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Solving the Multi-Depot Location-Routing Problem using Augmented Lagrangian Relaxation

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AGENDA

- PROBLEM DEFINITION
- LITERATURE REVIEW
- MDLRP MODEL
- SOLUTION METHOD
- COMPUTATIONAL RESULTS
- CONCLUSION

Location-Routing Problem(LRP)

- Multi depot LRP involves
  - determining optimal numbers and locations of depots
  - allocating customers to depots
  - determining the vehicle routes to visit all customers

- Objective: Minimize total of
  - Depot opening cost
  - Depot operating cost
  - Vehicle acquisition cost
  - Total travelling cost

Problem Definition

- Which depots to open among candidate depots?
- Which depots to close among present depots?
- Which customers will be served by which depot?
- How many vehicles will be acquired by each depot?
- What will be the routes of vehicles?

- Facility Location-Allocation Problem (FLAP)
  (Strategic Level)
- Vehicle Routing Problem (MDVRP)
  (Operational Level)

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Literature Review

- The interdependence of facility location-allocation and vehicle routing part of the problem is known for long.
  - Effect of ignoring routes when locating depots stated by Salhi and Rand *(EJOR, 1989)*
- But due to complexity of the problem these two steps were solved separately.
- In recent years both heuristic and exact solution methods are proposed for the combined LRP.

Literature Review

- Literature Surveys on the LRP
  - Nagy and Salhi *(EJOR, 2007)*
  - Ahipaşaoğlu et al. *(Working Paper, Bilkent Uni, 2005)*
  - Min, Jayaraman, and Srivastava *(EJOR, 1998)*

- Exact Solution Approaches
  - Laporte, Nobert, and Taillefer *(Trans Sci, 1988)*
    - transform the MDVRP and LRP into constrained assignment problem
    - solve by the branch-and-bound method

Literature Review

- Heuristic Solution Approaches
  - Perl and Daskin *(J of Business Logistics, 1984)*
  - Perl and Daskin *(Trans Res-B, 1985)*
    - Route first, location-allocation second
  - Hansen, Hegedahl, Hjortkjær, and Obel *(EJOR, 1994)*
  - Tüzün and Burke *(EJOR, 1999)*
    - Two phase tabu search
  - Wu, Low, and Bai *(Comp. & OR, 2002)*
    - Decompose into FLAP and VRP
    - Solve with simulated annealing

Literature Review

- Albareda-Sambola, Diaz, and Fernandez *(Comp. & OR, 2005)*
  - Generate lower bound from linear relaxation
  - Tabu search
- Melechovsky, Prins, and Calvo *(J of Heuristics, 2005)*
  - Nonlinear depot operating cost
  - Hybrid metaheuristic,
    tabu search + variable neighborhood search

- Math. Prog. Formulation of a 4-tier (3-echelon) LRP
  - Ambrosino and Scutellà *(EJOR, 2005)*

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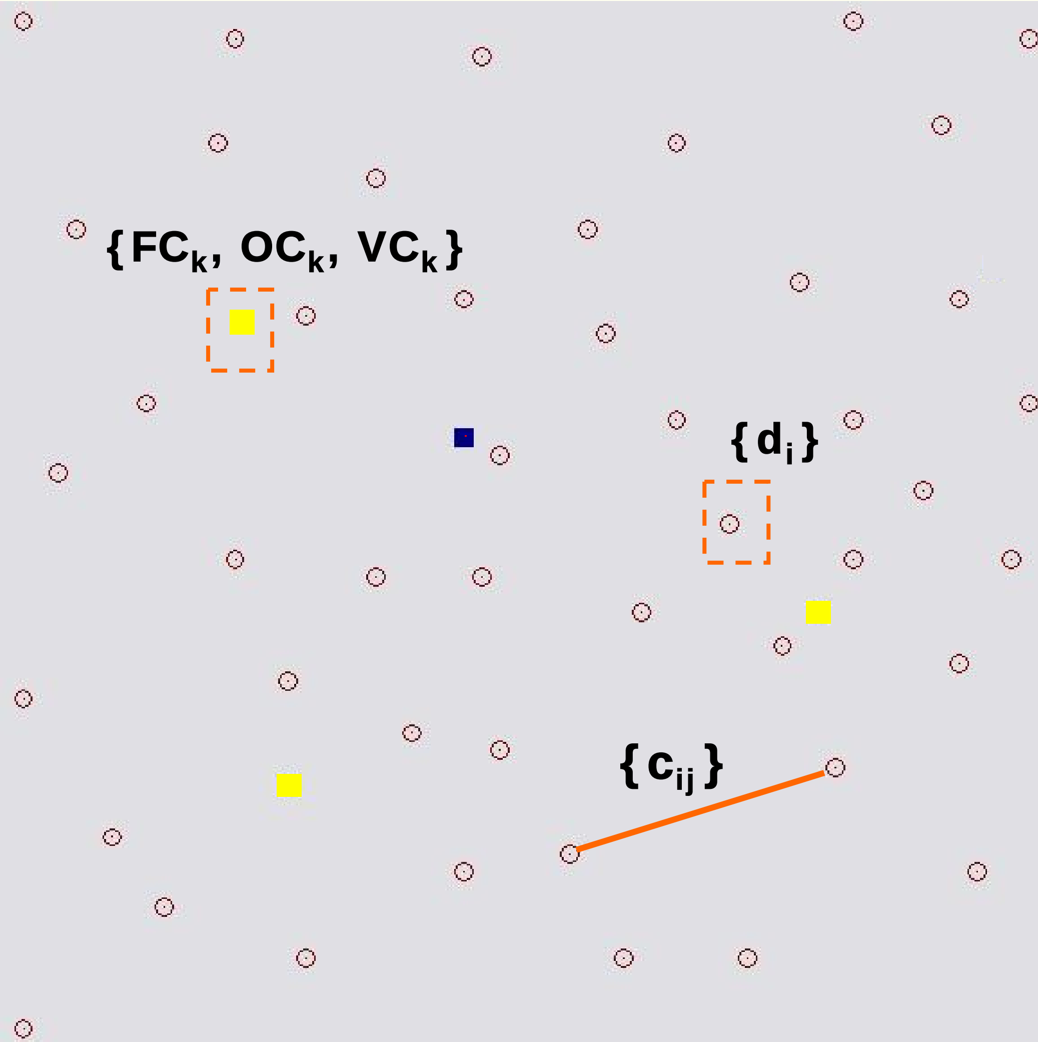
MDLRP

