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CONVERSION FROM BRICK-AND-MORTAR TO CLICK-AND-MORTAR BUSINESS MODEL IN RETAILING

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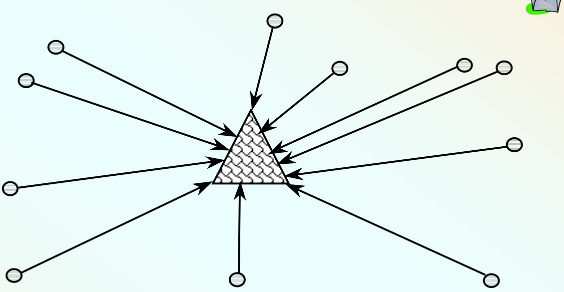
Conversion from BM to CM

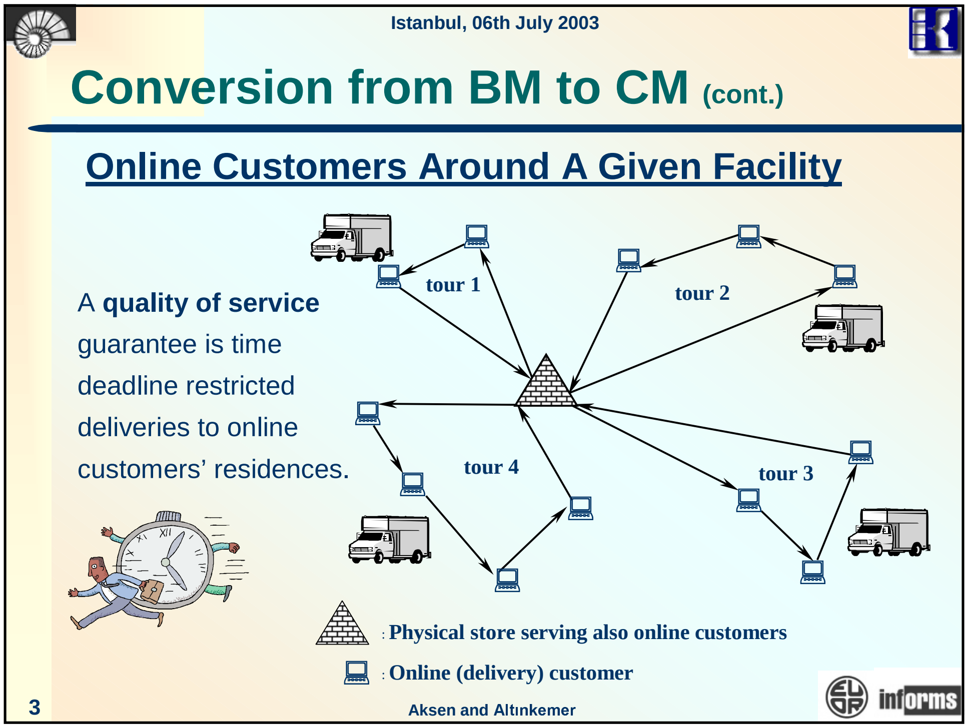
Walk-in Customers Around A Physical Store

An approximate metric of **proximity** and **quality of service** of stores for walk-in customers



△ : Physical store

○ : Walk-in customer’s residence



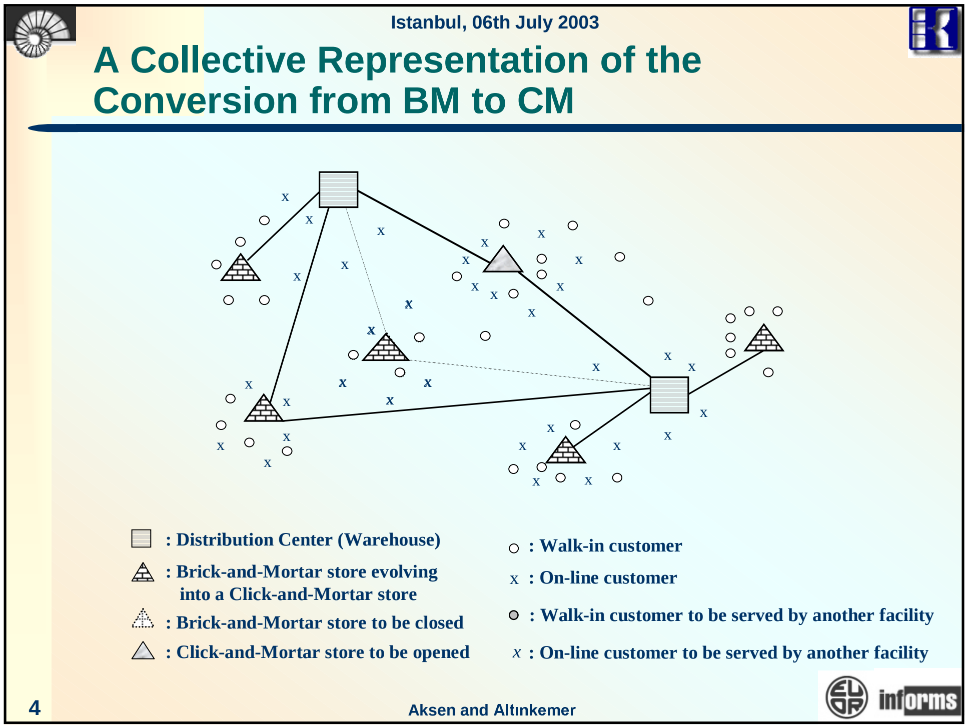
Conversion from BM to CM (cont.)

Online Customers Around A Given Facility

A **quality of service** guarantee is time deadline restricted deliveries to online customers' residences.

▲ : Physical store serving also online customers

🖥 : Online (delivery) customer



A Collective Representation of the Conversion from BM to CM

□ : Distribution Center (Warehouse)

▲ : Brick-and-Mortar store evolving into a Click-and-Mortar store

◬ : Brick-and-Mortar store to be closed

△ : Click-and-Mortar store to be opened

○ : Walk-in customer

x : On-line customer

● : Walk-in customer to be served by another facility

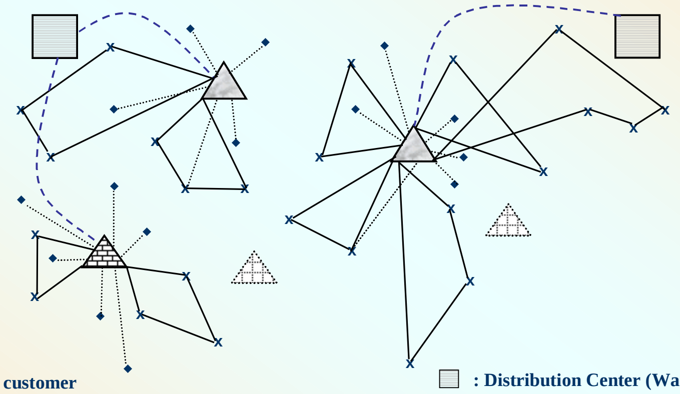
x : On-line customer to be served by another facility

Research Problem and Methods

- A combination of discrete multi-source facility location / customer allocation problem (**FLAP**) with a static multi-depot time deadline constrained vehicle routing problem (**MDVRP-TD**).
- Both problems are shown to be $\mathcal{NP}$-*hard*.
- State-of-the-art solution methods required:
  - *Decomposition through Lagrangian Relaxation*
  - Heuristic techniques

Description of a Static Composite Mathematical Model

A picture visually exhibiting a feasible solution to the problem CMBM-Static



◆ : Walk-in customer

x : Online customer

········ : Driving path of a walk-in customer

– – – : Transfer of goods from warehouses to stores

—— : Vehicle route to an online customer

□ : Distribution Center (Warehouse)

△ : Brick-and-Mortar store evolving into a Click-and-Mortar store

△ : Brick-and-Mortar store to be closed

△ : Click-and-Mortar store to be opened

An Approximation of an Efficient Frontier to CMBM-S

- **Approximation because… heuristic, not optimal.**
- **Given any arbitrary location plan:**
  - Check whether there is at least one open store for each online customer within his/her “time deadline (**TD**) satisfying” reach.
    - ⇨ **Preliminary Feasibility Check**
- **If the given location plan is feasible:**
  - Assign each walk-in customer to the nearest open store within his/her maximum walking distance.
    - If there is no such open store, consider his/her demand **lost**.

