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Budget Constrained Fortification and Service Capacity Expansion in the Bilevel *r*-Interdiction Median Problem

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AGENDA

- PROBLEM DEFINITION and DESCRIPTION
- LITERATURE REVIEW
- BCRIMF-CE MODEL
- SOLUTION METHOD
- COMPUTATIONAL RESULTS
- CONCLUSION and ONGOING WORK

PROBLEM DEFINITION

Interdiction of a Geographically Distributed Service or Supply Network System

- **Definition**:

  An intentional strike against a network system which is anticipated in the form of a man-made threat

  Examples: sabotage, riot, enemy attack, terrorist action

- **Target**:

  Critical infrastructure elements – the ones which, if lost, would pose a significant threat to needed supplies, services, and communications or a significant loss of service coverage or significant loss of efficiency in service delivery.

  Examples:

  - Needed supplies: food, energy, medicines, water
  - Needed services: police, fire, emergency medical services, transportation (airports, railways, ports), purification and sanitation
  - Needed communications: antenna towers, base stations, switch offices, radars

PROBLEM DEFINITION

Interdiction of a Geographically Distributed Service or Supply Network System

- **Consequences:**
  - Disruption in the service provision/delivery
  - Decay in service system efficiencies

r-Interdiction Median Problem (RIM)

r-Interdiction Coverage Problem (RIC)

PROBLEM DEFINITION: RIM

r - Interdiction Median Problem

- <u>Definition</u>:

  Of the *p* different locations of supply, find the subset of *r* facilities, which when removed, yields the highest level of demand-weighted distance (cost) in total.

- <u>Purpose</u>:
  - Identification of *r* most critical facilities in a network consisting of *p* service/supply and *n* demand nodes.
  - Prediction of the most disruptive reaction of an Attacker.

PROBLEM DESCRIPTION: RIM

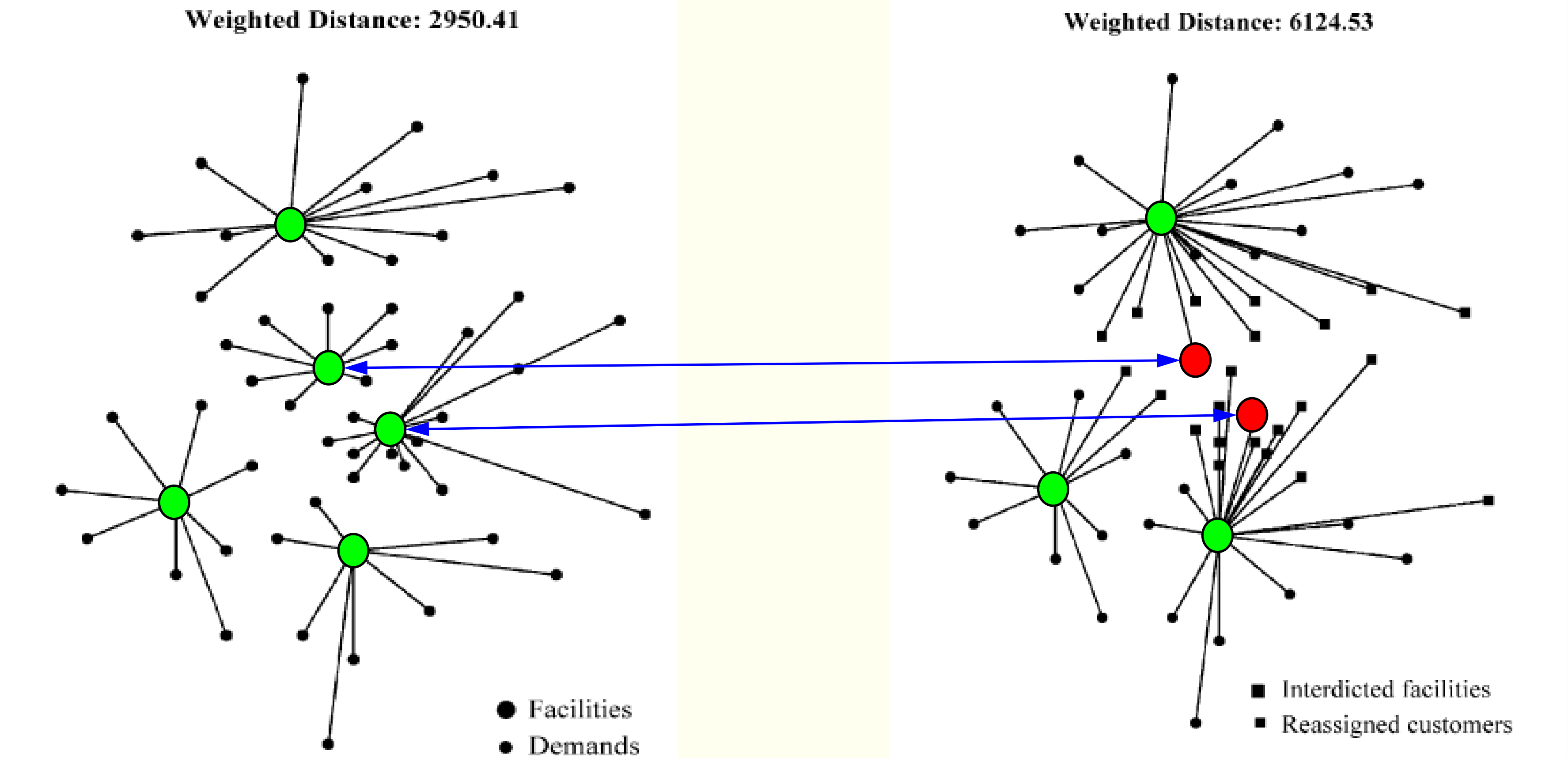

Figure 2a. Optimal solution to the p-median problem ($p = 5$).