- Depot sites:
  - Candidate - Present
  - Opening - Closing cost
  - Operating cost
- Customer nodes:
  - Demand
- Vehicles
  - Homogeneous
  - Aquisition cost



Notation

- Sets:

  IC : set of customers

  ID : set of depots

  ID_{pres}: set of already existing depots, ID_{cand}: set of candidate depots

  I : set of all nodes (IC $\cup$ ID)

- Binary Decision Variables:

  x_{ijk} : 1 if node j is visited after node i on a route originating from depot k

  y_k : 1 if depot k is opened for $k \in ID_{cand}$, if depot k is preserved for $k \in ID_{pres}$

  δ_{ik} : 1 if customer i is assigned to depot k

Notation

- Parameters:

FC_k	: the opening or closing cost of depot k
OC_k	: the operating cost of depot k
VC_k	: vehicle acquisition cost for depot k
c_{ij}	: traveling cost of one vehicle from node i to node j
Q	: vehicle capacity
M	: a very big number

MDLRP Model

- Objective Function:

$$\mathbf{P}: \quad \min \sum_{k \in ID} OC_k y_k + \sum_{k \in ID_{cand}} FC_k y_k + \sum_{k \in ID_{pres}} FC_k (1 - y_k) +$$

depot operating cost

depot opening - closing cost

$$\sum_{k \in ID} \sum_{j \in IC} VC_k x_{kjk} + \sum_{k \in ID} \sum_{i \in I} \sum_{\substack{j \in I \\ i \neq j}} c_{ij} x_{ijk}$$

vehicle acquisition cost

travelling cost

- Combination of objectives of:
  - Facility location-allocation problem (FLAP)
  - Multi-depot vehicle routing problem (MDVRP)

Model

- Allocation of customers to depots

$$\sum_{k \in ID} \delta_{ik} = 1 \qquad \forall i \in IC \qquad (1)$$

- Flow conservation constraints

$$\sum_{\substack{j \in IC \cup \{k\} \\ j \neq i}} x_{ijk} = \delta_{ik} \qquad \forall i \in IC, \forall k \in ID \quad (2)$$

$$\sum_{\substack{j \in IC \cup \{k\} \\ j \neq i}} x_{jik} = \delta_{ik} \qquad \forall i \in IC, \forall k \in ID \quad (3)$$

$$\sum_{i \in IC} x_{ikk} = \sum_{i \in IC} x_{kik} \qquad \forall i \in ID, \forall k \in ID \quad (4)$$

Model

- Connectivity and Subtour elimination constraints

$$\sum_{k\in ID}\sum_{i\in IC} x_{kik} + \sum_{k\in ID}\sum_{i\in IC}\sum_{\substack{j\in IC \\ i\neq j}} x_{ijk} = |I| \qquad (5)$$

$$\sum_{k\in ID}\sum_{i\in S}\sum_{j\in S} x_{ijk} \leq |S| - L(S) \qquad \forall S \subseteq IC,\ L(S) = \left\lceil \frac{1}{Q}\sum_{i\in S} d_i \right\rceil \qquad (6)$$

- Constraints to avoid assignment of customers to unestablished depots

$$\sum_{i\in IC} \delta_{ik} \leq |IC|\, y_k \qquad \forall k \in ID \qquad (7)$$

Model

- Integrality and nonnegativity constraints

$$x_{ijk}, \delta_{ik}, y_k \in \{0,1\} \qquad \forall i \in I, \forall j \in I, \forall k \in ID$$

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Lagrangian Relaxation

- Decomposition approach used for solving a variety of $\mathcal{NP}$-hard problems
- True optimal solution is bracketed between a lower bound and an upper bound
- In case of minimization
  - Lower bound: Solution of the Lagrangian Relaxed Problem
  - Upper bound: A good feasible solution obtained by a heuristic method
- Quality of this heuristic solution is measured with the percentage gap between lower and upper bound

Lagrangian Relaxation

- Some of the constraints are multiplied with "Lagrangian multipliers" (λ) and added to objective function

$$\text{P}: \min \quad f(x) \qquad \qquad \text{P}': \min \quad f(x,\lambda)$$
$$\text{s.t.} \quad Ax \le b \qquad \qquad \text{s.t.} \quad A'x \le b'$$