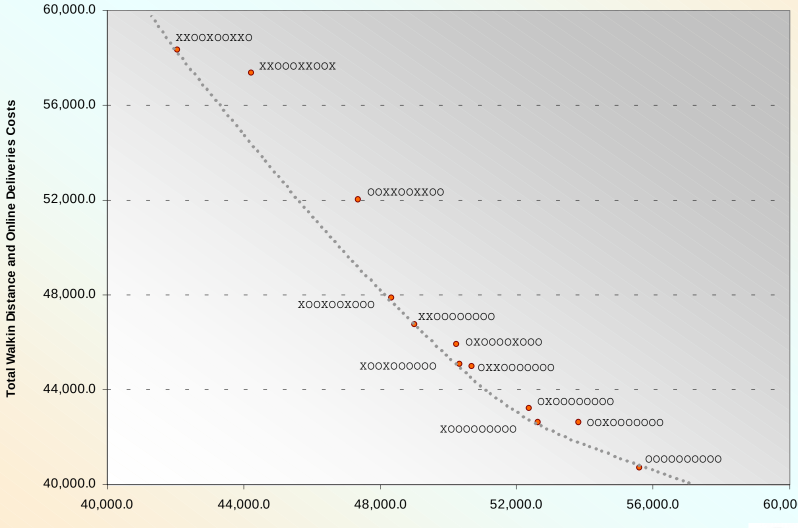
An Approximation of an Efficient Frontier to CMBM-S (cont.)

- Taking the opened CM and converted BM stores as depots, solve a **Multi-Depot Vehicle Routing Problem (MDVRP-TD)** for the online customers.
  - We use a hybrid **PUSH-FORWARD INSERTION / NEAREST NEIGHBOURHOOD** search heuristic (**PFIH-NN**) modified to account for TD constraints and complemented by a **1-0 exchange Local Post Optimization** (**LPO**) routine.
- Calculate each open store’s total sales to customers, and supply this amount from the nearest warehouse.

An Approximation of an Efficient Frontier to CMBM-S (cont.)

- Implemented on a large-scale problem:
  - 250 online and 250 walk-in customers,
  - 1 warehouse,
  - 10 candidate CM store locations,
  - no currently open BM stores.
- Problem Space: 2000 x 1000 distance units [**EUCLIDEAN**]
  - WH in the center of the problem space.
  - Walk-in customers clustered heterogeneously in 10 regions of the problem space with equidistant centers.
  - Online customers dispersed uniformly.
  - Candidate CM store locations coincide with the centers of the 10 clusters.

An Approximation of an Efficient Frontier to CMBM-S (cont.)



An Initial Heuristic Feasible Solution (Z_{ub}) in polynomial time

- First, find a "feasible" location plan in polynomial time:
  - Convert any BM store into CM if it is the nearest store for any customer.
  - Open a new CM store if and only if its opening is indispensable to the fulfillment of one or more online customers' QoS guarantees. Otherwise, do not open a new CM store.
- Then, the associated "best" allocation plan :
  - Assign each customer to the nearest appropriate Open Store.
  - Solve for each store serving online customers a separate single depot VRP with time deadlines (VRP-TD).

Lagrangian Relaxation for CMBM-S

LR Approach to the comprehensive static problem

Min. z = *Total Costs with all traveling distances converted into $$$*

s.t. :

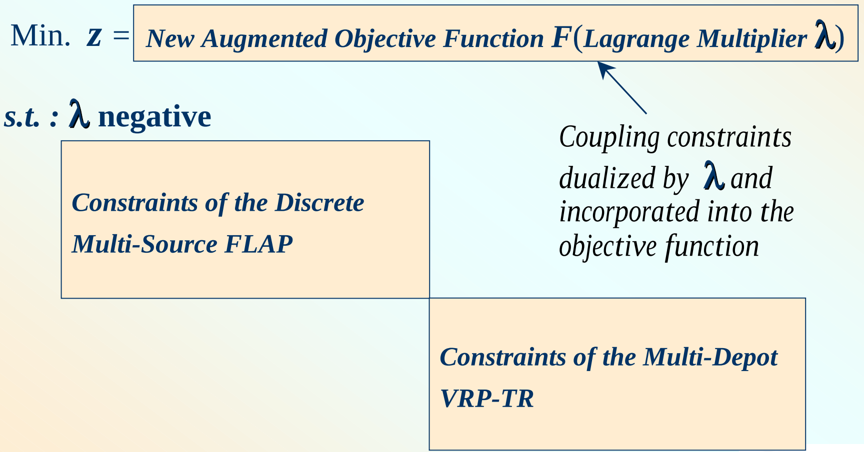
Constraints of the Discrete Multi-Source facility location/allocation problem

Coupling constraints linking the two subproblems $\geq$ *RHS values*

Constraints of the Multi-Depot vehicle routing problem with time deadlines

Lagrangian Relaxation (cont.)

Lagrangian Relaxed Problem



Min. z = *New Augmented Objective Function* F(*Lagrange Multiplier* λ)

s.t. : λ **negative**

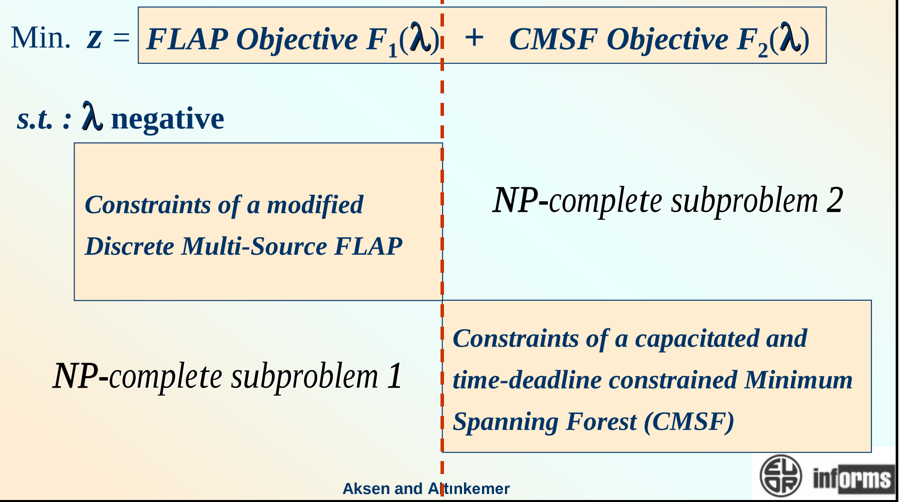
Coupling constraints dualized by λ *and incorporated into the objective function*

Constraints of the Discrete Multi-Source FLAP

Constraints of the Multi-Depot VRP-TR

Lagrangian Relaxation (cont.)

Lagrangian Relaxed Problem is now separable.