Figure 2b. Optimal interdiction of the p-median solution given in Figure 2a ($r = 2$).

Source: *Church RL, Scaparra MP, Middleton RS (2004). Identifying critical infrastructure: the median and covering facility interdiction problems. Annals of the Association of American Geographers 94(3):491–502.*

PROBLEM DEFINITION: IMF

r - Interdiction Median Problem with Fortification

- <u>Definition</u>:

  Of the *p* different locations of supply or service response, find the subset of *q* facilities, which when fortified, provides the best protection against a subsequent optimal *(worst-case)* *r*-interdiction strike leading to the definitive loss of *r* non-fortified facilities.

  $(q + r \leq p)$

- <u>Objective</u>:

  Minimize the post-attack cost of accessibility from demand nodes to the <u>**CLOSEST AVAILABLE**</u> (non-interdicted) supply/service nodes.

PROBLEM DEFINITION: RIMF

The Bilevel Programming (Stackelberg Game) Formulation of the *r* - Interdiction Median Problem with Fortification

DEFENDER

Obj.: Minimize *H(X)* = The total demand-weighted distance between demand nodes and their *CLOSEST AVAILABLE* facilities by fortifying (protecting) *q* of *p* facilities.

Apply the protection plan

RIM

Obj. Maximize *H(X) given the protection plan of the Defender*

Report

the interdiction (loss) of *r* unprotected facilities

ATTACKER

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LITERATURE REVIEW: quick & dirty

Table 1. Interdiction Problems: References and Structural Characteristics

Reference	Objective	Decision	Constraint	Underlying Model
Wollmer (1964)	Minimize network flow capacity	Complete interdiction on arcs	Cardinality	Maximum flow through planar networks
Wollmer (1970)	Maximize minimum-cost flow	Complete interdiction on arcs	Cardinality	Minimum cost flow through networks
McMasters and Mustin (1970)	Minimize network flow capacity	Interdiction on arc capacities by units	Budget	Maximum flow through planar networks
Ghare, Montgomery, and Turner (1971)	Minimize network flow capacity	Complete interdiction on arcs	Budget	Maximum flow through networks
Corley and Chang (1974)	Minimize network flow capacity	Complete interdiction on nodes and incident arcs	Cardinality	Maximum flow through networks
Ratliff, Sicilia, and Lubore (1975)	Minimize network flow capacity	Complete interdiction on arcs	Cardinality	Maximum flow through networks
Fulkerson and Harding (1977)	Maximize shortest source-sink path	Interdiction on arc lengths by units	Budget	Minimum cost flow through networks
Golden (1978)	Minimize interdiction costs	Interdiction on arc lengths by units	Disruption Level	Minimum cost flow through networks
Corley and Sha (1982) Ball, Golden, and Vohra (1989) Malik, Mittal, and Gupta (1989)	Maximize shortest source-sink path	Complete interdiction on arcs	Cardinality	Shortest path through networks
Phillips (1993)	Minimize network flow capacity	Interdiction on arc capacities by units	Budget	Maximum flow through outerplanar and planar networks
Wood (1993)	Minimize network flow capacity	Complete interdiction on arcs Interdiction on arc capacities by units	Budget Cardinality	Maximum flow through general networks and multi-commodity networks
Cormican, Morton, and Wood (1998)	Minimize expected maximum flow	Interdiction attempt on arcs	Budget	Maximum flow through networks
Whiteman (1999)	Minimize interdiction costs	Complete and partial interdiction on nodes	Disruption level	Maximum flow through multi-commodity networks
Israeli and Wood (2002)	Maximize shortest source-sink path	Complete interdiction on arcs	Budget	Shortest path through networks
Burch et al. (2003)	Minimize network flow capacity	Complete interdiction on arcs	Budget	Maximum flow through nonplanar networks
Hemmecke, Schultz, and Woodruff (2002) Held et al. (2003)	Maximize the probability of given disruption level	Complete interdiction on arcs	Budget	Shortest path through uncertain networks

LITERATURE REVIEW

Review of critical infrastructure, reliability and disruption models for facility location and supply chain networks

Snyder LV, Daskin MS. Reliability models for facility location: the expected failure cost case. *(Trans Sci 2005;39)*

Snyder LV, Scaparra MP, Daskin MS, Church RL *(2006)*. Planning for disruptions in supply chain networks. In: Greenberg HK (ed), *(TutORials in Operations Research, INFORMS, Baltimore)*

Murray AT, Grubesic TH (eds) *(2007)*. *Critical infrastructure: reliability and vulnerability. Advances in spatial sciences.* Springer-Verlag, Berlin, Heidelberg.

Sources of inspiration for our research

- **RIM + RIC + an annotated bibliography**

Church RL, Scaparra MP, Middleton RS (2004). Identifying critical infrastructure: the median and covering facility interdiction problems. Annals of the Association of American Geographers 94(3):491-502.

- **IMF : Single level problem solved with Cplex 7.0**

Church RL, Scaparra MP (2007). Protecting critical assets: the r-interdiction median problem with fortification. Geographical Analysis 39(2):129-146.

- **Maximal Covering Problem with Precedence Constraints (MCPC): an alternative way of reformulating IMF**

Scaparra MP, Church RL (2008a). An exact solution approach for the interdiction median problem with fortification. European Journal of Operational Research 189(1):76-92.

Sources of inspiration for our research

MCPC

- ✓ allows a specialized model reduction process, providing upper and lower bounds that can be used to reduce the size of the original model.
- ✓ can be solved easily to optimality by the commercial MIP solver Cplex.
- ✗ requires a complete enumeration of all $\binom{p}{r}$ possible ways of interdicting r out of the p facilities.