- Let optimal solution to P be Z^*

$$f_{\min}(x,\lambda) \le Z^*$$

- If we find λ maximizing $f_{min}(x,\lambda)$

$$f_{\min}(x,\lambda^*) = \max_{\lambda} f(x,\lambda)$$

$$f_{\min}(x,\lambda) \le f_{\min}(x,\lambda^*) \le Z^*$$

Lagrangian Relaxation to MDLRP

- Relax constraints that combine FLAP and MDVRP

$$\sum_{\substack{j \in IC \cup \{k\} \\ j \neq i}} x_{ijk} = \delta_{ik} \qquad \forall i \in IC \quad \leftarrow \lambda$$

$$\sum_{\substack{j \in IC \cup \{k\} \\ j \neq i}} x_{jik} = \delta_{ik} \qquad \forall i \in IC \quad \leftarrow \mu$$

$$ZLR(\lambda, \mu) = \sum_{k \in ID} OC_k y_k + \sum_{k \in ID_{cand}} FC_k y_k + \sum_{k \in ID_{pres}} FC_k (1 - y_k) +$$

$$\sum_{i \in ID} \sum_{j \in IC} VC_i x_{iji} + \sum_{k \in ID} \sum_{i \in I} \sum_{\substack{j \in I \\ j \neq i}} c_{ij} x_{ijk} +$$

$$\sum_{i \in IC} \sum_{k \in ID} \delta_{ik} (\mu_{ik} + \lambda_{ik}) \qquad \sum_{k \in ID} \sum_{i \in IC} \sum_{\substack{j \in IC \cup \{k\} \\ j \neq i}} (\delta_{ik} - x_{ijk}) \lambda_{ik} + \sum_{k \in ID} \sum_{j \in IC} \sum_{\substack{i \in IC \cup \{k\} \\ i \neq j}} (\delta_{jk} - x_{ijk}) \mu_{jk}$$

Relaxed Problem(LR)

- The terms in the objective function is rearranged to obtain a more compact form and objective of two subproblems separately
- Separate one part of the augmented terms

$$\sum_{i \in IC} \sum_{k \in ID} \delta_{ik} (\mu_{ik} + \lambda_{ik})$$

- 3-dimensional depot dependent cost matrix C_{new}

i. $(i, j) \in IC \times IC,\ i \neq j,\ k \in ID$: $(c_{ijk})_{new} = c_{ij} - \lambda_{ik} - \mu_{jk}$

ii. $(i, j) \in IC \times ID$: $(c_{ijj})_{new} = c_{ij} - \lambda_{ij}$

iii. $(i, j) \in ID \times IC$: $(c_{iji})_{new} = c_{ij} - \mu_{ji} + VC_i$

iv. $(i, j) \in ID \times IC,\ k \in ID,\ i \neq k$: $(c_{ijk})_{new} = +\infty$

v. $(i, j) \in IC \times ID,\ k \in ID,\ j \neq k$: $(c_{ijk})_{new} = +\infty$

Relaxed Problem(LR)

- Objective Function:

$$ZLR(\lambda,\mu)=\sum_{k\in ID} OC_k y_k + \sum_{k\in ID_{cand}} FC_k y_k + \sum_{k\in ID_{pres}} FC_k (1-y_k) +$$

$$\sum_{k\in ID}\sum_{i\in IC} \delta_{ik}(\lambda_{ik}+\mu_{ik}) + \sum_{k\in ID}\sum_{i\in I}\sum_{\substack{j\in I\\ j\neq i}} (c_{ijk})_{new}\, x_{ijk}$$