Min. z = ***FLAP Objective*** $F_1(\lambda)$ + ***CMSF Objective*** $F_2(\lambda)$

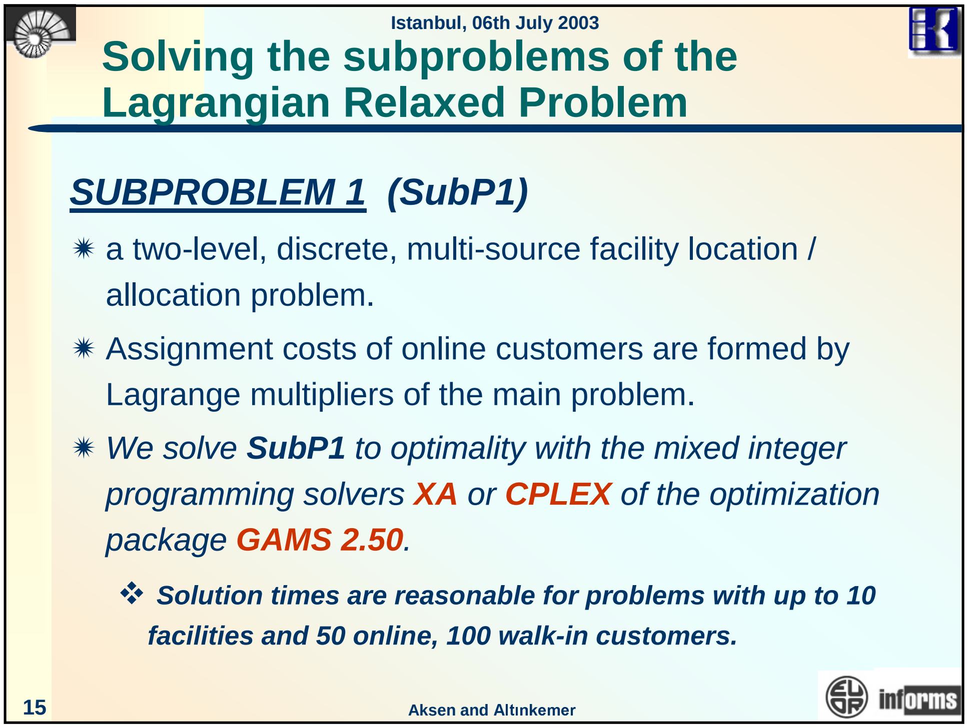
s.t. : λ **negative**

Constraints of a modified Discrete Multi-Source FLAP

NP-complete subproblem 2

NP-complete subproblem 1

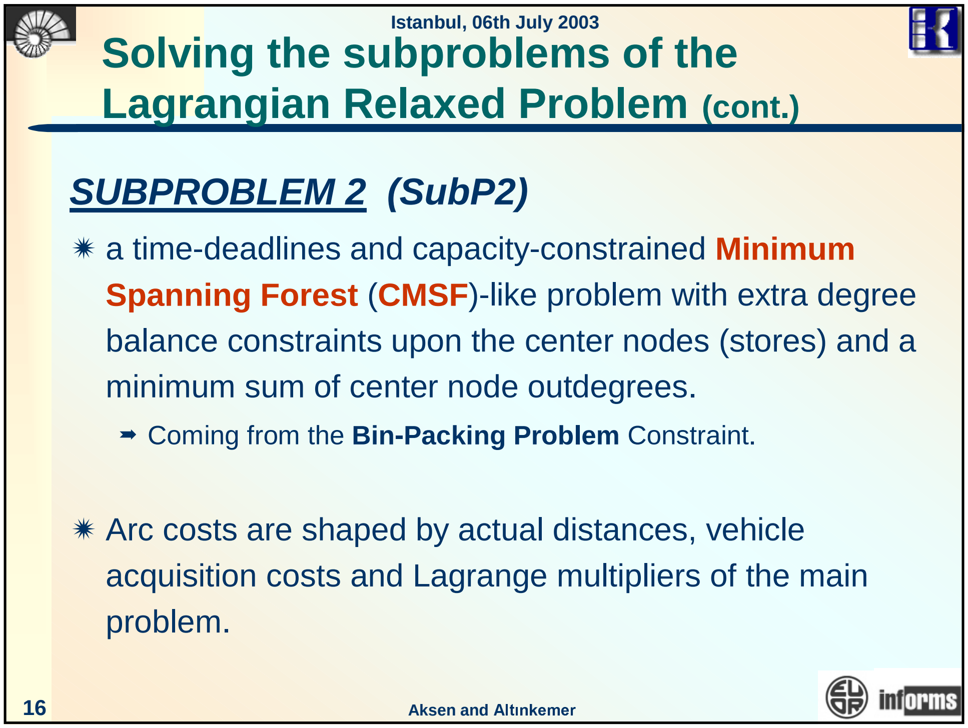
Constraints of a capacitated and time-deadline constrained Minimum Spanning Forest (CMSF)



Solving the subproblems of the Lagrangian Relaxed Problem

SUBPROBLEM 1 ***(SubP1)***

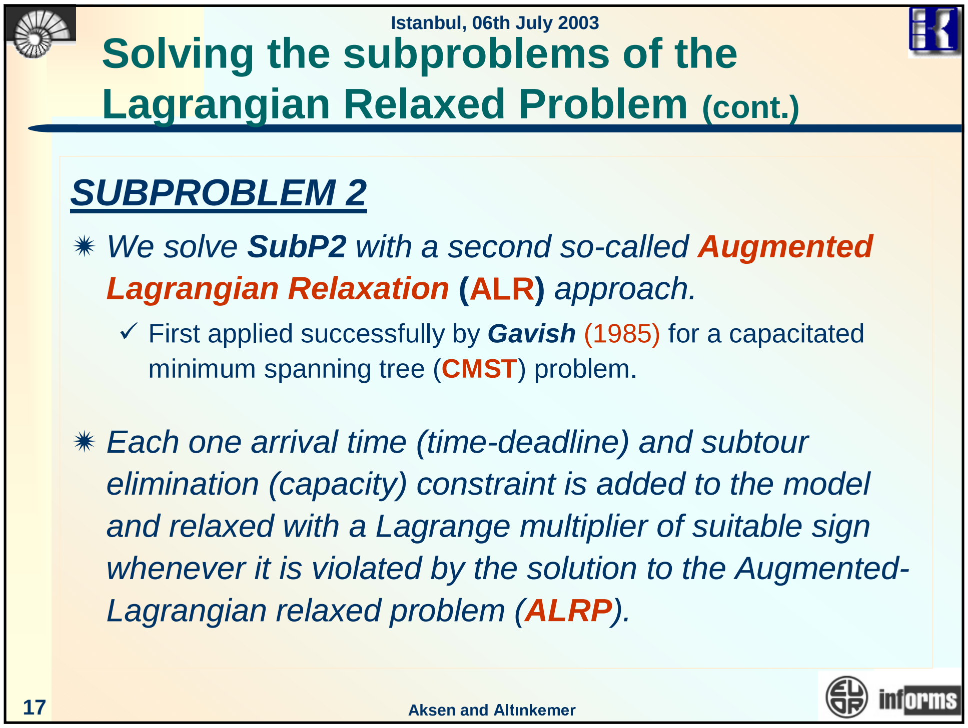
- a two-level, discrete, multi-source facility location / allocation problem.
- Assignment costs of online customers are formed by Lagrange multipliers of the main problem.
- *We solve **SubP1** to optimality with the mixed integer programming solvers **XA** or **CPLEX** of the optimization package **GAMS 2.50**.*
  - ***Solution times are reasonable for problems with up to 10 facilities and 50 online, 100 walk-in customers.***



Solving the subproblems of the Lagrangian Relaxed Problem (cont.)