- **RIMF: Bilevel problem solved with implicit enumeration and Cplex 9.0**

Scaparra MP, Church RL (2008b). A bilevel mixed integer program for critical infrastructure protection planning. Computers & Operations Research 35(6):1905-1923.

  - *Requires at most $(r^{q+1}-1)/(r-1)$ RIM problems to be solved conditional on the fortification plan of the Defender.*

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NEW RIMF MODEL: BCRIMF-CE

A Defender-Attacker Game for the Budget Constrained *r*-Interdiction Median Problem with Capacity Expansion

- The number of fortifications is not fixed, but subject to a fortification budget where each facility has a different fortification cost.
- Each demand node reassignment due to interdiction is associated with an added capacity expansion cost proportional to the demand of that node and to the marginal capacity expansion cost of its new facility.

NEW RIMF MODEL: BCRIMF-CE

Assumption of capacity expansion at non-interdicted facilities:

- Initially (before the attack) all *p* facilities operate at full capacity, i.e., none of them has spare capacity to serve additional demand nodes.

Remaining assumptions of the original RIMF model preserved:

- System reliability due to common component failure is not included.
- The *maximum* number of interdictions by the Attacker, namely *r*, is known with certainty.
- Once a facility is fortified, it cannot be interdicted anymore. It becomes fully protected.
- The Attacker knows which facilities are protected by the Defender.
- If the Attacker interdicts an unprotected facility, it is destroyed beyond repair and shuts down with certainty.

BCRIMF-CE :: Notation

- Sets:

  I = *Set of demand nodes (customers),* $\boldsymbol{I = \{1,\ldots, n\}}$

  J = *Set of facility locations,* $\boldsymbol{J = \{1,\ldots, p\}}$

- Parameters

d_{ij} = traveling cost (shortest distance cost) between facility location j and demand node i.

d_{max} = maximum traveling cost between facility location j and demand node i, i.e.,
$d_{max} = \max_{(i,j)}\{d_{ij}\}$.

q_i = demand at node i.

c_j = marginal cost of expanding the capacity of facility at location j.

b_j = fortification cost of the facility at location j.

b_{tot} = total fortification budget (protective resources) of the defender.

r = the number of facilities that can be interdicted within the offensive resources of the attacker.

BCRIMF-CE :: Notation

- Parameters *(cont.)*:

$$\delta_{ij} = \begin{cases} 1 & \text{if demand node } i \text{ is initially assigned to the facility at location } j, \\ 0 & \text{otherwise.} \end{cases}$$

- Binary Decision Variables:

$$Z_j = \begin{cases} 1 & \text{if the facility at location } j \text{ is fortified}, \\ 0 & \text{otherwise.} \end{cases}$$

$$S_j = \begin{cases} 1 & \text{if the facility at location } j \text{ is lost due to an interdiction}, \\ 0 & \text{otherwise.} \end{cases}$$

$$X_{ij} = \begin{cases} 1 & \text{if demand node } i \text{ is assigned to the facility at location } j, \\ 0 & \text{otherwise.} \end{cases}$$

BCRIMF-CE Model

fortification budget constraint

BCRIMF-CE : $\min\ H(\mathbf{Z})$ (1)

Subject to (2)

$Z_j \in \{0,1\} \quad \forall j \in \mathbf{J}$ (3)

Defender's (Upper Level) Problem

demand-weighted total traveling cost

where $\mathbf{Z}$ solve:

$H(\mathbf{Z}) = \max$ + (4)

capacity expansion cost

Attacker's (Lower Level) Problem

Subject to $\quad \forall i \in \mathbf{I}$, (5)

(6)

$\forall j \in \mathbf{J}$, (7)

$\forall i \in \mathbf{I},\ \forall j,k \in \mathbf{J},\ j \neq k.$ (8)

$\forall j \in \mathbf{J}$, (9)

$S_j \in \{0,1\} \quad \forall j \in \mathbf{J}$ (10)

$X_{ij} \in \{0,1\} \quad \forall i \in \mathbf{I},\ \forall j \in \mathbf{J}.$ (11)

Closest Assignment (CA)

- ## Rojeski-ReVelle Constraints:

*Rojeski P, ReVelle CS **(1970)**. Central facilities location under an investment constraint. **Geographical Analysis 2(4):343-360**.*

$$
\left.
\begin{aligned}
x_{ii_1} &= y_{i_1}, \\
x_{ii_2} &\geqslant y_{i_2} - y_{i_1}, \\
x_{ii_3} &\geqslant y_{i_3} - (y_{i_2} + y_{i_1}), \\
&\vdots \\
x_{ii_m} &\geqslant y_{i_m} - (y_{i_{m-1}} + \cdots + y_{i_2} + y_{i_1}),
\end{aligned}
\right\} \quad i \in I.
$$

Let $i_1 \in J$ represent the closest alternative site to population center i. This additional requirement can be represented as $x_{ii_1} = y_{i_1}$. Similarly, population center i should be assigned to its second closest alternative site $i_2 \in J$, if facility i_1 is not open and there is a facility at i_2. This can be imposed by $x_{ii_2} \geq y_{i_2} - y_{i_1}$. The argument can be extended to third, fourth etc. closest alternative sites.

Closest Assignment (CA)

- Rojeski-ReVelle (RR) Constraints for the BCRIMF-CE

$$x_{ij} \geq (1 - s_j) - \sum_{h \in C_{ij}} (1 - s_h) \quad \text{for all } i \in \boldsymbol{I} \text{ and for all } j \in \boldsymbol{J}_i$$

J_i : the set of r closest facilities to demand node i before interdiction

C_{ij} : the set of all the facilities h which are closer to demand i than facility j, but not further than the $(r+1)^{st}$ closest to i.