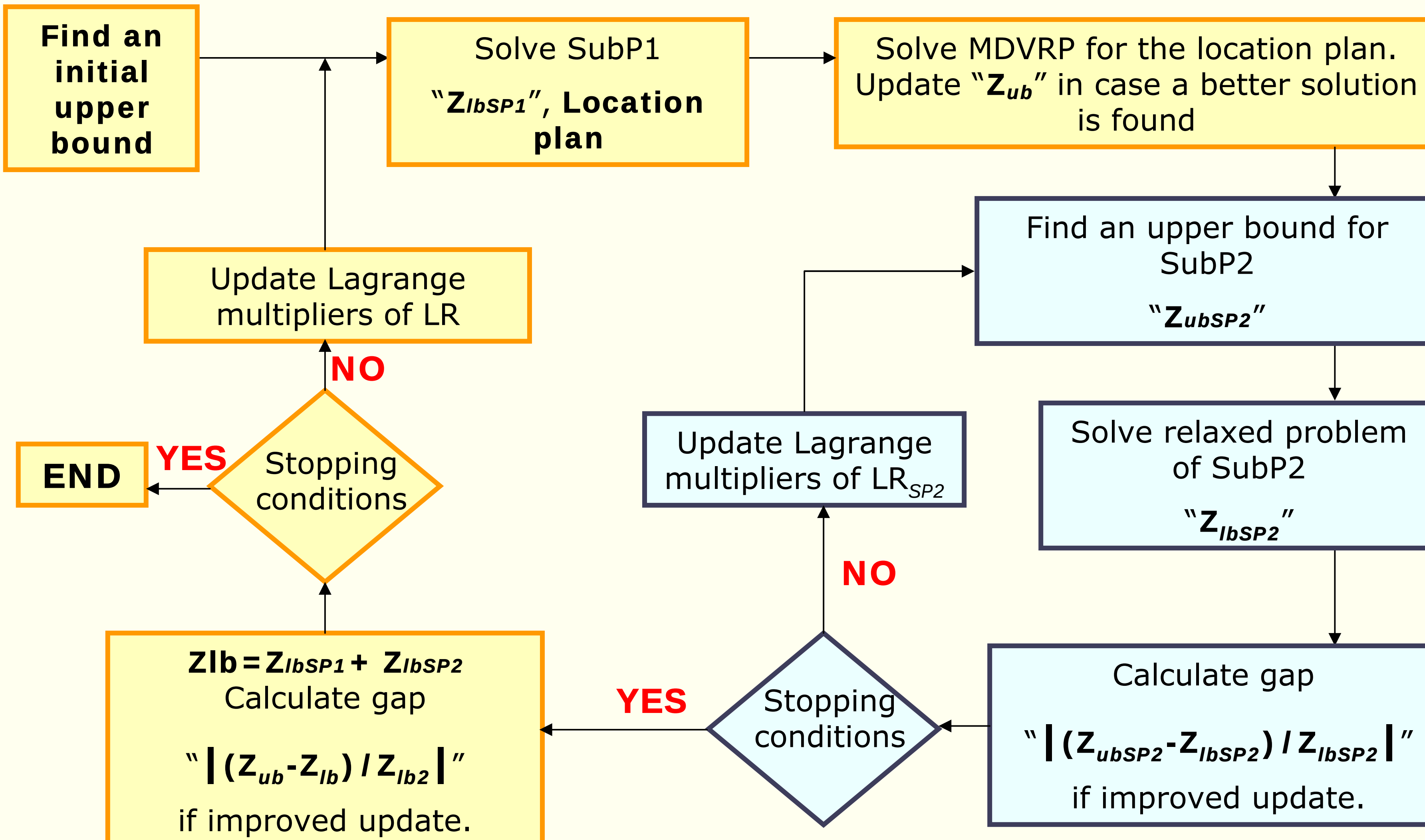
- Two subproblems:
  - **FLAP-like subproblem SubP1**
  - **Degree and Time Constrained MSF-like subproblem SubP2**

- Min $\sum$ Augmented FLAP obj + $\sum$ Augmented DCMSF obj

  s.t. Pure FLAP constraints (1),(7)

  Pure DCMSF constraints (2),(5),(6)

Solving LR



Find an initial upper bound
Solve SubP1
"Z_{lbSP1}", Location plan
Solve MDVRP for the location plan. Update "Z_{ub}" in case a better solution is found
Find an upper bound for SubP2
"Z_{ubSP2}"
Solve relaxed problem of SubP2
"Z_{lbSP2}"
Calculate gap
"$|(Z_{ubSP2}-Z_{lbSP2})/Z_{lbSP2}|$"
if improved update.
Stopping conditions
NO
Update Lagrange multipliers of LR_{SP2}
YES
$Zlb = Z_{lbSP1} + Z_{lbSP2}$
Calculate gap
"$|(Z_{ub}-Z_{lb})/Z_{lb2}|$"
if improved update.
Stopping conditions
NO
Update Lagrange multipliers of LR
YES
END

Subgradient Optimization in LR

- $Z_{lb} = Z_{lbSP1} + Z_{lbSP2}$
- Z_{ub} comes from heuristic solution: Tabu Search

$$(SG_{ik}^{\lambda})^q = (\delta_{ik})^q - \sum_{\substack{j \in IC \cup \{k\} \\ j \neq i}} (x_{ijk})^q \qquad \forall i \in IC, \forall k \in ID$$

$$(SG_{ik}^{\mu})^q = (\delta_{ik})^q - \sum_{\substack{j \in IC \cup \{k\} \\ j \neq i}} (x_{jik})^q \qquad \forall i \in IC, \forall k \in ID$$

$$\left\| SG \right\|^2 = \left\| SG^{\lambda} \right\|^2 + \left\| SG^{\mu} \right\|^2$$

$$s^q = \Lambda^q \frac{Z_{ub}^q - Z_{LR}^q(\lambda, \mu)}{\left\| SG \right\|^2}$$

$$(\lambda_{ik})^{q+1} = (\lambda_{ik})^q + s^q (SG_{ik})^q \qquad \forall i \in IC, \forall k \in ID$$

$$(\mu_{ik})^{q+1} = (\mu_{ik})^q + s^q (SG_{ik})^q \qquad \forall i \in IC, \forall k \in ID$$

SubP1: FLAP like subproblem

- SubP1: Uncapacitated Facility Location Allocation Problem
  - Lagrange multipliers → allocation costs
  - Unimodularity ⇒ δ_{ik} can be defined as continuous variables

$$Z_{SubP1} = \sum_{k \in ID} OC_k y_k + \sum_{k \in ID_{cand}} FC_k y_k + \sum_{k \in ID_{pres}} FC_k (1 - y_k) +$$

$$\sum_{k \in ID} \sum_{i \in IC} \delta_{ik} (\lambda_{ik} + \mu_{ik})$$

$$\sum_{k \in ID} \delta_{ik} = 1 \qquad \forall i \in IC$$

$$\sum_{i \in IC} \delta_{ik} \le |IC| \, y_k \quad \forall k \in ID$$

⇒

$$\delta_{ik} \le 1 \qquad \forall i \in IC, \forall k \in ID$$

$$\delta_{ik} \le y_k \qquad \forall i \in IC, \forall k \in ID$$

$$\delta_{ik} \ge 0, \; y_{ik} \in \{0,1\} \; \forall i \in IC, \forall k \in ID$$