SUBPROBLEM 2 ***(SubP2)***

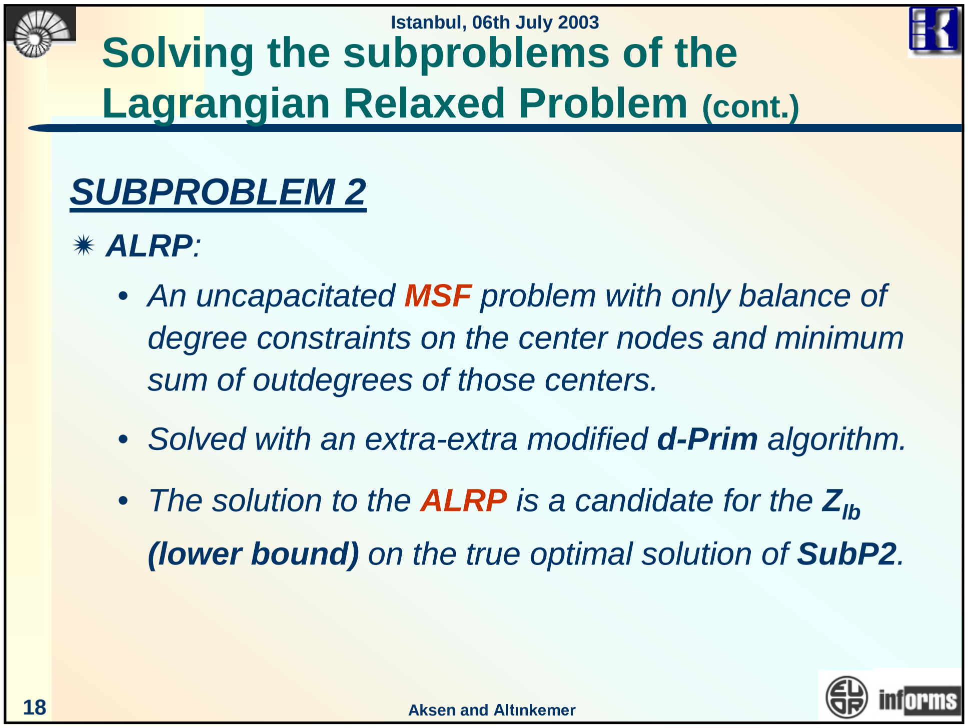
- a time-deadlines and capacity-constrained **Minimum Spanning Forest** (**CMSF**)-like problem with extra degree balance constraints upon the center nodes (stores) and a minimum sum of center node outdegrees.
  - Coming from the **Bin-Packing Problem** Constraint.
- Arc costs are shaped by actual distances, vehicle acquisition costs and Lagrange multipliers of the main problem.



Solving the subproblems of the Lagrangian Relaxed Problem (cont.)

SUBPROBLEM 2

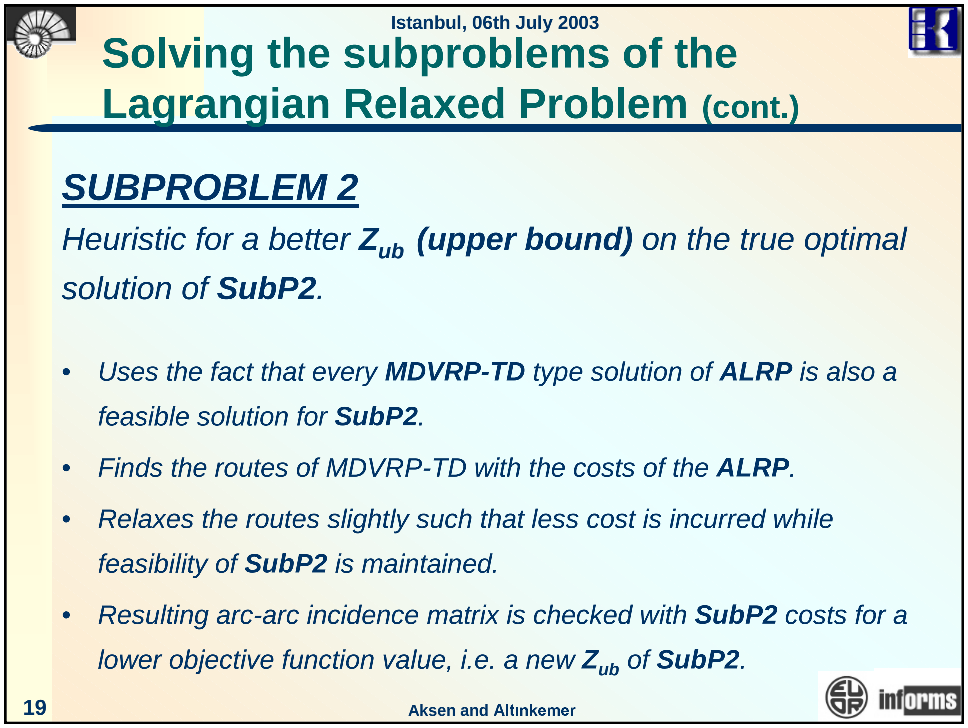
- *We solve **SubP2** with a second so-called **Augmented Lagrangian Relaxation** (**ALR**) approach.*
  - ✓ First applied successfully by ***Gavish*** (1985) for a capacitated minimum spanning tree (**CMST**) problem.
- *Each one arrival time (time-deadline) and subtour elimination (capacity) constraint is added to the model and relaxed with a Lagrange multiplier of suitable sign whenever it is violated by the solution to the Augmented-Lagrangian relaxed problem (**ALRP**).*



Solving the subproblems of the Lagrangian Relaxed Problem (cont.)

SUBPROBLEM 2

- ***ALRP**:*
  - *An uncapacitated **MSF** problem with only balance of degree constraints on the center nodes and minimum sum of outdegrees of those centers.*
  - *Solved with an extra-extra modified **d-Prim** algorithm.*
  - *The solution to the **ALRP** is a candidate for the $\boldsymbol{Z_{lb}}$ **(lower bound)** on the true optimal solution of **SubP2**.*

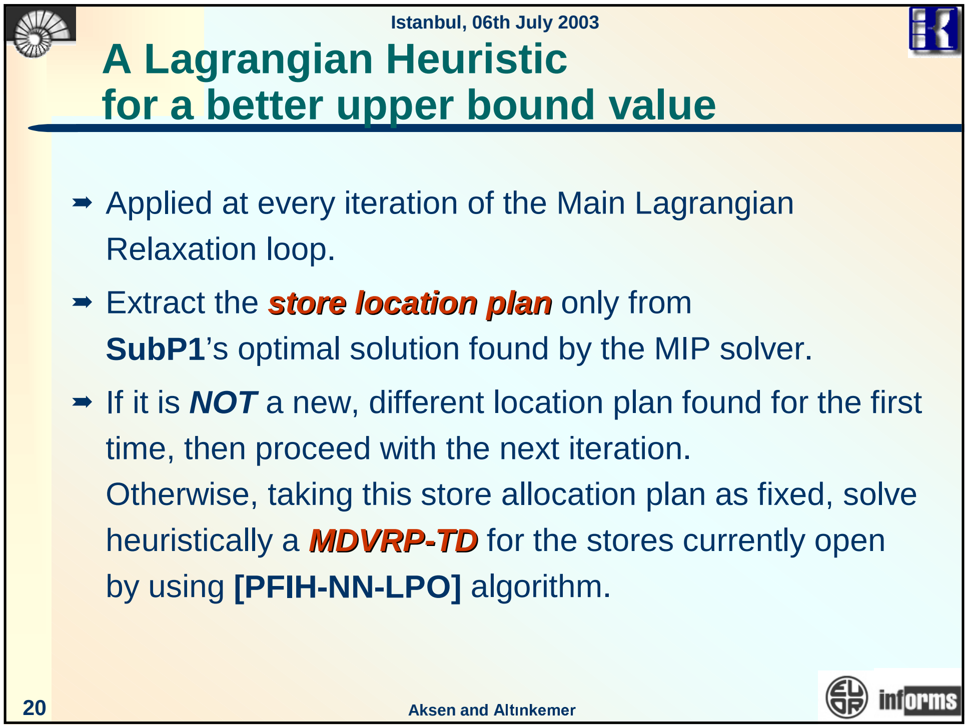


Solving the subproblems of the Lagrangian Relaxed Problem (cont.)

