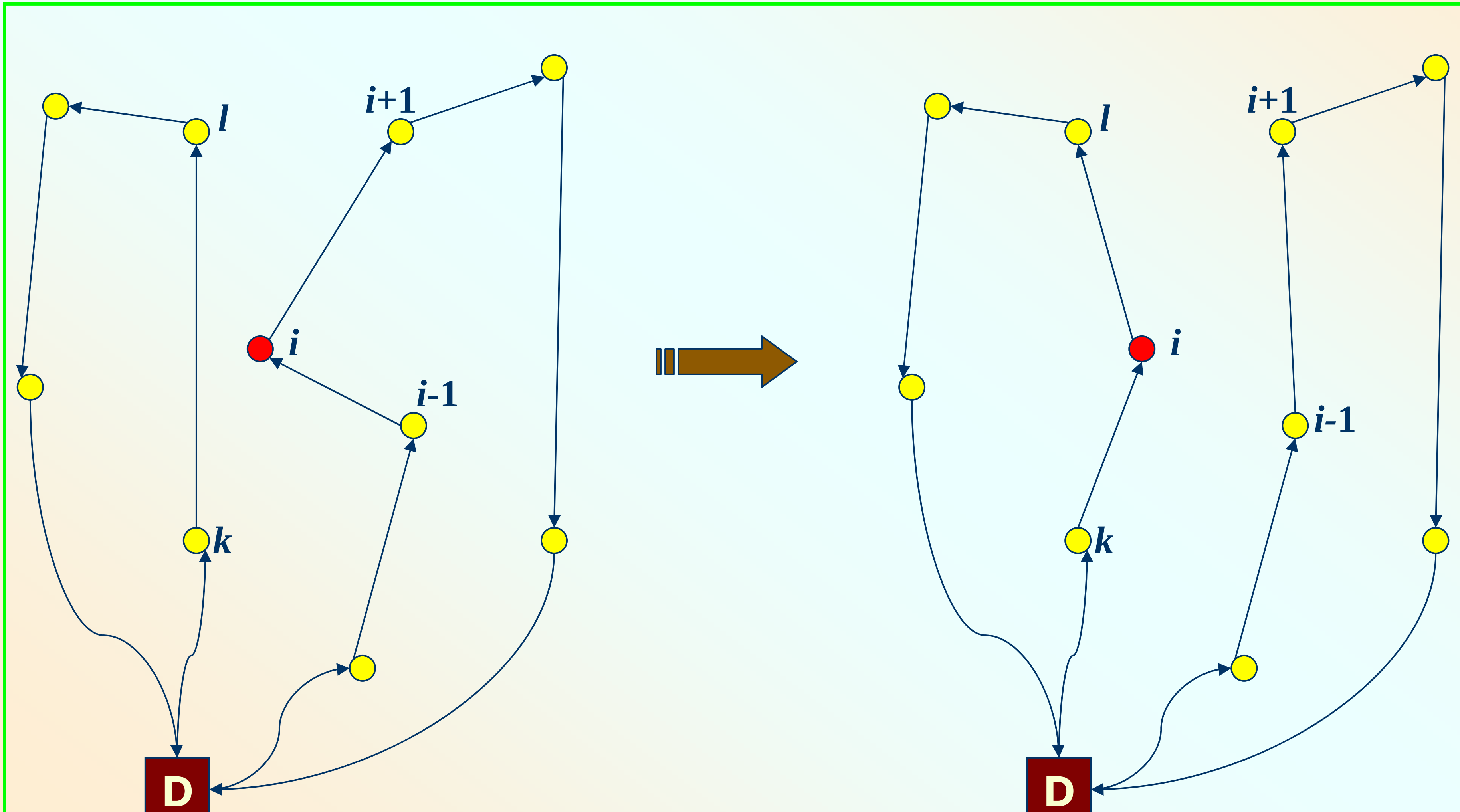
VRPPTD: Maximize the NET PROFIT (cont.)

1-0 move LPO on the same route



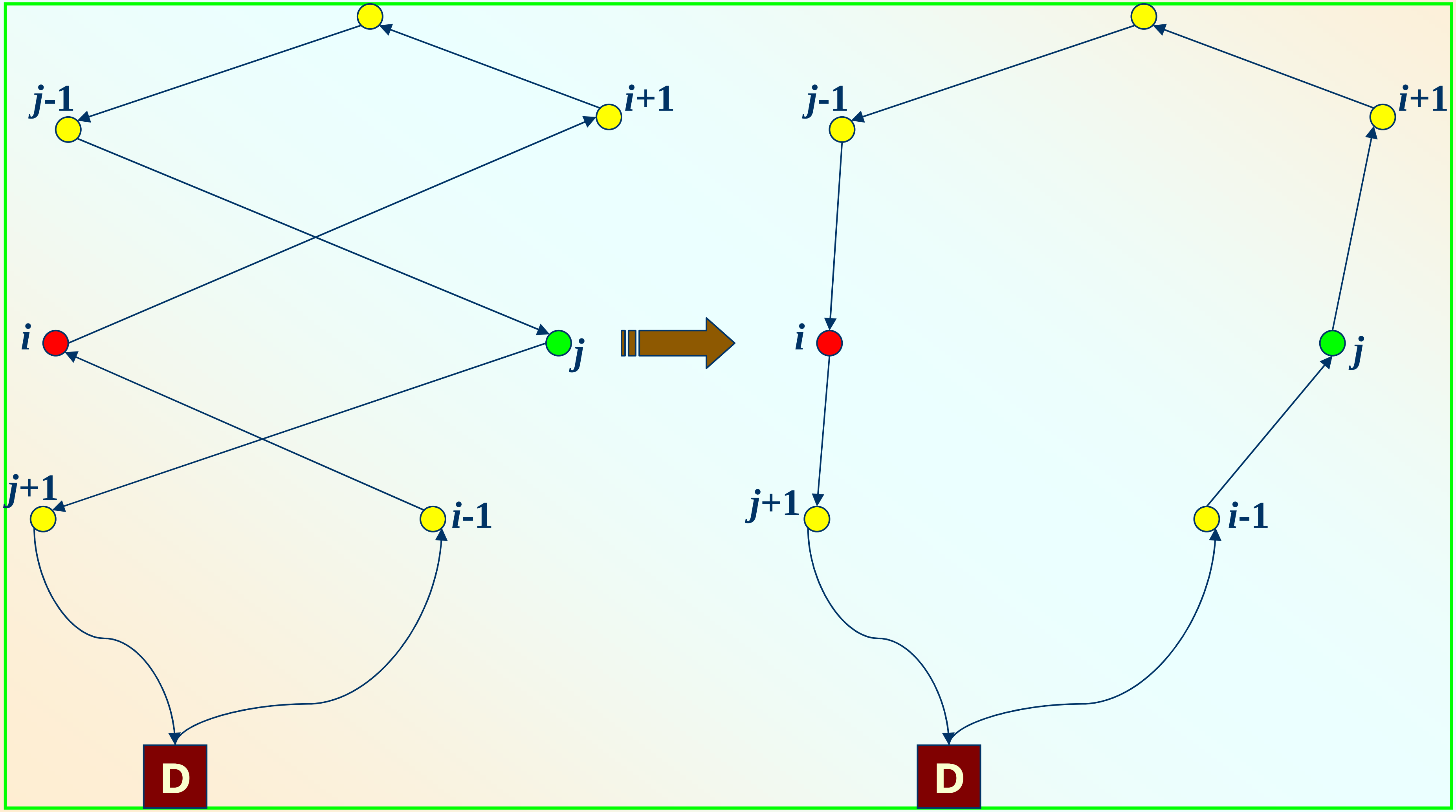
VRPPTD: Maximize the NET PROFIT (cont.)