- Church-Cohon (CC) Constraints

Church RL, Cohon JL (1976). Multiobjective location analysis of regional energy facility sitting problems. Report prepared for the US Energy Research and Development Administration (BNL 50567).

Closest Assignment (CA)

- Church-Cohon (CC) Constraints for the BCRIMF-CE (v1.0)

$$\sum_{k \in \mathbf{J}:\, d_{ik} > d_{ij}} X_{ik} \leq S_j \qquad \forall i \in \boldsymbol{I},\ \forall j \in \boldsymbol{J}$$

*Scaparra MP, Church RL (2008b). A bilevel mixed integer program for critical infrastructure protection planning. **Computers & Operations Research 35(6):1905-1923**.*

- Church-Cohon Constraints for public facility location (v2.0)

$$\sum_{k \in \mathbf{J}:\, d_{ik} \leq d_{ij}} X_{ik} \geq Y_j \qquad \forall i \in \boldsymbol{I},\ \forall j \in \boldsymbol{J}$$

*Teixeira JC, Antunes AP (2008). A hierarchical location model for public facility planning. **European Journal of Operational Research 185(1):92-104**.*

Closest Assignment (CA)

- Dobson-Karmarkar (DK) Constraints for the location of preventive health care facilities or for competitive facility location

$$X_{ij} \leq 1 - Y_l \qquad \forall l \in \boldsymbol{S}_{ij}, \ \forall i \in \boldsymbol{I}, \ \forall j \in \boldsymbol{J}$$

S_{ij} : the set of alternative facility sites – indexed by l, that are closer to population center i than site j , i.e., $S_{ij} = \{l \mid d_{il} < d_{ij}\}$.

*Verter V, Lapierre SD (2002). Location of preventive health care facilities. **Annals of Operations Research 110(1/4):123-132**.*

*Dobson G, Karmarkar US (1987). Competitive location on a network. **Operations Research 35 (4):565-574**.*

Closest Assignment (CA)

- Our novel CA constraints: proposed for the first time

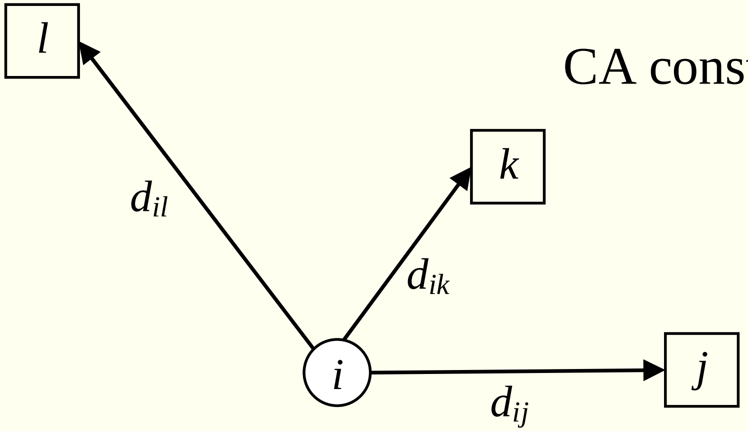
$$d_{ij} \leq d_{ik} + d_{max}(1 + S_j + S_k - X_{ij}) \qquad \forall i \in \mathbf{I},\ \forall j,k \in \mathbf{J},\ j \neq k.$$

d_{max} : $d_{max} = \max_{(i,j)}\{d_{ij}\}$.

Distance order: $d_{ik} < d_{ij} < d_{il}$

CA constraint for $\{j,k\}$: $d_{ij} \leq d_{ik} + d_{max}(1 + S_j + S_k - X_{ij})$

CA constraint for $\{k,j\}$: $d_{ik} \leq d_{ij} + d_{max}(1 + S_k + S_j - X_{ik})$



Drawback: $n \times m \times (m-1)$
many constraints necessary

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Sample binary search tree for $p = 5$, $r = 2$

A = optimal solution set for the lower level attacker's problem (RIM) in accordance with the current node's fortification plan.

z_j = fortification variable (1 if the facility j is fortified, 0 otherwise.)

H_a = The Attacker's objective value.

H_d = The Defender's objective value.

B = Remaining fortification budget at the current node

Root node

Parent node

Leaf node

Fortification Costs of Facilities

f_1= 5 m.u.