SubP1: FLAP like subproblem

Solving SubP1

- At each iteration of the subgradient optimization, allocation cost changes acc. to Lagrange multipliers
- Sub1 is solved with CPLEX 9.1 (or CPLEX 10.0) in acceptable time
  - 1000 customers and 20 facilities in 2.84 seconds

SubP2: DCMSF-like subproblem

$$\text{Min } Z_{SubP2} = \sum_{k \in ID} \sum_{i \in I} \sum_{\substack{j \in I \\ j \neq i}} (c_{ijk})_{new} \, x_{ijk}$$

- Balance of degree constraints at depot nodes
- Connectivity constraint
- Subtour elimination constraints

- Minimum Spanning Forest Problem
  - Minimum outdegree at center nodes
  - Balance of in- and out-degrees at center nodes

SubP2: DCMSF-like subproblem

- SubP2 is also $\mathcal{NP}$-hard!
  - Garey and Johnson (1979) - The general *DCMST* with arbitrary degree constraints on nodes other than the center is $\mathcal{NP}$-hard.
- An augmented Lagrangian relaxation is applied to SubP2
  - One part of the subtour elimination constraints

$$\sum_{k \in ID} \sum_{i \in S} \sum_{\substack{j \in S \\ j \neq i}} x_{ijk} \leq |S| - 1 \qquad \forall S \subseteq IC \;\; \exists |S| \geq 2$$

$$\sum_{k \in ID} \sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - \mathrm{L}_S \quad \forall S \subseteq IC, |S| \geq 2$$

$$\sum_{k \in ID} \sum_{i \in IC} x_{kik} \geq \mathrm{L}_I$$

SubP2: DCMSF-like subproblem

$$\sum_{k\in ID}\sum_{i\in S}\sum_{j\in S} x_{ijk} \le |S| - \mathrm{L}_S \qquad \forall S \subseteq IC, |S| \ge 2 \quad \longleftarrow \quad \alpha$$

$$\text{Min } Z_{LR}^{SubP2}(\alpha) = \sum_{k\in ID}\sum_{i\in IC}\left[\left(c_{kik}\right)^{new}\right] x_{kik} + \sum_{k\in ID}\sum_{i\in IC}\left[\left(c_{ikk}\right)^{new}\right] x_{ikk} + \sum_{k\in ID}\sum_{i\in IC}\sum_{\substack{j\in IC \\ j\neq i}}\left[\left(c_{ijk}\right)^{new} - \sum_{s\in G_{ij}} \alpha_s\right] x_{ijk} + \sum_{s\in\psi}\left(|S| - \mathrm{L}_S\right)\alpha_s$$

SubP2: DCMSF-like subproblem

- Solving the Lagrangean Relaxed SubP2
  - Modified Prim's algorithm
  - Adaptation to MSF
  - Minimum out-degree constraint at center nodes
  - Balance of degrees at center nodes

- Construct a MSF using Prim's Algorithm
- Control minimum number of outgoing arcs on depot nodes
- Add arcs returning to depots as many as outgoing arcs

Constraints are relaxed and augmented in the objected function as they are violated by the solution of the Lagrangian relaxed problem

SubP2: DCMSF-like subproblem

$$\left(\Upsilon_r^{\alpha}\right)^q = \left(\left|S_r\right| - \mathrm{L}_{S_r}\right) - \sum_{k \in ID} \sum_{i \in S_r} \sum_{\substack{j \in S_r \\ j \neq i}} \left(x_{ijk}\right)^q \qquad \forall r \in G^q$$

$$s_{ALR}^q = \Lambda_{ALR}^q \frac{Z_{ub(SubP2)}^q - Z_{ALR(SubP2)}^q(\alpha)}{\left\|\Upsilon\right\|^2}$$

$$\left(\alpha_r\right)^{q+1} = \min\left\{0, \left(\alpha_r\right)^q + s_{ALR}^q \left(\Upsilon_r^{\alpha}\right)^q\right\} \qquad \forall r \in G^q$$

Obtaining the upper bound

- The objective function value of a feasible solution is an upper bound for Z^*.
- Upper bound is obtained as follows:
  - Optimal solution of SubP1 (FLAP) is used to determine the depots to be opened
  - The remainder of the problem becomes a MDVRP. This problem is solved with tabu search.
- Each time a new location plan comes from solution of SubP1, MDVRP is solved with Tabu Search
  - *Aksen D., Özyurt Z. and Aras N., "Open Vehicle Routing Problem with Driver Nodes and Time Deadlines," to appear in JORS*

Tabu Search for MDVRP

- Initial Solution: Push Forward Insertion-Nearest Neighborhood Heuristic (PFIH-NN)
  - Assign each customer to the nearest depot
  - Calculate a C_{ij} depending on the Euclidian distances between customer i and depot j
  - Append customer i with the lowest C_{ij} value to a current route conforming to the capacity constraint
  - At each iteration, append the customer with lowest insertion cost to the solution
  - When spare capacity is not enough to satisfy the demand of any unrouted customer, start a new route