1-0 move LPO on 2 routes



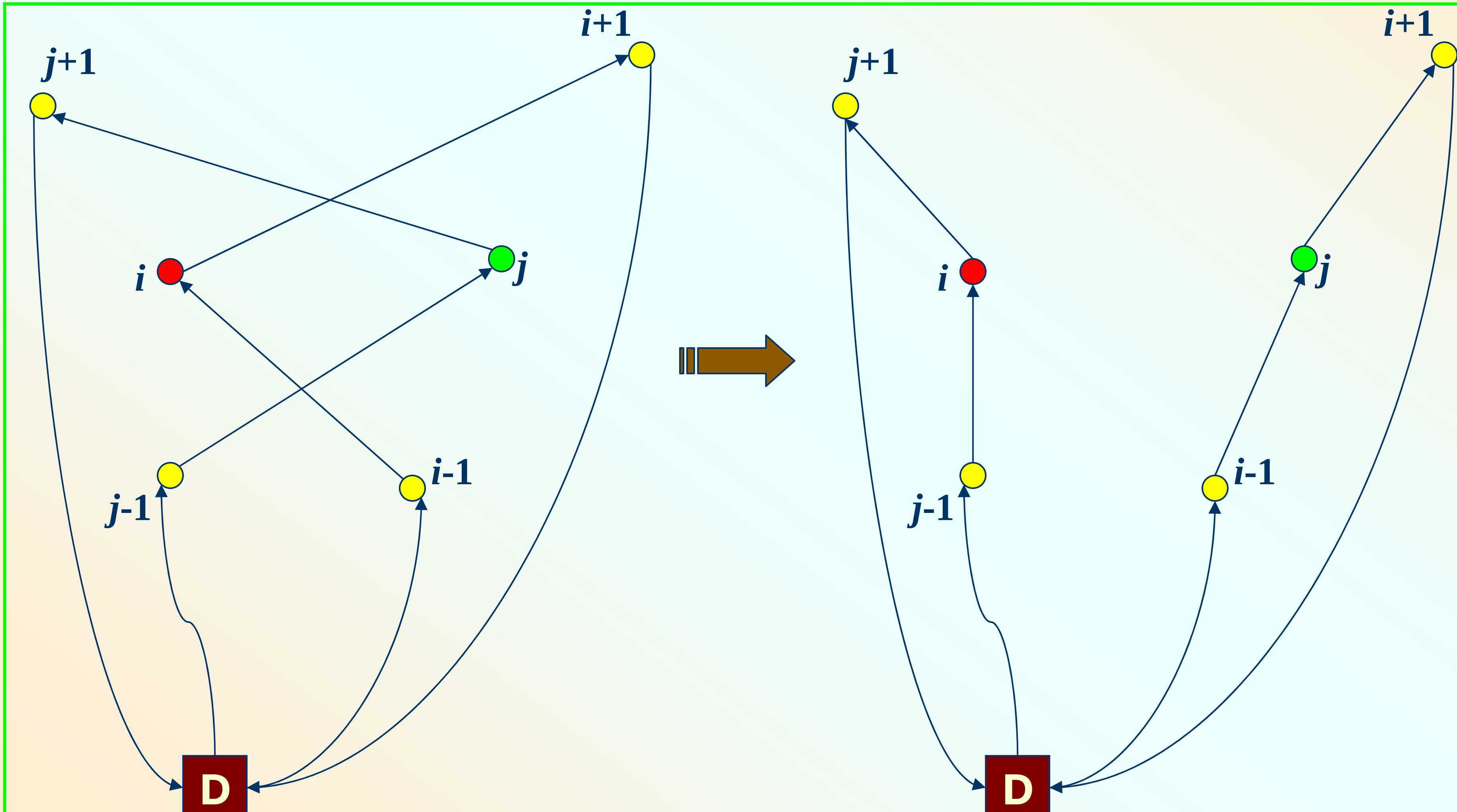
VRPPTD: Maximize the NET PROFIT (cont.)