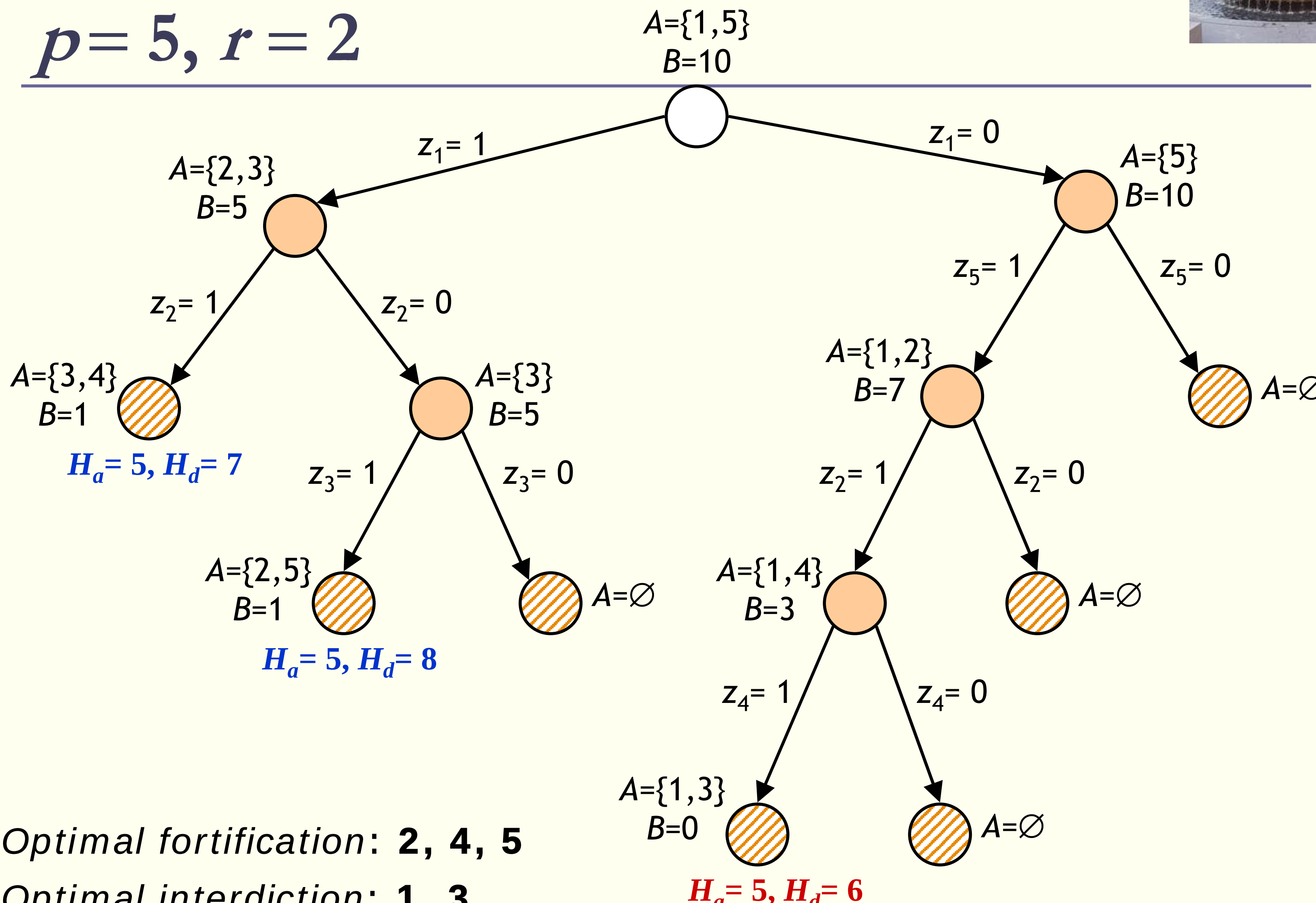
f_2= 4 m.u.

f_3= 4 m.u.

f_4= 3 m.u.

f_5= 3 m.u.

Sample binary search tree for $p = 5$, $r = 2$



Optimal fortification: **2, 4, 5**

Optimal interdiction: **1, 3**

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Computational Results

- **Computing Platform**
  - Microsoft Visual C++ 2005
  - Desktop PC with AMD Athlon 64x2 Dual Core 2.40 GHz processor and 4 GB RAM.
  - The RIM instance conditional on the present fortification plan obtained at each node of the binary search tree solved to optimality with Cplex 11.
- **Randomly Generated Test Problems**: 12 sets each with:
  - 2 levels of fortification budget b_{tot} ,
  - 4 levels of the interdiction parameter r.

Computational Results

Random Instance Generation

- Customer demand data and coordinates of customer and facility locations from Aras and Aksen ***(IJPE, 2008)***.
- Marginal capacity expansion costs from a discrete uniform distribution between 50 and 100 m.u.
- Fortification costs randomly assigned between 1,000 and 10,000 m.u. as integer multiples of 250.
- Low budget level was equal to $[\![\frac{1}{1{,}250} \sum_j b_j]\!] \times 250$.
- High budget level was taken as $[\![\frac{3}{1{,}250} \sum_j b_j]\!] \times 250$;

  however, it was reduced as necessary to ensure that $q+r \leq p$.

Computational Results

The characteristics of 96 test instances of BCRIMF-CE

Prob. ID	Problem Set	# Dem. nodes n	# Facilities p	r	Budget level low	Budget level high
1	**n100p10**	100	10	2, 3, 4, 5	9,750	29,250
2	**n100p15**	100	15	2, 3, 4, 5	12,250	36,500
3	**n100p20**	100	20	2, 3, 4, 5	15,500	36,500
4	**n100p25**	100	25	2, 3, 4, 5	15,000	36,500
5	**n150p10**	150	10	2, 3, 4, 5	9,750	29,250
6	**n150p15**	150	15	2, 3, 4, 5	12,250	22,500
7	**n150p20**	150	20	2, 3, 4, 5	16,500	35,250
8	**n150p25**	150	25	2, 3, 4, 5	16,000	39,500
9	**n200p10**	200	10	2, 3, 4, 5	10,500	29,250
10	**n200p15**	200	15	2, 3, 4, 5	12,250	36,500
11	**n200p20**	200	20	2, 3, 4, 5	16,500	39,500
12	**n200p25**	200	25	2, 3, 4, 5	16,500	39,500