Tabu Search for MDVRP

- Evaluation of Solutions

  Since infeasible solutions are allowed objective function comprises two parts:

  - Traveling and vehicle acquisition costs
  - Penalty term for over capacity

- Why are infeasible solutions allowed?
  - Good solutions are expected to exist in the boundary of feasible and infeasible solutions
  - Penalty is intended to prevent the algorithm from spending too much time with exploring the infeasible regions of the search space

Tabu Search for MDVRP

- Neighborhood Generation:
  - 1-1 exchange
  - 1-0 move
  - 2-2 exchange
  - 2-Opt
- Tabu attributes
  - For 1-0 exchange ID of moved customer
  - For 1-1 and 2-2 exchange IDs of swapped customers
  - For 2-Opt IDs of two customers selected for the move
- Tabu duration: Random between [5,15]

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Computational Results

Two sets of randomly generated test problems +

36 problems earlier solved and reported in Tüzün and Burke *(EJOR, 1999)*

- **Test bed R1**: n = 15, 20, 25, 30, 35 ➡ 30 prob's
  m = 2, 3, 4, 5, 6
- **Test bed R2**: n = 40, 50, 60, 80, 100 ➡ 30 prob's
  m = 4, 5, 6
- **Test bed TB**: n = 100, 150, 200 ➡ 36 prob's
  m = 10, 20

Randomly generated test problems

prob id	N_c	N_{pd}	N_{cd}	$dist_c$
R1-1	15	1	2	U
R1-2	15	1	2	RU
R1-3	15	1	2	C
R1-4	15	-	2	U
R1-5	15	-	2	RU
R1-6	15	-	2	C
R1-7	20	1	3	U
R1-8	20	1	3	RU
R1-9	20	1	3	C
R1-10	20	-	3	U
R1-11	20	-	3	RU
R1-12	20	-	3	C
R1-13	25	1	2	U
R1-14	25	1	2	RU
R1-15	25	1	2	C
R1-16	25	1	4	U
R1-17	25	1	4	RU
R1-18	25	1	4	C
R1-19	30	1	4	U
R1-20	30	1	4	RU
R1-21	30	1	4	C
R1-22	30	-	4	U
R1-23	30	-	4	RU
R1-24	30	-	4	C
R1-25	35	1	4	U
R1-26	35	1	4	RU
R1-27	35	1	4	C
R1-28	35	-	6	U
R1-29	35	-	6	RU
R1-30	35	-	6	C

Randomly generated test problems

prob id	N_c	N_{pd}	N_{cd}	$dist_c$	*prob id*	N_c	N_{pd}	N_{cd}	$dist_c$
R2-1	40	1	4	C	R2-16	60	-	5	C
R2-2	40	1	4	RU	R2-17	60	-	5	RU
R2-3	40	1	4	U	R2-18	60	-	5	U
R2-4	40	-	4	C	R2-19	80	2	4	C
R2-5	40	-	4	RU	R2-20	80	2	4	RU
R2-6	40	-	4	U	R2-21	80	2	4	U
R2-7	50	1	4	C	R2-22	80	-	6	C
R2-8	50	1	4	RU	R2-23	80	-	6	RU
R2-9	50	1	4	U	R2-24	80	-	6	U
R2-10	50	-	5	C	R2-25	100	2	4	C
R2-11	50	-	5	RU	R2-26	100	2	4	RU
R2-12	50	-	5	U	R2-27	100	2	4	U
R2-13	60	1	4	C	R2-28	100	-	6	C
R2-14	60	1	4	RU	R2-29	100	-	6	RU
R2-15	60	1	4	U	R2-30	100	-	6	U

Results for R1

prob id	Z_{Cplex}	%GAP1	Z_{lb}	Z_{ub}	%GAP2	CPU (s)	%GAP3
R1-1	1127,84	0,10%	1075,58	1127,84	4,86%	54,69	0,00%
R1-2	994,92	0,10%	994,92	994,92	0,00%	27,56	0,00%
R1-3	909,72	0,10%	805,16	909,72	12,99%	44,96	0,00%
R1-4	1024,19	1,01%	975,28	1027,14	5,32%	53,49	0,29%
R1-5	1032,08	1,01%	1031,51	1074,68	4,19%	48,46	4,13%
R1-6	1280,85	1,01%	1274,75	1291,96	1,35%	29,23	0,87%
R1-7	1136,52	0,10%	1128,51	1148,72	1,79%	38,15	1,07%
R1-8	1285,05	10,75%	1262,14	1317,96	4,42%	96,66	2,56%
R1-9	1138,72	9,83%	1114,10	1138,72	2,21%	158,09	0,00%
R1-10	1442,47	5,58%	1435,11	1442,48	0,51%	155,10	0,00%
R1-11	1442,47	5,57%	953,04	1022,49	7,29%	74,44	-29,12%
R1-12	1220,48	5,26%	1220,38	1220,48	0,00%	52,53	0,00%
R1-13	1407,29	12,72%	1321,13	1402,44	6,15%	261,56	-0,34%
R1-14	1271,85	15,98%	1244,53	1286,85	3,40%	161,76	1,18%
R1-15	1210,09	11,52%	1204,01	1204,41	0,03%	95,46	-0,47%