1-1 exchange LPO on the same route



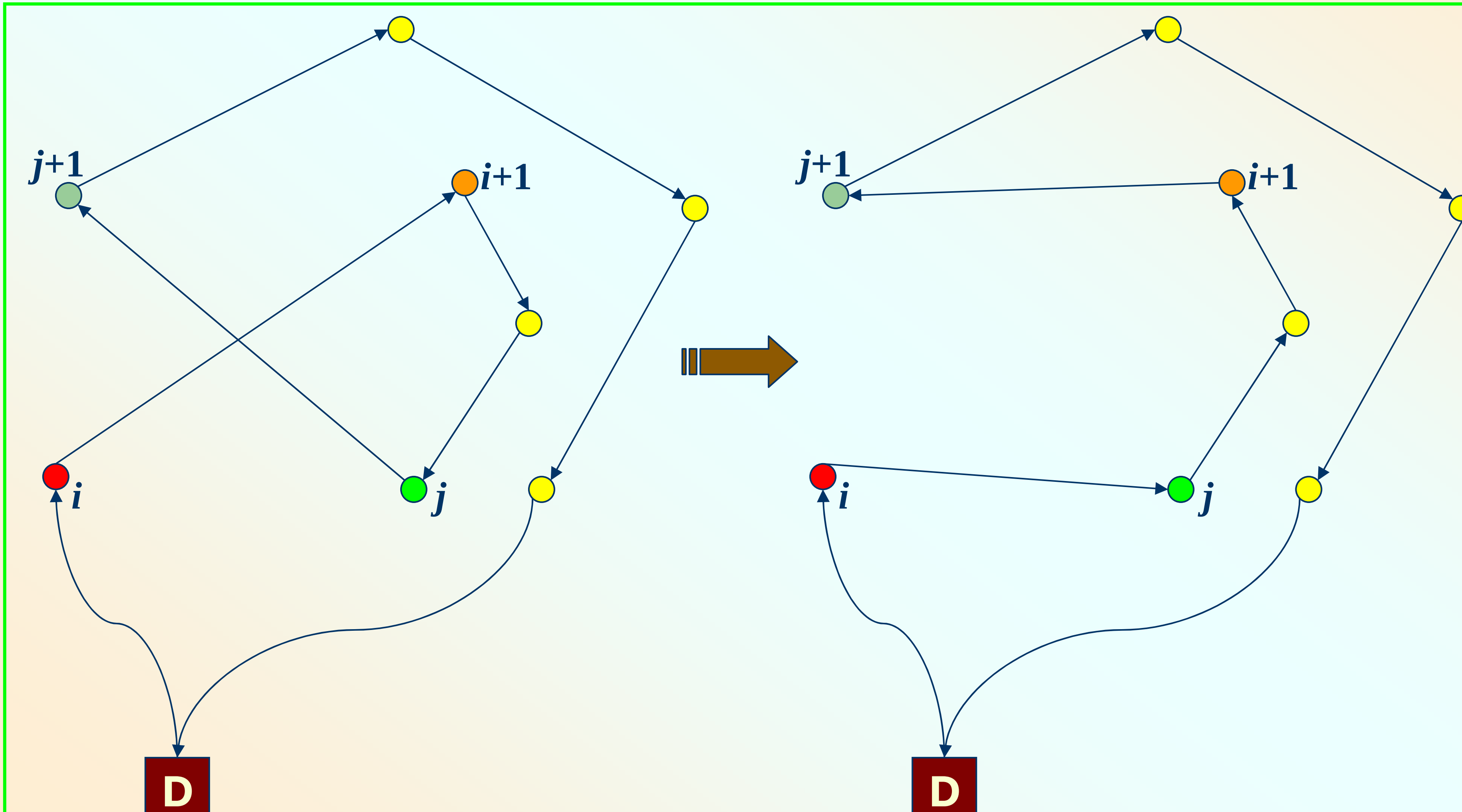
VRPPTD: Maximize the NET PROFIT (cont.)

1-1 exchange LPO on 2 routes



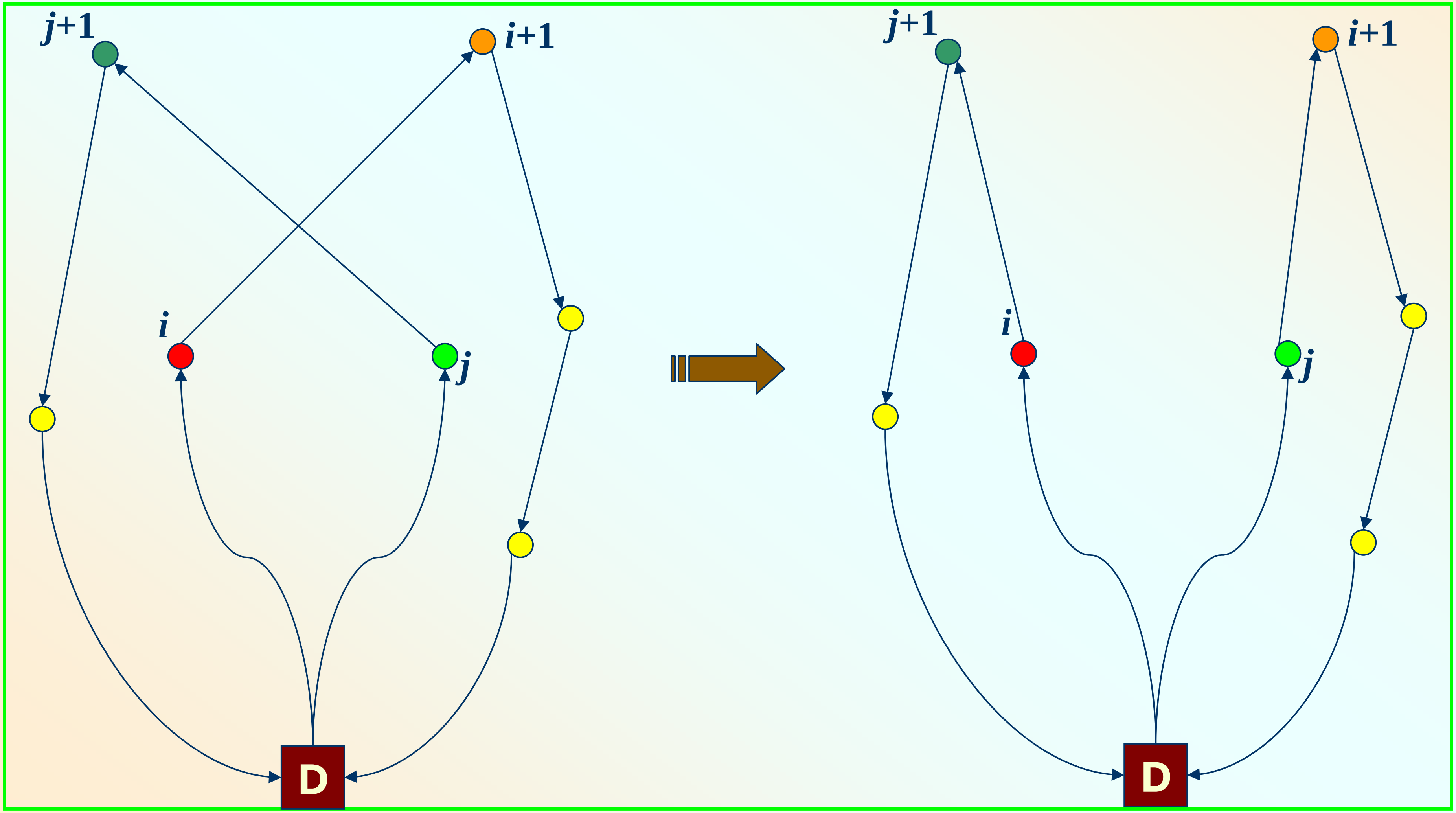
VRPPTD: Maximize the NET PROFIT (cont.)

2-Opt LPO on the same route



VRPPTD: Maximize the NET PROFIT (cont.)