Results for problem sets 1–6

Prob. No.	r	Budget level: low				Budget level: high			
		Budget Available	Budget used (%)	# Fortified fac.	Obj. Value	Budget Available	Budget used (%)	# Fortified fac.	Obj. Value
1	2	9,750	69.2	2	140,590	29,250	96.6	6	81,030
	3		69.2	2	169,220		99.1	7	85,399
	4		92.3	3	197,186		96.6	6	97,114
	5		82.1	3	224,811		88.9	5	129,789
2	2	12,250	93.9	3	114,336	36,500	99.3	10	37,192
	3		93.9	3	143,604		99.3	10	45,446
	4		100.0	5	161,811		99.3	10	50,608
	5		100.0	5	170,667		99.3	10	53,989
3	2	15,500	96.8	4	101,678	36,500	96.6	8	62,497
	3		96.8	4	120,359		96.6	8	70,841
	4		96.8	4	138,192		96.6	8	78,578
	5		96.8	4	154,062		96.6	8	84,222
4	2	15,000	100.0	5	75,576	36,500	93.2	8	59,555
	3		100.0	5	94,257		93.2	8	71,383
	4		100.0	5	112,090		97.9	9	79,141
	5		100.0	5	127,057		97.9	9	86,011
5	2	9,750	71.8	2	222,078	29,250	93.2	6	160,834
	3		92.3	3	260,455		93.2	6	183,736
	4		92.3	3	296,642		93.2	6	202,788
	5		82.1	3	324,102		88.9	5	231,784
6	2	12,250	85.7	4	160,519	22,500	100.0	7	146,535
	3		85.7	4	193,360		100.0	7	164,230
	4		95.9	4	215,910		100.0	7	181,781
	5		95.9	4	235,448		100.0	7	197,853

Results for problem sets 7–12

Prob. No.	r	Budget level: low				Budget level: high			
		Budget Available	Budget used (%)	# Fortified fac.	Obj. Value	Budget Available	Budget used (%)	# Fortified fac.	Obj. Value
7	2	16,500	97.0	4	168,198	35,250	87.2	9	146,837
	3		97.0	4	195,647		100.0	11	158,096
	4		97.0	4	218,092		100.0	11	168,636
	5		100.0	6	237,116		100.0	11	179,895
8	2	16,000	79.7	4	176,549	39,500	96.8	9	144,037
	3		90.6	5	194,788		96.8	9	156,671
	4		90.6	5	209,987		96.8	9	167,930
	5		90.6	5	224,464		96.8	9	178,051
9	2	10,500	66.7	2	277,896	29,250	94.0	5	216,174
	3		85.7	3	328,439		98.3	6	249,521
	4		97.6	4	365,191		98.3	6	275,889
	5		97.6	4	393,538		94.0	5	313,280
10	2	12,250	85.7	4	209,459	36,500	95.2	9	169,611
	3		85.7	4	249,116		95.2	9	185,424
	4		95.9	5	277,829		95.2	9	200,418
	5		95.9	5	305,552		99.3	10	211,367
11	2	16,500	97.0	4	198,652	39,500	95.5	8	181,792
	3		97.0	4	231,890		99.4	11	196,915
	4		97.0	4	263,113		99.4	11	218,607
	5		97.0	4	296,097		98.7	13	229,502
12	2	16,500	80.3	4	232,508	39,500	92.4	8	184,554
	3		87.9	5	258,783		92.4	8	199,546
	4		87.9	5	282,903		97.5	9	213,846
	5		87.9	5	305,531		97.5	9	227,443