Results for R1

prob id	Z_{Cplex}	%GAP1	Z_{lb}	Z_{ub}	%GAP2	CPU (s)	%GAP3
R1-16	1424,57	14,73%	1370,51	1418,18	3,48%	241,80	-0,45%
R1-17	1368,62	18,05%	1367,03	1370,47	0,25%	241,31	0,14%
R1-18	1050,80	19,40%	975,77	1029,39	5,50%	223,86	-2,04%
R1-19	1629,90	28,58%	1471,51	1525,03	3,64%	356,78	-6,43%
R1-20	1432,56	30,06%	1348,73	1441,94	6,91%	640,78	0,65%
R1-21	1175,44	19,58%	1093,31	1198,73	9,64%	514,11	1,98%
R1-22	1599,46	23,98%	1511,43	1543,86	2,15%	232,63	-3,48%
R1-23	1619,42	26,79%	1555,81	1611,87	3,60%	482,01	-0,47%
R1-24	1472,70	14,57%	1386,08	1442,53	4,07%	392,95	-2,05%
R1-25	1909,93	31,26%	1735,65	1812,81	4,45%	945,08	-5,08%
R1-26	1408,74	31,75%	1362,82	1386,02	1,70%	285,20	-1,61%
R1-27	1289,18	26,58%	1140,06	1255,75	10,15%	978,43	-2,59%
R1-28	1844,70	25,45%	1658,61	1801,37	8,61%	682,30	-2,35%
R1-29	1730,64	31,43%	1556,13	1634,60	5,04%	582,43	-5,55%
R1-30	1244,89	24,54%	1081,08	1185,35	9.64%	420,44	-4,78%
	1337,54	**14,91%**	**1255,16**	**1308,96**	**4,27%**	**285,74**	**-1,80%**

Results for R2

prob id	Z_{lb}	Z_{ub}	%GAP $_{Zlb\text{-}Zub}$	CPU (s)
R2-1	1206,44	1428,56	18,41%	706,09
R2-2	1549,84	1649,62	6,44%	1006,56
R2-3	1903,06	2144,55	12,69%	825,27
R2-4	1708,44	1877,78	9,91%	1025,87
R2-5	1770,79	1947,74	9,99%	937,99
R2-6	1897,92	2221,40	17,04%	1052,37
R2-7	1532,22	1700,76	11,00%	3031,87
R2-8	1772,49	1997,27	12,68%	2797,84
R2-9	2112,81	2357,29	11,57%	2114,86
R2-10	1778,51	1973,15	10,94%	754,82
R2-11	1635,76	1990,79	21,70%	1084,03
R2-12	2114,33	2306,43	9,09%	1660,26
R2-13	1617,15	1921,78	18,84%	3017,17
R2-14	2084,90	2336,22	12,05%	5425,50
R2-15	2130,51	2735,82	28,41%	1594,54

Results for R2

prob id	Z_{lb}	Z_{ub}	%GAP $_{Zlb\text{-}Zub}$	CPU (s)
R2-16	2103,90	2200,74	4,60%	3468,76
R2-17	1890,00	2339,18	23,77%	1984,47
R2-18	2115,01	2751,57	30,10%	1231,72
R2-19	1781,52	1996,10	12,04%	6285,83
R2-20	2286,75	2642,04	15,54%	4571,53
R2-21	2585,78	3076,79	18,99%	8182,53
R2-22	1945,81	2190,70	12,59%	1909,62
R2-23	2308,77	2670,56	15,67%	5315,75
R2-24	2488,13	3050,53	22,60%	4343,42
R2-25	1952,80	2184,96	11,89%	8800,01
R2-26	2369,93	2893,82	22,11%	11337,67
R2-27	2691,79	3320,89	23,37%	12650,38
R2-28	2032,33	2448,95	20,50%	5026,88
R2-29	2324,22	2896,34	24,62%	14666,70
R2-30	2741,77	3258,81	18,86%	11207,85
	2014,46	**2350,37**	**16,27%**	**4267,27**

Results for R1-R2

N_C	Avg_{ZCplex}	Avg_{GAP1}	Avg_{Zlb}	Avg_{Zub}	Avg_{GAP2}	Avg_{CPU}
15	1061,60	0,88%	1026,20	1071,04	4,79%	43,06
20	1277,62	-4,25%	1185,55	1215,14	2,70%	95,83
25	1288,87	-0,33%	1247,16	1285,29	3,14%	204,29
30	1488,25	-1,63%	1394,48	1460,66	5,00%	436,54
35	1571,35	-3,66%	1422,39	1512,65	5,99%	648,98

N_C	Avg_{Zlb}	Avg_{Zub}	Avg_{GAP2}	Avg_{CPU}
40	1672,75	1878,28	12,41%	925,69
50	1824,35	2054,28	12,83%	1907,28
60	1990,25	2380,89	19,63%	2787,03
80	2232,79	2604,45	16,24%	5101,44
100	2352,14	2833,96	20,23%	10614,92