2-Opt LPO on 2 routes

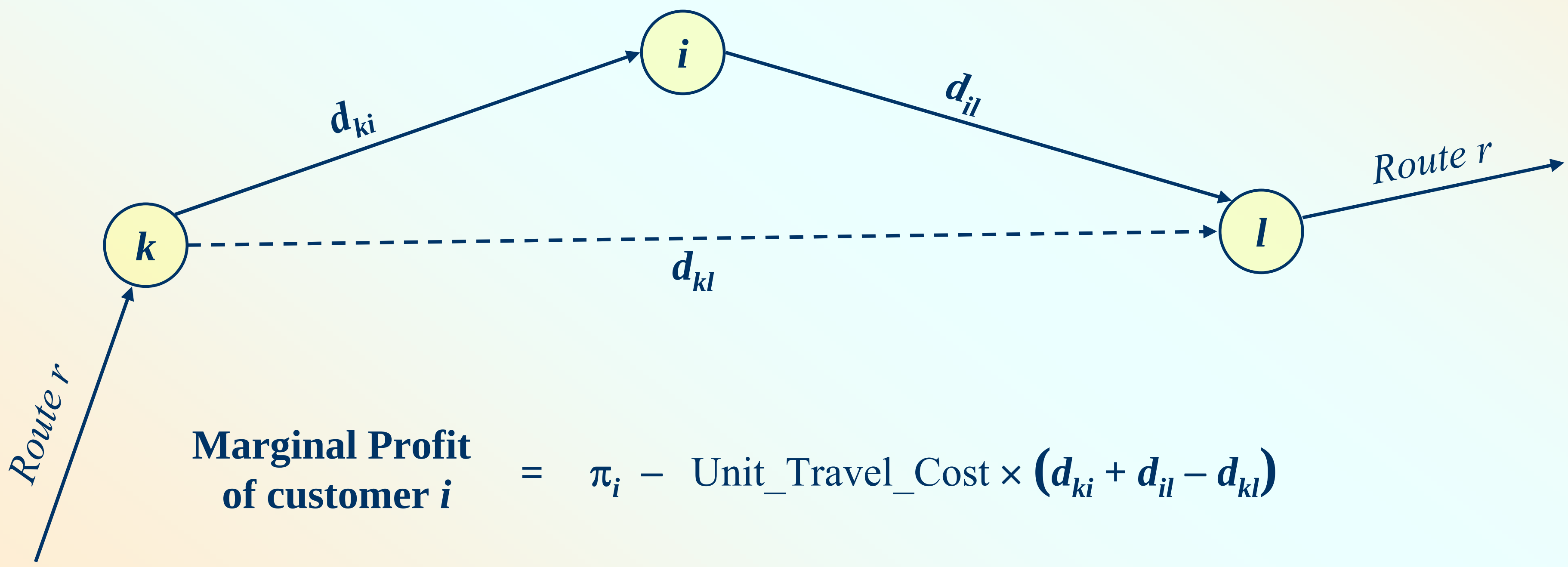


VRPPTD: Maximize the NET PROFIT (cont.)

Is customer *i* worth keeping (inserting) between nodes *k* and *l* ?

Triangular Inequality holds true even if distances are not perfectly Euclidean.

Predecessor of customer node i : node k
Successor of customer node i : node l



Marginal Profit of customer *i* $= \pi_i - \text{Unit\_Travel\_Cost} \times (d_{ki} + d_{il} - d_{kl})$

Algorithm iterative_Marginal_Profit_Analysis [iMPA]

Initialization

For the current set of routes ***R*** compute the marginal profit value $\mathbf{M}\pi_i$ for each customer in the present set of customers ***C*** . Sort the $\mathbf{M}\pi_i$ values in ***nondecreasing order*** and obtain a sorted stack $\mathbf{M}\pi_{[i]}$.

Step 2

If $\mathbf{M}\pi_{[0]}$, the 1st (*thus*: **LOWEST**) marginal profit in the stack, which belongs to customer $i_{[0]}$ is positive then go to ***Step 5***! O/w go to ***Step 3*.**

Step 3

Let $succ_i$ and $pred_i$ be the successor and predecessor nodes of customer $i_{[0]}$ on its current route r_i, resp. Delete $i_{[0]}$ from r_i. Discard it also from ***C***. Update the $\mathbf{M}\pi$ values of $succ_i$ and $pred_i$ if they are customer nodes. Cancel route r_i and update ***R*** if $i_{[0]}$ was the only customer on it.

Step 4

Set $\mathbf{M}\pi$ value for $i_{[0]}$ to INFINITY such that it is put at the end of the stack.
Restore the nondecreasing order of $\mathbf{M}\pi$ values in the stack and go to ***Step 2***.

Step 5

Stop [iMPA].

Solving the VRPPTD with [iMPA]

Initialization

Solve the given problem instance as a total distance minimization VRPTD with the present (***nondeleted***) customers set ***C*** in it. Apply to the total distance minimizing set of routes ***R*** an LPO procedure of choice. In the resulting solution all customers in ***C*** will be visited.

Step 1

For the current set of routes ***R*** apply one round of the Marginal_Profit_Analysis [**MPA**].

Step 2

If [**MPA**] has not modified the current solution (*i.e.* [**MPA**] *has not discarded any customers from* ***C*** and has not changed *the current routes* ***R***,*)*
then go to ***Step 4***! O/w go to ***Step 3*****.**