SUBPROBLEM 2

Heuristic for a better ***Z_{ub} (upper bound)*** *on the true optimal solution of* ***SubP2****.*

- *Uses the fact that every* ***MDVRP-TD*** *type solution of* ***ALRP*** *is also a feasible solution for* ***SubP2****.*
- *Finds the routes of MDVRP-TD with the costs of the* ***ALRP****.*
- *Relaxes the routes slightly such that less cost is incurred while feasibility of* ***SubP2*** *is maintained.*
- *Resulting arc-arc incidence matrix is checked with* ***SubP2*** *costs for a lower objective function value, i.e. a new* ***Z_{ub}*** *of* ***SubP2****.*



A Lagrangian Heuristic for a better upper bound value

- Applied at every iteration of the Main Lagrangian Relaxation loop.
- Extract the ***store location plan*** only from **SubP1**'s optimal solution found by the MIP solver.
- If it is ***NOT*** a new, different location plan found for the first time, then proceed with the next iteration.
  Otherwise, taking this store allocation plan as fixed, solve heuristically a ***MDVRP-TD*** for the stores currently open by using **[PFIH-NN-LPO]** algorithm.

How are lower and upper bounds on CMBM-S obtained?

LOWER BOUND Z_{lb} on CMBM-S

Initial Value = $-\infty$

Candidate Value = $Z_{SubP1} + Zlb_{SubP2}$

*The optimal objective value of **FLAP-like SubP1***

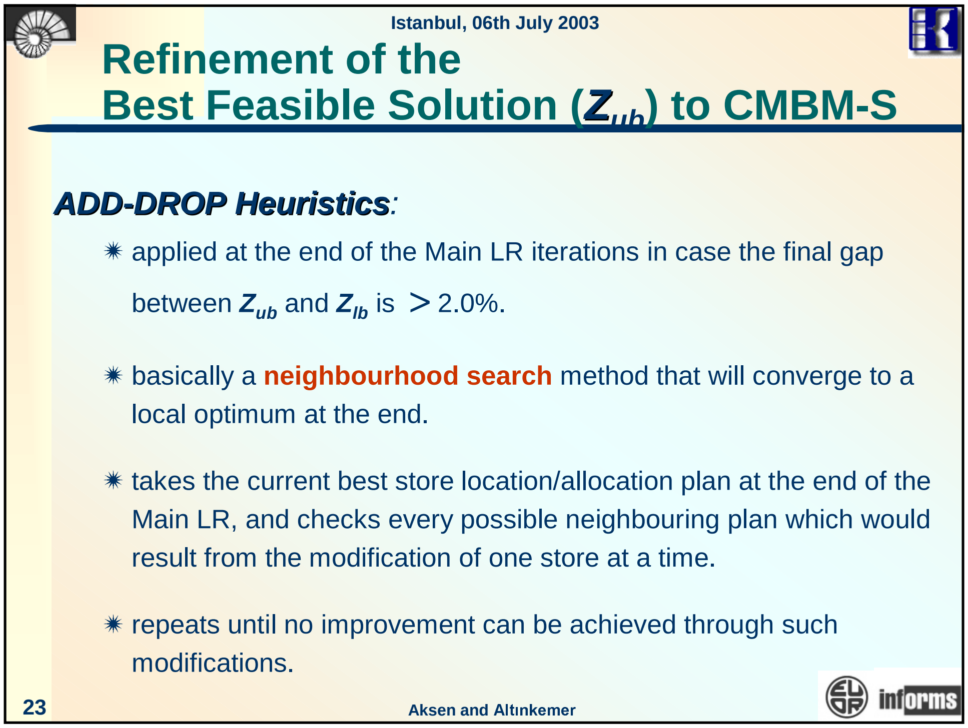
*The final lower bound on the objective function of **CMSF-like SubP2** found at the end of Augmented Lagrangian Relaxation iterations*

How are lower and upper bounds on CMBM-S obtained? (cont.)

UPPER BOUND Z_{ub} on CMBM-S

Initial Value = Objective value of the initial heuristic feasible solution which converts all BM stores into CM stores, opens a new CM store only if necessary to meet *TD's,* and solves a separate VRP-TD for each opened CM and converted BM store.

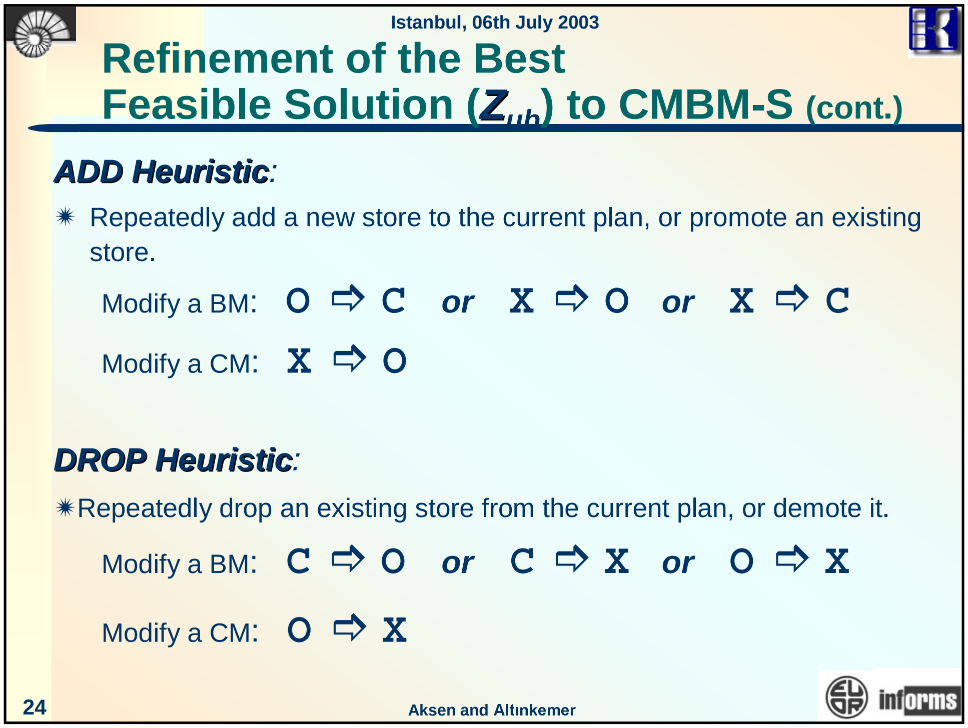
Candidate Value = Objective value of the ***MDVRP-TD*** solved by [**PFIH-NN-LPO**] heuristic with respect to a ***NEW optimal*** store location/allocation plan coming from FLAP-like ***SubP1***.



Refinement of the Best Feasible Solution (Z_{ub}) to CMBM-S

ADD-DROP Heuristics:

- applied at the end of the Main LR iterations in case the final gap between Z_{ub} and Z_{lb} is $>$ 2.0%.
- basically a **neighbourhood search** method that will converge to a local optimum at the end.
- takes the current best store location/allocation plan at the end of the Main LR, and checks every possible neighbouring plan which would result from the modification of one store at a time.
- repeats until no improvement can be achieved through such modifications.



Refinement of the Best Feasible Solution (Z_{ub}) to CMBM-S (cont.)

ADD Heuristic:

- Repeatedly add a new store to the current plan, or promote an existing store.

Modify a BM: O ⇨ C *or* X ⇨ O *or* X ⇨ C

Modify a CM: X ⇨ O

DROP Heuristic:

- Repeatedly drop an existing store from the current plan, or demote it.