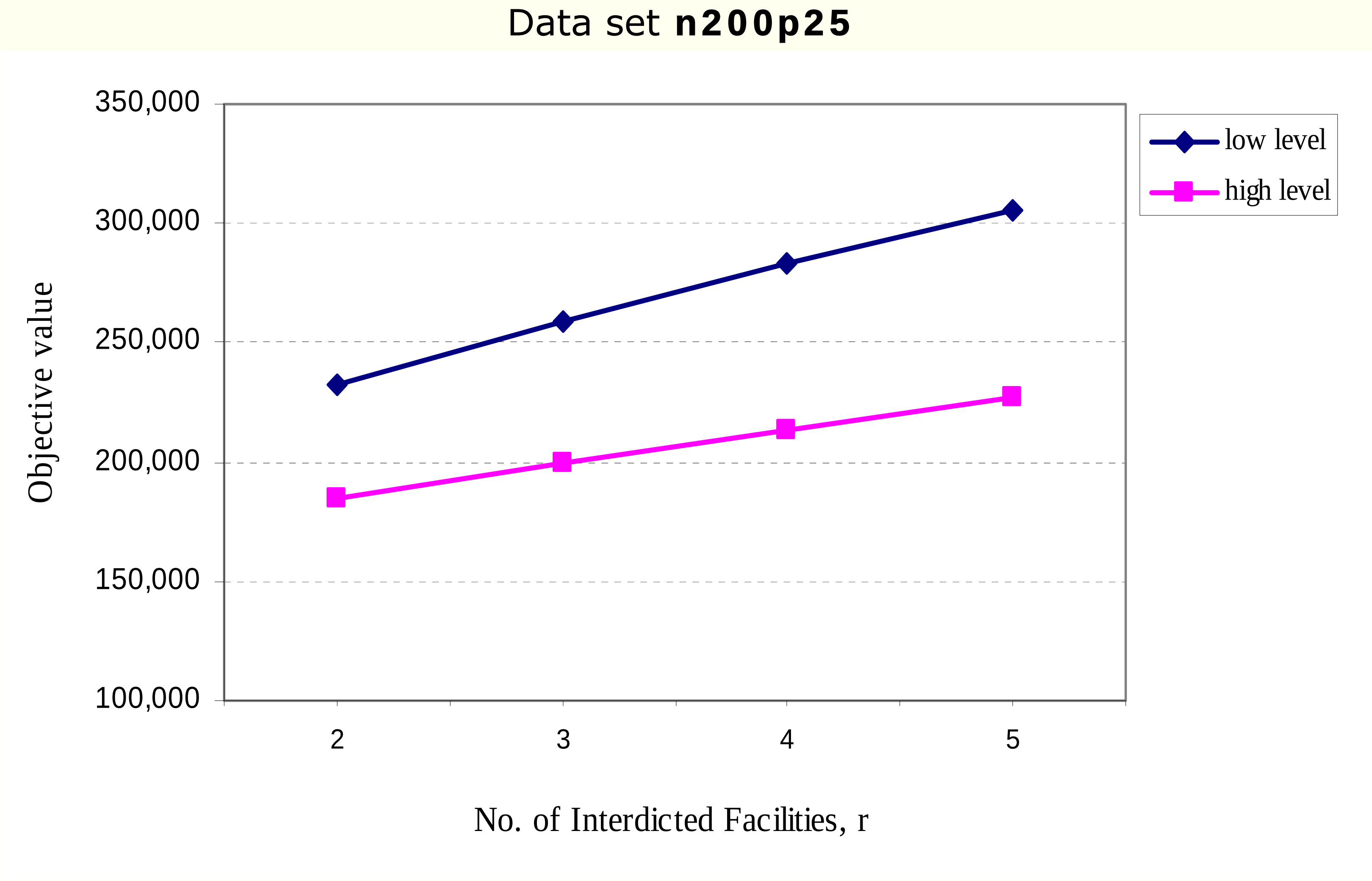
Details for high-budget sets 1–6

Prob. No.	r	Budget level: high			
		Fortified Facilities	Interdicted Facilities	No. Cplex	CPU time (sec)
1	2	2, 4, 5, 6, 7, 9	8, 10	22	8.5
	3	2, 3, 4, 5, 7, 8, 9	1, 6, 10	60	23.0
	4	2, 4, 5, 6, 7, 9	1, 3, 8, 10	142	54.5
	5	2, 5, 6, 7, 9	1, 3, 4, 8, 10	225	85.9
2	2	2, 3, 4, 6, 7, 9, 11, 13, 14, 15	10, 12	64	27.6
	3	2, 3, 4, 6, 7, 9, 11, 13, 14, 15	5, 10, 12	221	94.7
	4	2, 3, 4, 6, 7, 9, 11, 13, 14, 15	5, 8, 10, 12	710	306.1
	5	2, 3, 4, 6, 7, 9, 11, 13, 14, 15	1, 5, 8, 10, 12	2,089	896.6
3	2	2, 4, 6, 7, 9, 16, 17, 19	13, 15	46	22.9
	3	2, 4, 6, 7, 9, 16, 17, 19	12, 13, 15	215	104.4
	4	2, 4, 6, 7, 9, 16, 17, 19	11, 12, 13, 15	742	364.8
	5	2, 4, 6, 7, 9, 16, 17, 19	8, 11, 12, 13, 15	2,213	1075.8
4	2	4, 9, 16, 17, 19, 21, 24, 25	6, 15	45	25.4
	3	4, 9, 16, 17, 19, 21, 24, 25	6, 15, 23	165	92.6
	4	4, 9, 13, 16, 17, 19, 21, 24, 25	6, 11, 15, 23	554	308.3
	5	4, 9, 13, 16, 17, 19, 21, 24, 25	1, 6, 11, 15, 23	1,825	1019.8
5	2	1, 3, 7, 8, 9, 10	2, 5	23	10.1
	3	1, 3, 7, 8, 9, 10	2, 4, 5	80	33.9
	4	1, 3, 7, 8, 9, 10	2, 4, 5, 6	170	71.6
	5	1, 7, 8, 9, 10	2, 3, 4, 5, 6	242	102.5
6	2	1, 2, 3, 7, 9, 12, 13	10, 15	20	11.8
	3	1, 2, 3, 7, 9, 12, 13	8, 10, 15	78	44.2
	4	1, 2, 3, 7, 9, 12, 13	8, 10, 11, 15	143	82.5
	5	1, 2, 3, 7, 9, 12, 13	6, 8, 10, 11, 15	270	149.6