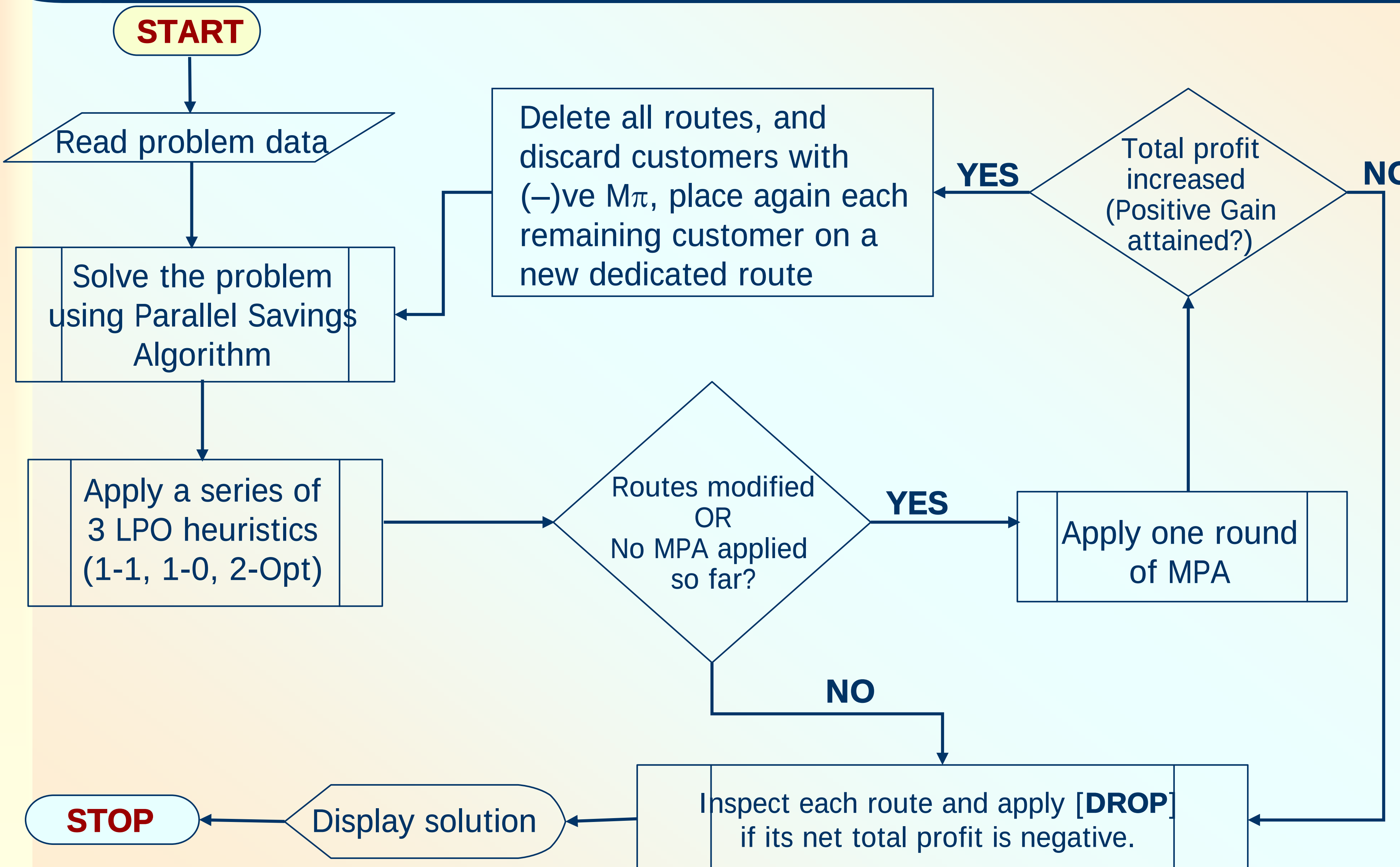
Step 3

Reset all routes and marginal profit values found in the previous steps. Delete the customer nodes that are found to be deleted in ***Step 2*** and go to ***Initialization*** to resolve the problem.

Step 4

Scan each final route and check if its net total profit value is positive. If not, then correct the route by using **DROP** heuristic until its net total profit becomes positive AND the disposal of the lowest marginal profit customer on it does not improve its net total profit anymore.

Flow Chart of solving VRPPTD with [iMPA]



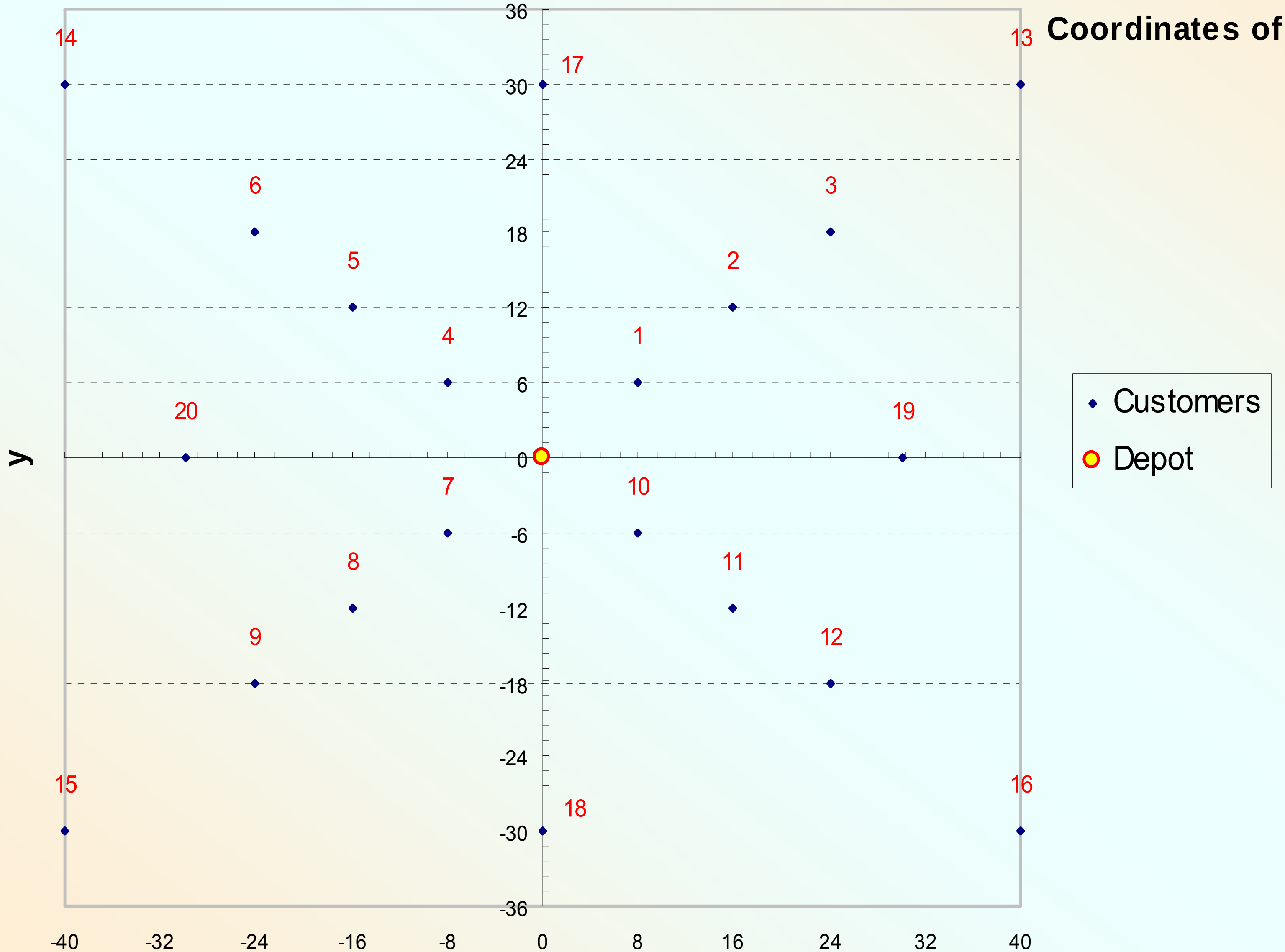
A Numerical Example solved: test0.txt

CUST#	XCOORD.	YCOORD.	DEMAND	READY	DUE	SERVICE	UNIT PROFIT
0	0	0	0	0	100000	0	0
1	8	6	1	0	100000	3	20
2	16	12	1	0	100000	3	20
3	24	18	1	0	100000	3	20
4	-8	6	1	0	100000	3	20
5	-16	12	1	0	100000	3	20
6	-24	18	1	0	100000	3	20
7	-8	-6	1	0	100000	3	20
8	-16	-12	1	0	100000	3	20
9	-24	-18	1	0	100000	3	20
10	8	-6	1	0	100000	3	20
11	16	-12	1	0	100000	3	20
12	24	-18	1	0	100000	3	20
13	40	30	1	0	100000	3	45
14	-40	30	1	0	100000	3	45
15	-40	-30	1	0	100000	3	45
16	40	-30	1	0	100000	3	45
17	0	30	1	0	100000	3	15
18	0	-30	1	0	100000	3	15
19	30	0	1	0	100000	3	15
20	-30	0	1	0	100000	3	15

Uniform Vehicle Capacity: 6

A Numerical Example: test0.txt (cont.)