Modify a BM: C ⇨ O *or* C ⇨ X *or* O ⇨ X

Modify a CM: O ⇨ X

Coding and Testing of `ANA-cmbm.C` and its variants/accessories

`ana (`*`n.`*`): 1.` *`mother, Godmother.`* `2.` *`main, primary.`*

- ✓ Codes written in ANSI C making them portable to any platform, and compiled with MS-Visual C++ 6.0 (© 1994-98 Microsoft Corporation).
- ✓ Tests and coding performed on a Compaq Presario *5700T* Pentium III 500 MHz PC with 384 MB RAM. GAMS 2.50 models written and run on the same computer.
- ✓ All-subsets-generator code used for the explicit representation of ($2^N - N - 1$) subtour elimination and capacity constraints.

  The pseudo-code adopted from
  "Combinatorial Algorithms: Theory & Practice,"
  Edward M. Reingold, Jürg Nievergelt, Narsingh Deo, Prentice-Hall 1977.

Tests Results

Different Random Number Seed w/ Euclidean and Rectilinear Distances

Distance Measure	Seed	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
Euclidean	519	C | XX	1854.89	1991.00	7.34%	69
	622	C | XX	1563.35	1589.00	1.64%	300
	1204	C | XX	1883.63	1994.00	5.86%	299
	2002	C | XX	1752.00	1857.00	5.99%	273
Rectilinear	519	C | XX	2203.12	2399.00	8.89%	299
	622	C | XX	1820.36	1848.00	1.52%	249
	1204	C | XX	2209.88	2369.00	7.20%	297
	2002	C | XX	2059.10	2258.00	9.66%	253

Tests Results (cont.)

Different No. Customers w/ 1 WH, 1 BM and 2 CMs

Distance Measure	No. Customers	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
Euclidean	(10, 10)	C | XX	1232.00	1232.00	0.00%	26
	(15, 15)	C | XX	1563.35	1589.00	1.64%	300
	(20, 20)	C | XX	2005.21	2127.00	6.07%	265
	(25, 25)	C | XX	2834.64	3021.00	6.57%	126
Rectilinear	(10, 10)	C | XX	1372.99	1373.00	0.00%	146
	(15, 15)	C | XX	1990.31	2038.00	2.40%	23
	(20, 20)	C | XX	2523.29	2601.00	3.08%	298
	(25, 25)	C | XX	3225.92	3473.00	7.66%	83

Tests Results (cont.)

`ANA-cmbm` on 8 medium-size problems with Euclidean Distances

Prob. Name	No. Facilities [WH, BM, CM]	No. Customers [Online, Walk-in]	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
Le01	(1, 2, 3)	(18, 32)	CO | XXX	6055.05	6373.00	5.25%	63
Le02	(1, 2, 3)	(20, 30)	CO | XXX	6123.21	6581.30	7.48%	300
Le03	(1, 2, 3)	(20, 30)	CO | XXX	9429.19	9747.00	3.37%	295
Le04	(1, 2, 2)	(20, 30)	OC | XX	9114.46	9445.00	3.63%	92
Le05	(1, 2, 2)	(20, 30)	CC | XX	9220.40	9614.00	4.27%	198
Le06	(1, 2, 3)	(25, 25)	CC | XXX	6297.11	6663.75	5.82%	293
Le07	(1, 2, 3)	(25, 25)	CC | XXX	6279.68	6717.75	6.98%	117
Le08	(1, 2, 3)	(25, 25)	CC | XXX	6154.97	6505.75	5.70%	73

Tests Results (cont.)

`ANA-cmbm` on 6 large-size problems with Euclidean Distances and erratic cost data

No. Facilities [WH, BM, CM]	No. Customers [Online, Walk-in]	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
(2, 4, 2)	(25, 75)	CXCC | OO	23001.69	24272.42	5.52%	2 Add-Drops
(2, 4, 2)	(30, 70)	CXOC | OO	23312.56	24581.75	5.44%	1 Add-Drop
(1, 2, 3)	(40, 40)	XC | XXO	31249.56	35327.01	13.05%	79
(1, 4, 5)	(40, 40)	XXXX | XXXOX	27444.97	32265.29	17.56%	155
(2, 5, 3)	(50, 50)	CXXXX | XXX	32716.91	43387.00	32.61%	61
(1, 2, 2)	(100, 200)	CC | XX	187717.93	205635.96	9.55%	43

Sensitivity Analysis of Parameters

Test Problem : `Le03-20-50-6` on a rectangle 200 x 200

1 Warehouse, 2 BM and 3 candidate CM stores,

20 online, 30 walk-in customers, Unit demands, Moderate TD's,

Marginal cost of lost sales to walk-in customers: $750

(FC, OC, CC) of BMs: (-250), 500, 1500 $

(FC, OC) of CMs: 1750, 1250 $

Different Levels of *Time Deadline Stringency* on `Le03-20-50-6.ana`

Problem Name	K in U_i	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
`TD1-Le03`	1	CC | XXX	10307.51	10675.00	3.57%	300
`TD2-Le03`	4	CO | XXX	9429.19	9747.00	3.37%	295
`TD3-Le03`	6	CO | XXX	9416.71	9703.00	3.04%	39
`TD4-Le03`	10	CO | XXX	9418.69	9739.00	3.40%	39

Sensitivity Analysis of Parameters (cont.)

Different Levels of *Maximum Walking Distance* on `Le03-20-50-6.ana`

Maximum Walking Distance	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
50 dist. units	XO | OOX	15984.74	16216.00	1.45%	165
100	CO | XXX	9429.19	9747.00	3.37%	295
150	XC | XXX	8244.86	8518.00	3.31%	292
200	XC | XXX	8244.86	8518.00	3.31%	292

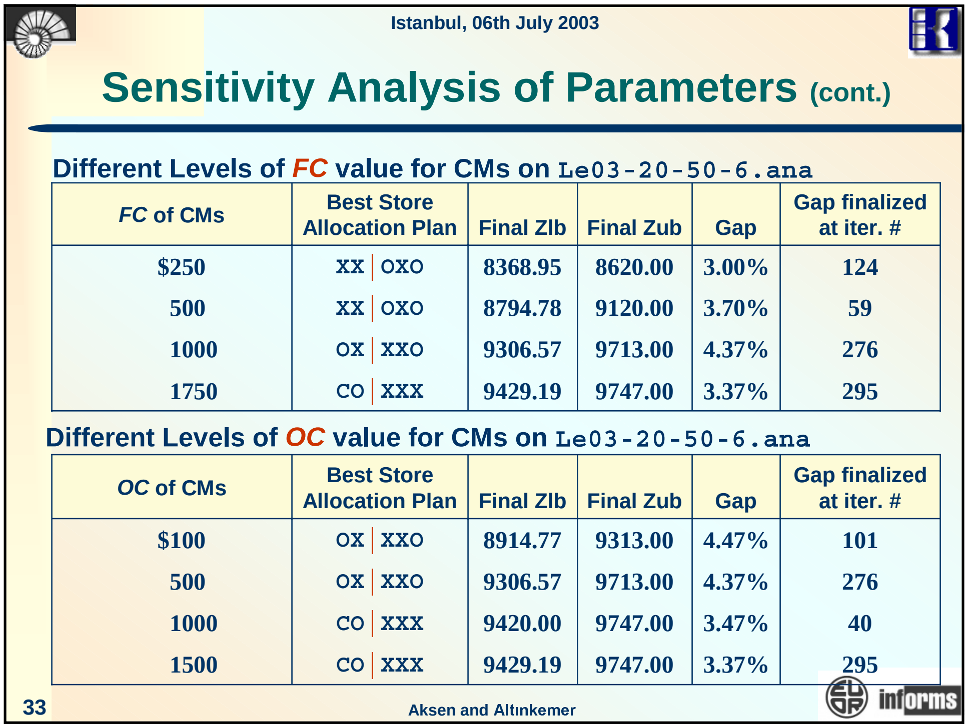
Different Levels of *Gross Marginal Profit from Sales* on `Le03-20-50-6`

Gross Marginal Profit from Walk-in Customers	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
$250	XC | XXX	8624.40	8873.00	2.88%	200
750	CO | XXX	9429.19	9747.00	3.37%	295
1500	CO | XXX	9429.19	9747.00	3.37%	295
2000	CO | XXX	9429.19	9747.00	3.37%	295

Sensitivity Analysis of Parameters (cont.)