Details for high-budget sets 7–12

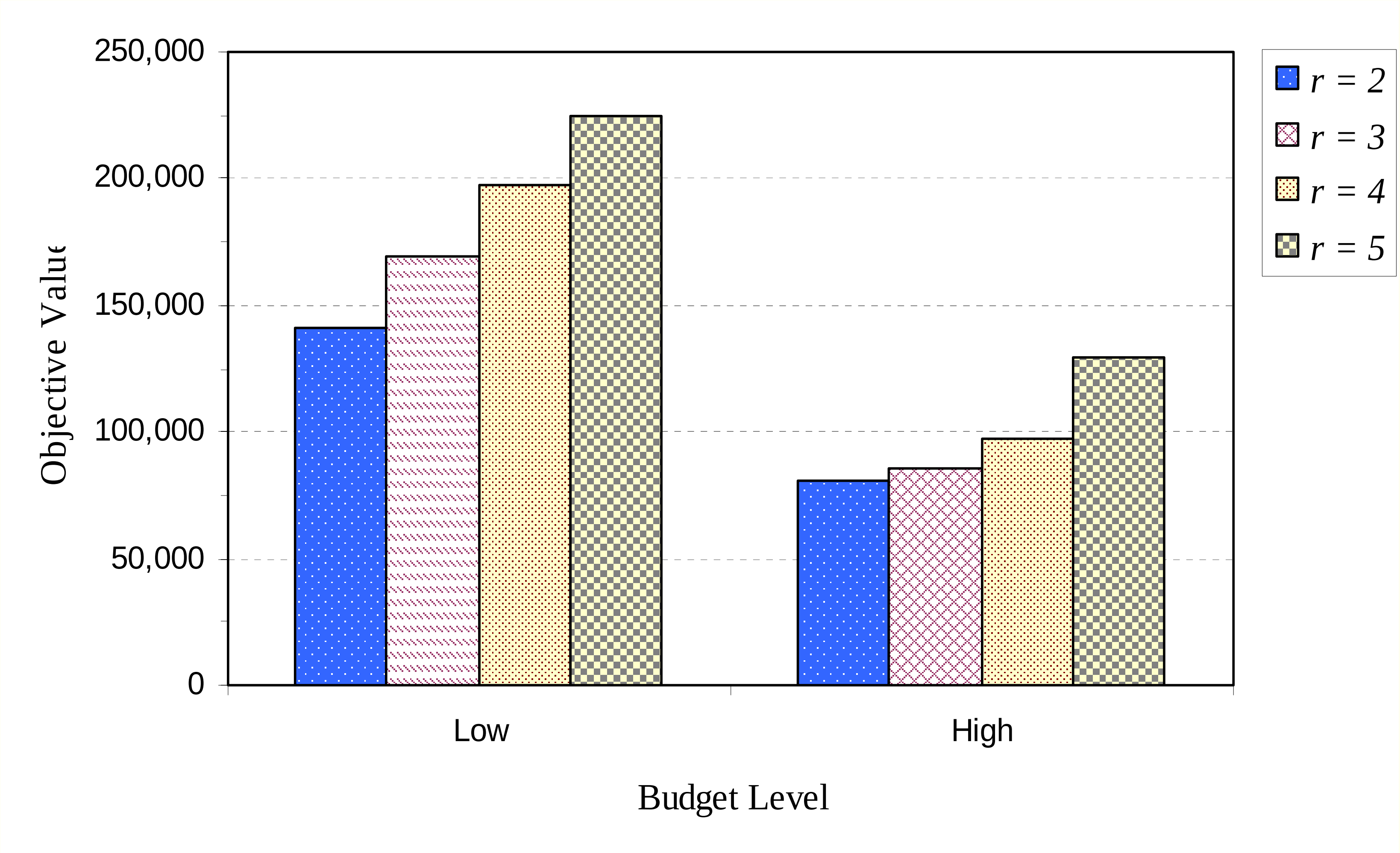
Prob. No.	*r*	Budget level: high			
		Fortified Facilities	Interdicted Facilities	No. Cplex	CPU time (sec)
7	2	1, 3, 6, 7, 9, 12, 13, 18, 19	2, 10	65	39.2
	3	1, 3, 4, 6, 7, 9, 12, 13, 17, 18, 19	2, 10, 15	231	135.6
	4	1, 3, 4, 6, 7, 9, 12, 13, 17, 18, 19	2, 5, 10, 14	662	403.4
	5	1, 3, 4, 6, 7, 9, 12, 13, 17, 18, 19	2, 5, 10, 14, 15	1,659	964.2
8	2	1, 3, 9, 10, 12, 13, 18, 19, 24	4, 6	47	37.7
	3	1, 3, 9, 10, 12, 13, 18, 19, 24	4, 6, 17	224	174.4
	4	1, 3, 9, 10, 12, 13, 18, 19, 24	4, 6, 15, 17	753	592.5
	5	1, 3, 9, 10, 12, 13, 18, 19, 24	4, 6, 15, 17, 25	2,161	1765.5
9	2	1, 2, 7, 9, 10	4, 8	28	14.5
	3	1, 2, 3, 7, 9, 10	4, 5, 8	90	44.6
	4	1, 2, 3, 7, 9, 10	4, 5, 6, 8	203	100.2
	5	1, 2, 7, 9, 10	3, 4, 5, 6, 8	267	135.9
10	2	1, 2, 3, 4, 7, 9, 10, 12, 13	6, 15	49	30.5
	3	1, 2, 3, 4, 7, 9, 10, 12, 13	6, 11, 15	208	126.0
	4	1, 2, 3, 4, 7, 9, 10, 12, 13	6, 8, 11, 15	580	345.0
	5	1, 2, 3, 4, 7, 9, 10, 12, 13, 14	5, 6, 8, 11, 15	1,615	964.6
11	2	1, 6, 7, 9, 10, 12, 13, 18	4, 19	51	38.1
	3	1, 3, 4, 6, 7, 9, 12, 13, 16, 18, 19	2, 10, 17	208	148.7
	4	1, 3, 4, 6, 7, 9, 12, 13, 16, 18, 19	2, 5, 10, 14	770	544.5
	5	1, 3, 4, 6, 7, 9, 11, 12, 13, 14, 17, 18, 19	2, 5, 10, 15, 16	2,203	1543.6
12	2	1, 3, 4, 10, 12, 13, 18, 24	6, 16	37	34.0
	3	1, 3, 4, 10, 12, 13, 18, 24	6, 16, 25	154	136.0
	4	1, 3, 4, 9, 10, 12, 13, 18, 24	6, 16, 19, 25	533	468.1
	5	1, 3, 4, 9, 10, 12, 13, 18, 24	6, 16, 19, 21, 25	1,667	1469.7

Effect of the interdiction parameter *r*



Effect of the fortification budget b_{tot}

Data set **n100p10**



AGENDA

- PROBLEM DEFINITION and DESCRIPTION
- LITERATURE REVIEW
- BCRIMF-CE MODEL
- SOLUTION METHOD
- COMPUTATIONAL RESULTS
- CONCLUSION and ONGOING WORK

Conclusion and Ongoing Work

- As was suggested in the future research directions of the inspiring studies in the literature, we added the **fortification budget constraint** and **capacity expansion / customer relocation costs** to the bilevel *r-Interdiction Median Problem with Fortification*.
- We proposed a novel set of **Closest Assignment** constraints that are applicable not only to critical infrastructure protection planning problems, but also public facility and competitive facility location problems.
- We are working on two new models:

1. **Minimizing the disruption of service efficiencies in critical networks: planning of less fragile facility locations in advance.**
2. **A Bilevel *p*-Median Problem for the Planning and Protection of Critical Infrastructure.**

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Questions & Comments?