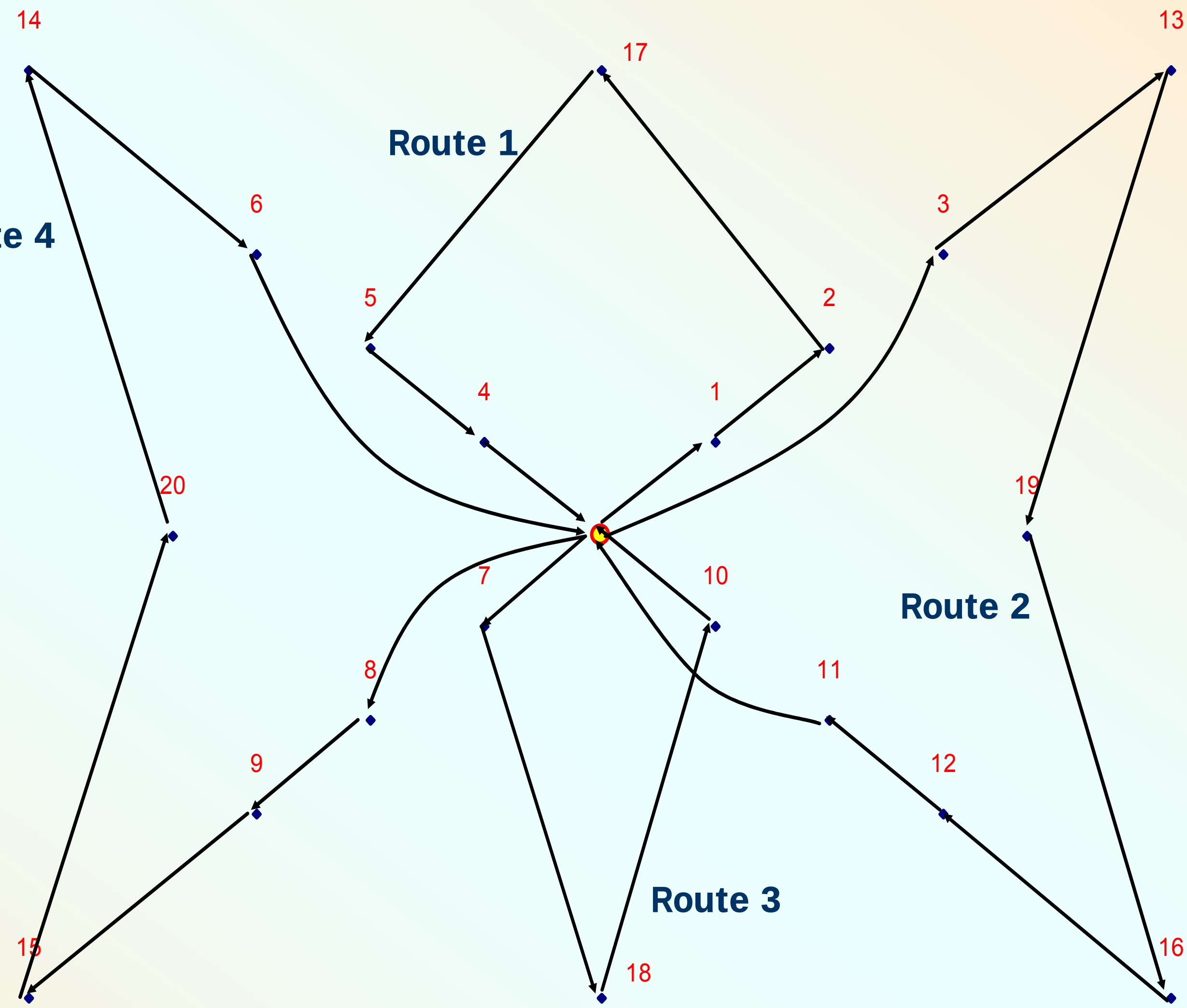
Coordinates of nodes



Aksen

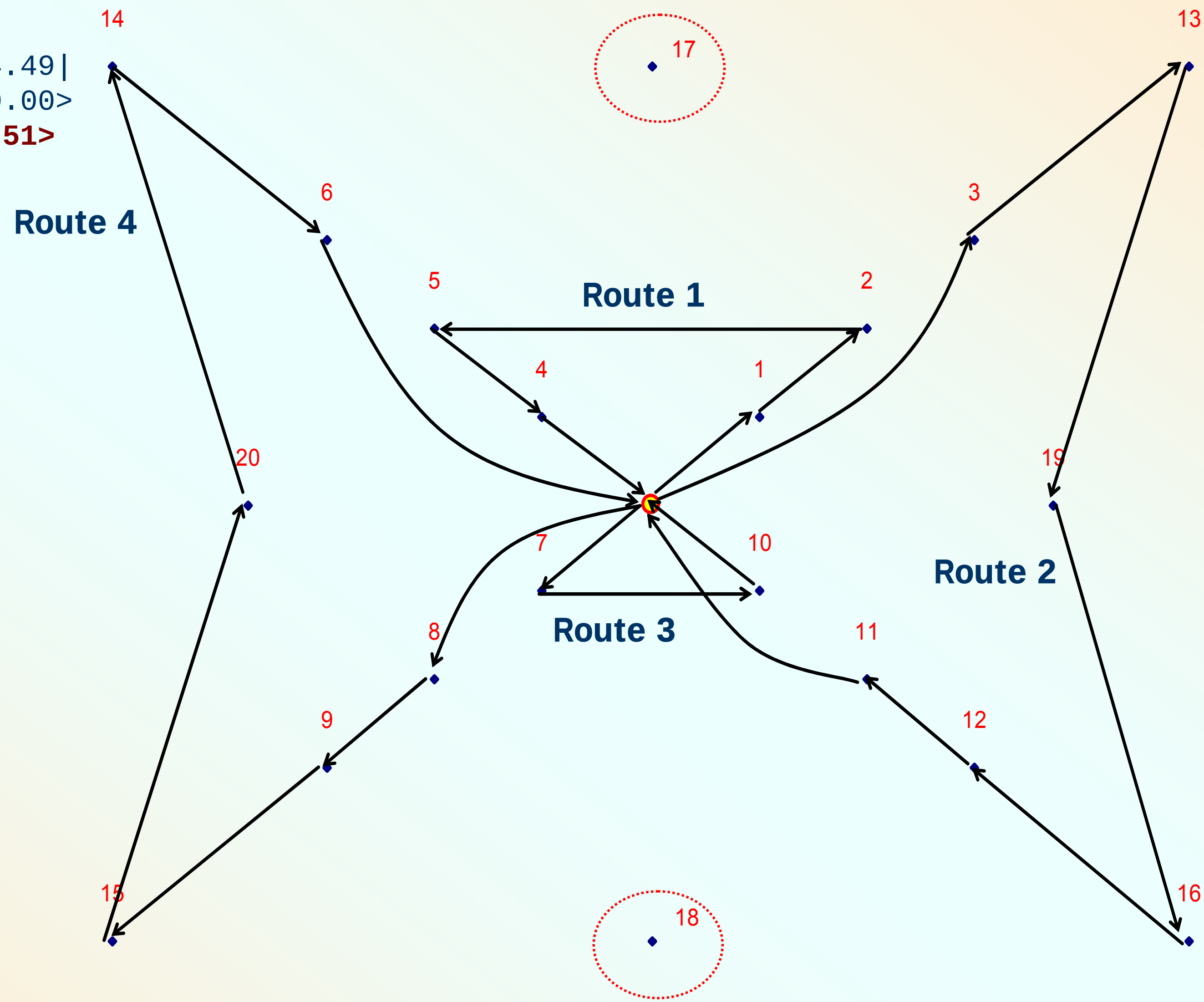
Before the MPA

No customers lost : (0)
No.required veh's : 4
Total Distance : |485.25|
Total Gross Profit : <480.00>