Different Levels of *Unit Costs of Traveling* on `Le03-20-50-6.ana`

WH-stores per unit good	Stores-Online Custs	Walk-in Custs - Stores	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
$ 1.00	$ 1.00	$ 1.00	CO | XXX	9429.19	9747.00	3.37%	295
0.10	0.10	0.10	CO | XXX	4815.71	4834.60	0.39%	296
5.00	5.00	5.00	CC | XXX	28851.80	30325.00	5.11%	258
0.01	1.00	1.00	CO | XXX	6947.39	7272.00	4.67%	50
0.01	1.00	0.10	CO | XXX	5480.01	5793.30	5.72%	297
0.01	2.50	0.10	CC | XXX	6922.48	7394.30	6.82%	300
0.01	5.00	0.10	CC | XXX	8823.97	9799.30	11.05%	289
0.10	1.00	0.25	CO | XXX	5937.73	6264.75	5.51%	45



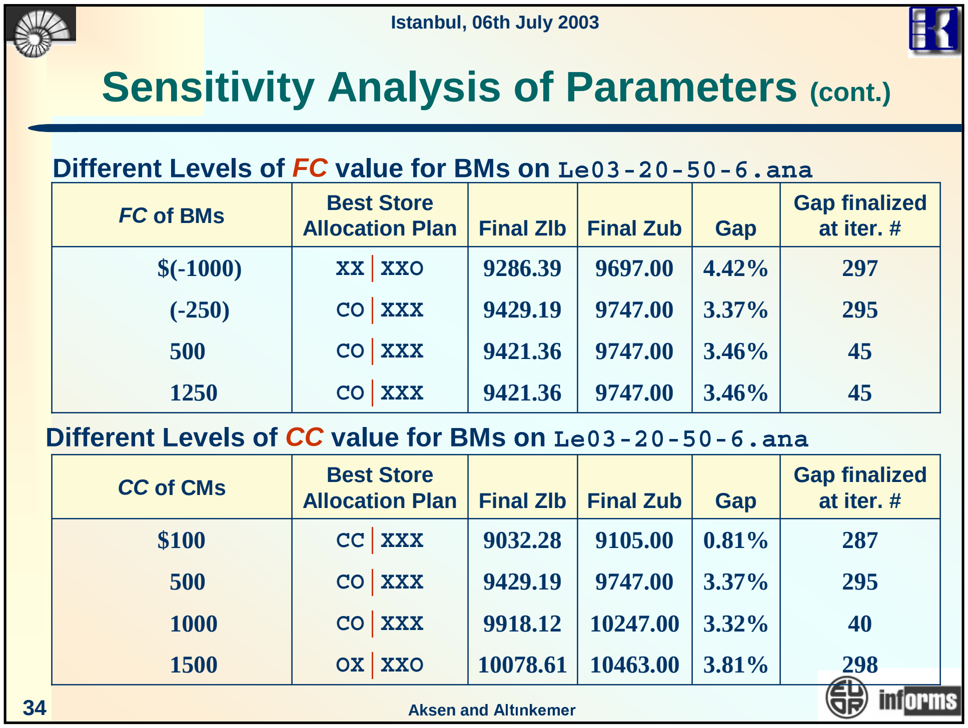
Sensitivity Analysis of Parameters (cont.)

Different Levels of *FC* value for CMs on `Le03-20-50-6.ana`

FC of CMs	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
$250	XX | OXO	8368.95	8620.00	3.00%	124
500	XX | OXO	8794.78	9120.00	3.70%	59
1000	OX | XXO	9306.57	9713.00	4.37%	276
1750	CO | XXX	9429.19	9747.00	3.37%	295

Different Levels of *OC* value for CMs on `Le03-20-50-6.ana`

OC of CMs	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
$100	OX | XXO	8914.77	9313.00	4.47%	101
500	OX | XXO	9306.57	9713.00	4.37%	276
1000	CO | XXX	9420.00	9747.00	3.47%	40
1500	CO | XXX	9429.19	9747.00	3.37%	295



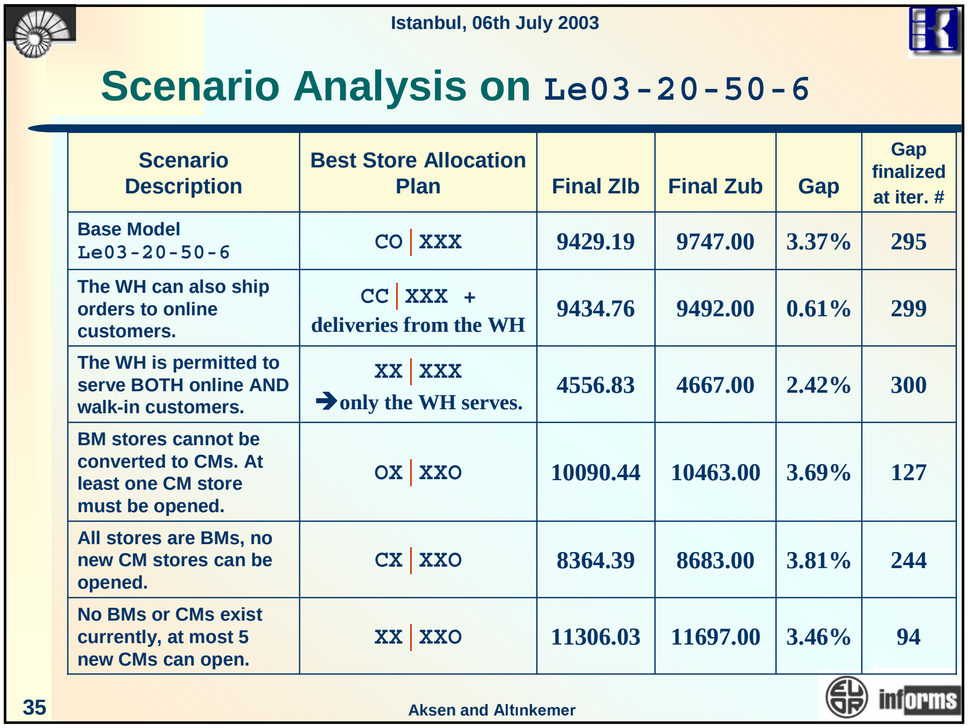
Sensitivity Analysis of Parameters (cont.)