Results for TB instances

prob id	N_{cd}	N_c	Z_{TB}	%GAP $_{ZTB-Zub}$	Z_{lb}	Z_{ub}	%GAP $_{Zlb-Zub}$	CPU(s)
P111112	100	10	1556,64	-8,95%	1283,09	1417,30	10,46%	19875,27
P111122	100	20	1531,88	-7,95%	1178,19	1410,04	19,68%	10554,93
P111212	100	10	1443,43	-2,57%	1140,54	1406,33	23,30%	9562,77
P111222	100	20	1511,39	-3,08%	1186,54	1464,84	23,45%	16420,19
P112112	100	10	1231,11	-1,72%	1079,16	1209,88	12,11%	14443,91
P112122	100	20	1132,02	-9,95%	925,16	1019,44	10,19%	18333,10
P112212	100	10	825,12	-11,95%	627,05	726,48	15,86%	7158,19
P112222	100	20	740,64	-0,31%	541,66	738,34	36,31%	15391,94
P113112	100	10	1316,98	-1,59%	1069,98	1296,04	21,13%	16432,57
P113122	100	20	1274,50	-8,98%	1055,33	1160,09	9,93%	12327,16
P113212	100	10	920,75	-1,30%	753,37	908,79	20,63%	6190,90
P113222	100	20	1042,21	-10,84%	780,93	929,22	18,99%	11696,95

Results for TB instances

prob id	N_{cd}	N_c	Z_{TB}	%GAP $_{ZTB-Zub}$	Z_{lb}	Z_{ub}	%GAP $_{Zlb-Zub}$	CPU(s)
P131112	150	10	2000,97	-6,57%	1561,25	1869,43	19,74%	52546,65
P131122	150	20	1892,84	0,35%	1465,80	1899,42	29,58%	54043,24
P131212	150	10	2022,11	3,83%	1589,11	2099,50	32,12%	43472,18
P131222	150	20	1854,97	-2,55%	1438,10	1807,63	25,70%	55900,30
P132112	150	10	1555,82	-4,34%	1151,67	1488,29	29,23%	42149,14
P132122	150	20	1478,80	1,58%	1144,07	1502,16	31,30%	59226,08
P132212	150	10	1231,34	0,26%	959,29	1234,50	28,69%	26122,60
P132222	150	20	948,28	-1,06%	742,16	938,22	26,42%	69757,69
P133112	150	10	1762,45	-5,38%	1232,78	1667,65	35,28%	10469,41
P133122	150	20	1488,34	-2,38%	1051,04	1452,97	38,24%	32540,27
P133212	150	10	1264,63	-7,22%	930,82	1173,29	26,05%	55394,52
P133222	150	20	1182,28	0,61%	973,35	1189,44	22,20%	26393,21

Results for TB instances

prob id	N_{cd}	N_c	Z_{TB}	%GAP $_{ZTB\text{-}Zub}$	Z_{lb}	Z_{ub}	%GAP $_{Zlb\text{-}Zub}$	CPU(s)
P121112	200	10	2379,47	-1,76%	1747,10	2337,60	33,80%	107893,15
P121122	200	20	2211,74	-1,58%	1639,88	2176,88	32,75%	75101,67
P121212	200	10	2288,17	-6,29%	1800,51	2144,31	19,09%	144487,62
P121222	200	20	2355,81	-2,23%	1683,70	2303,29	36,80%	122279,39
P122112	200	10	2158,60	-6,84%	1591,88	2011,02	26,33%	188714,78
P122122	200	20	1787,02	-1,65%	1320,11	1757,52	33,13%	206415,42
P122212	200	10	1549,79	-4,19%	1079,33	1484,87	37,57%	74098,24
P122222	200	20	1112,96	-1,64%	1001,98	1094,71	9,25%	76432,00
P123112	200	10	2056,11	-2,28%	1576,96	2009,21	27,41%	72359,66
P123122	200	20	2002,42	-5,82%	1433,07	1885,89	31,60%	130101,28
P123212	200	10	1877,30	-4,98%	1498,26	1783,77	19,06%	159535,92
P123222	200	20	1414,83	-3,67%	1064,47	1362,84	28,03%	61384,53

Results for TB instances

N_{cd}	N_c	Z_{TB}	%GAP $_{ZTB\text{-}Zub}$	Z_{lb}	Z_{ub}	%GAP $_{Zlb\text{-}Zub}$	CPU(s)
100	10	1215,67	-4,68%	992,20	1160,80	17,25%	12277,27
100	20	1205,44	-6,85%	944,64	1120,33	19,76%	14120,71
150	10	1639,55	-3,24%	1237,49	1588,78	28,52%	38359,08
150	20	1474,25	-0,58%	1135,75	1464,97	28,91%	49643,47
200	10	2051,57	-4,39%	1549,01	1961,80	27,21%	124514,89
200	20	1814,13	-2,77%	1357,20	1763,52	28,59%	111952,38
	averages	**1566,77**	**-3,75%**	**1202,71**	**1510,03**	**25,04%**	**58477,97**

AGENDA

- PROBLEM DEFINITION
- LITERATURE REVIEW
- MDLRP MODEL
- SOLUTION METHOD
- COMPUTATIONAL RESULTS
- CONCLUSION

Conclusion

- Solved: a test bed of 60 randomly generated problems + 36 test instances in Tüzün and Burke ***(EJOR, 1999)***
- Tüzün-Burke results improved by an average margin of 3.75%.
- CPU time needs to be improved - a faster implementation of the Modified Prim with less order of complexity.
- LRP with time restrictions (LRPTD and LRPTW) and LRP with depot capacities (CLRP) need to be addressed.

Questions & Comments?