Total Net Profit : <-5.25>



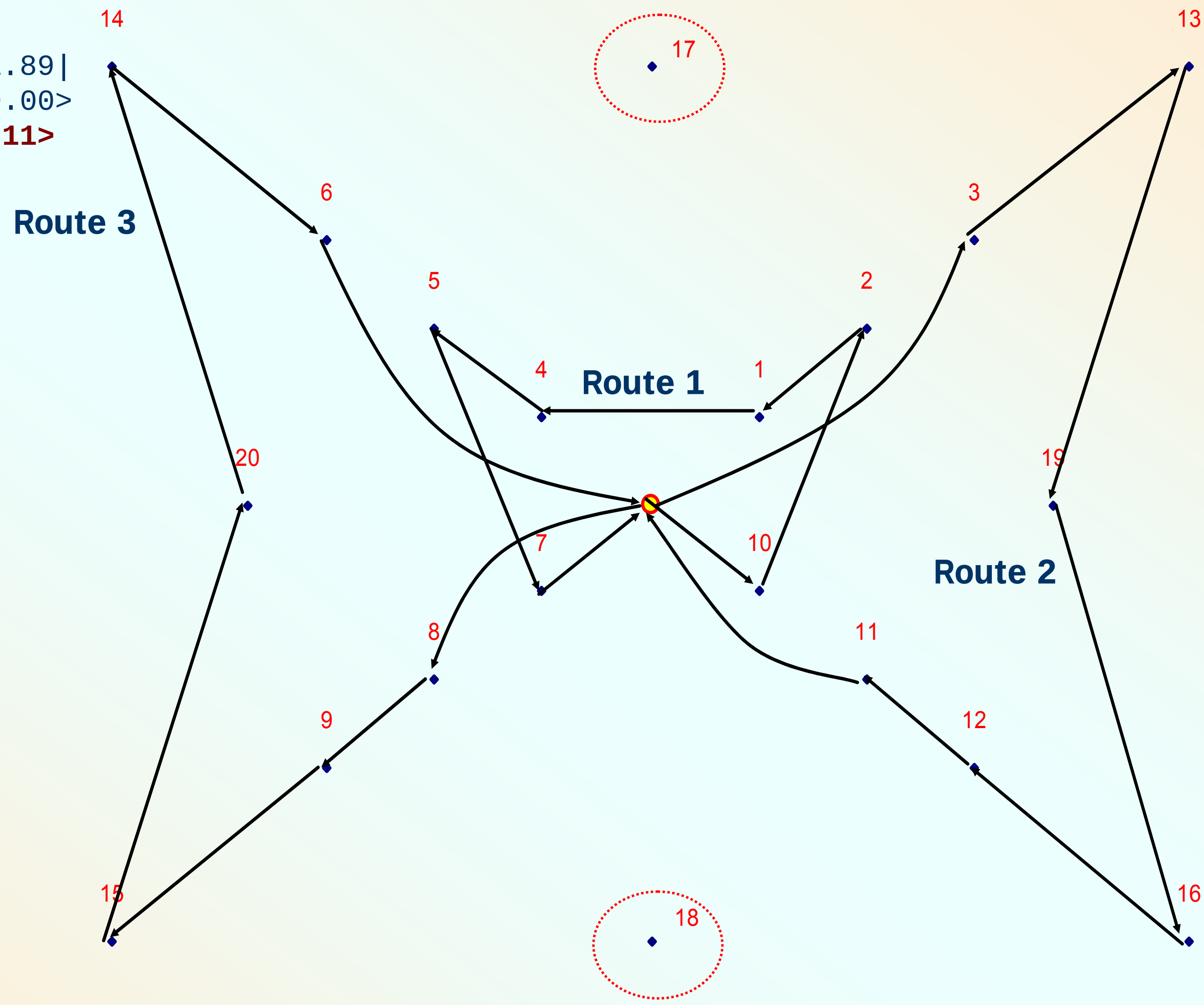
After one sweep of the MPA

```
No customers lost   : (2)
No.required veh's   : 4
Total Distance      : |434.49|
Total Gross Profit  : <450.00>
```
Total Net Profit : <15.51>



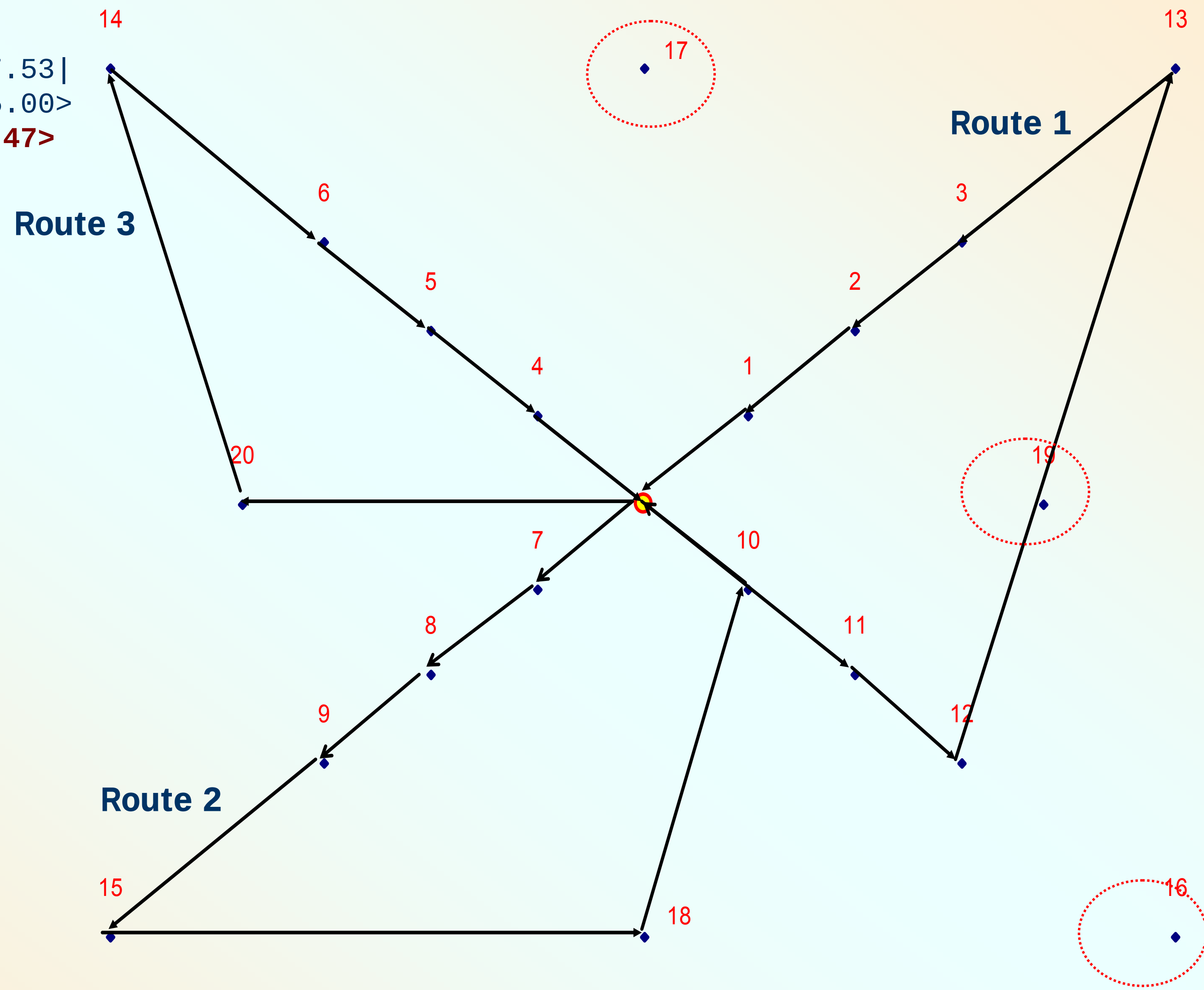
At the end of the MPA (2 sweeps)

```
No customers lost   : (2)
No.required veh's   : 3
Total Distance      : |421.89|
Total Gross Profit  : <450.00>
```
Total Net Profit : <28.11>



CPLEX 9.0 Optimal Solution
(148.9 sec on Pentium 4 3.40 GHz with 2 GB RAM)

```
No customers lost   : (2)
No.required veh's   : 3
Total Distance      : |367.53|
Total Gross Profit  : <405.00>
Total Net Profit    : <37.47>
```



Solving the VRP Part with Simulated Annealing

Prob. No.	No.cust. (n)	Best Profit by CPLEX 8.1	CPU Time by CPLEX [sec]	Proven Optimal?	[iMPA] w/ Parallel Savings	[iMPA] w/ SA
1	10	312.81	3.687	YES	308.68	312.81
2	10	100.41	10.281	YES	87.68	91.81
3	10	108.53	0.400	YES	97.99	97.99
4	10	191.06	0.300	YES	181.92	181.92
5	10	213.14	0.410	YES	185.48	194.55
6	10	25.54	0.330	YES	0.03	25.54
7	15	26.97	73.890	NO	24.92	24.928
8	15	45.10	17.015	NO	21.86	21.86
9	15	17.23	6.406	YES	11.56	11.56
10	20	90.72	40.578	NO	90.72	90.72
11	20	247.23	~ 2hr	NO	230.45	230.45
12	20	186.61	~ 2hr	NO	108.65	156.71

Future Research

Alternative Use of [iMPA] in a Greedy ADD-DROP Heuristic

Initialization- ADD *(starts with empty customer subset **C**)*

Preprocess customers by computing for each of them a ***preferability coefficient*** $\boldsymbol{PC_i}$.

$$PC_i = \alpha\left(\frac{\pi_i}{D_i}\right) + \beta\left(\pi_i - UNITCOST \times \min_{\substack{j\in N \\ i\neq j}}\{d_{ij}\}\right) + \gamma\left(\frac{\max_{j\in N\setminus\{0\}}\{d_{0j}\}}{d_{0i}}\right) + \delta\left(\frac{|NEIGH(i,R)|}{n}\right)$$

Sort the $\boldsymbol{PC_i}$ values in ***nonincreasing order*** and obtain a sorted stack $\boldsymbol{PC_{[i]}}$ with the customer [0] being the most *preferable* one for addition to subset ***C***. Set *i* = 0, BEST_PROFIT=0.

Step 1

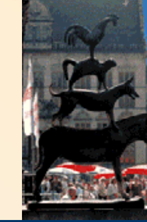
Add customer [*i*] of the stack $\boldsymbol{PC_{[i]}}$ to ***C***. On this current subset of customers, solve the VRPTD problem using Clarke-Wright Parallel Savings Algorithm with LPO to obtain the set of routes ***R***.

Step 2

On the current set of routes ***R***, apply the [iMPA] algorithm to find the best possible profit PROFIT realizable with ***C*** and ***R***. If PROFIT > BEST_PROFIT, then set BEST_PROFIT = PROFIT and record ***C*** and ***R***.

Step 3

Let *i* = *i* + 1. If *i* = *n*, then stop! Otherwise, go to **Step 2**.



Questions & Comments?

- Optimal Solution of test0.vrp
- Computational Results
- Algorithmic Flowchart
- No-Profit C-W Solution of test0.vrp