Different Levels of *FC* value for BMs on `Le03-20-50-6.ana`

FC of BMs	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
$(-1000)	XX | XXO	9286.39	9697.00	4.42%	297
(-250)	CO | XXX	9429.19	9747.00	3.37%	295
500	CO | XXX	9421.36	9747.00	3.46%	45
1250	CO | XXX	9421.36	9747.00	3.46%	45

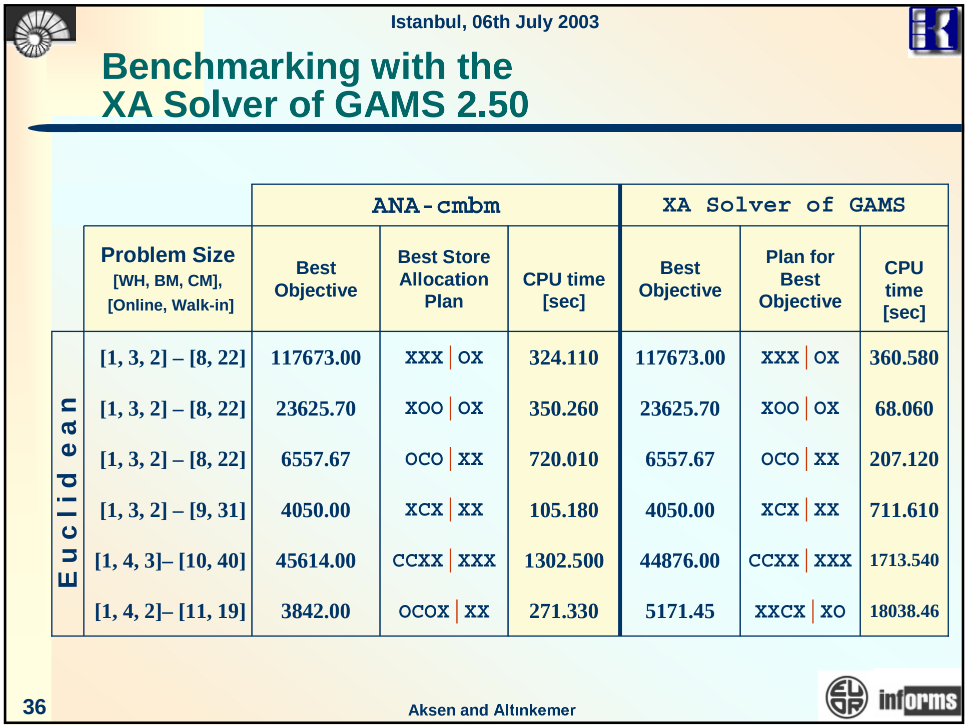
Different Levels of *CC* value for BMs on `Le03-20-50-6.ana`

CC of CMs	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
$100	CC | XXX	9032.28	9105.00	0.81%	287
500	CO | XXX	9429.19	9747.00	3.37%	295
1000	CO | XXX	9918.12	10247.00	3.32%	40
1500	OX | XXO	10078.61	10463.00	3.81%	298



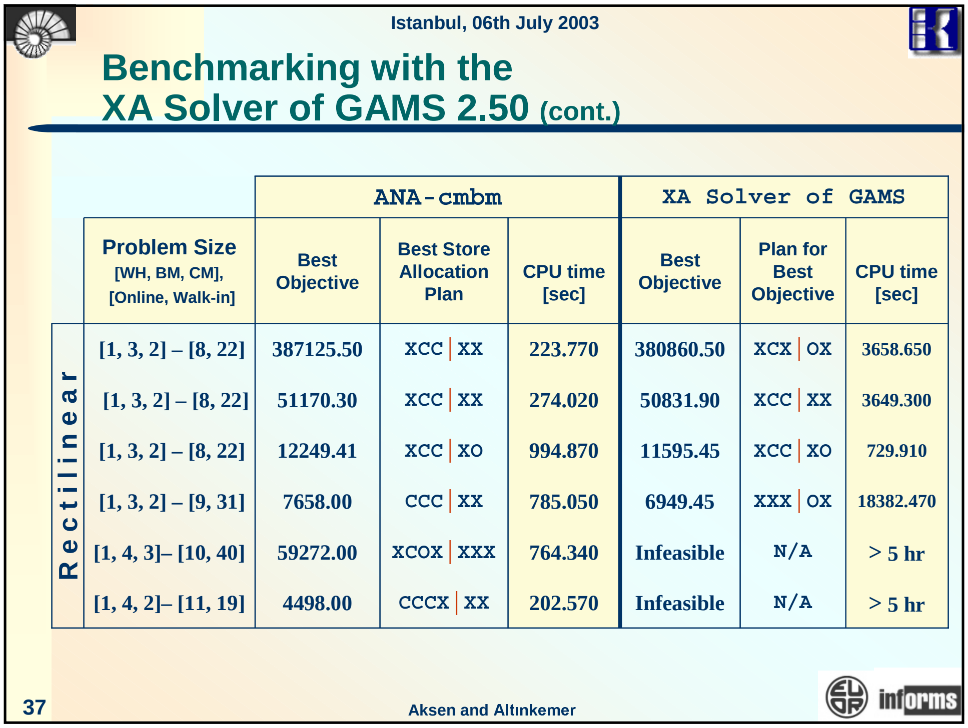
Scenario Analysis on `Le03-20-50-6`

Scenario Description	Best Store Allocation Plan	Final Zlb	Final Zub	Gap	Gap finalized at iter. #
Base Model `Le03-20-50-6`	CO | XXX	9429.19	9747.00	3.37%	295
The WH can also ship orders to online customers.	CC | XXX + deliveries from the WH	9434.76	9492.00	0.61%	299
The WH is permitted to serve BOTH online AND walk-in customers.	XX | XXX → only the WH serves.	4556.83	4667.00	2.42%	300
BM stores cannot be converted to CMs. At least one CM store must be opened.	OX | XXO	10090.44	10463.00	3.69%	127
All stores are BMs, no new CM stores can be opened.	CX | XXO	8364.39	8683.00	3.81%	244
No BMs or CMs exist currently, at most 5 new CMs can open.	XX | XXO	11306.03	11697.00	3.46%	94



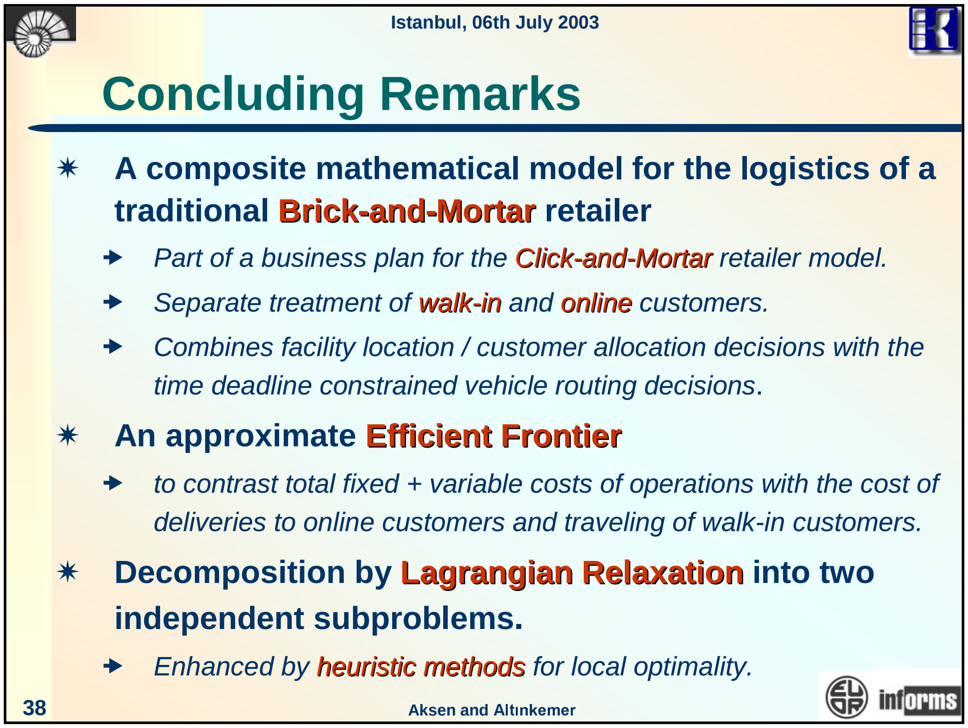
Benchmarking with the XA Solver of GAMS 2.50

		ANA-cmbm			XA Solver of GAMS		
	Problem Size [WH, BM, CM], [Online, Walk-in]	Best Objective	Best Store Allocation Plan	CPU time [sec]	Best Objective	Plan for Best Objective	CPU time [sec]
Euclidean	[1, 3, 2] – [8, 22]	117673.00	XXX | OX	324.110	117673.00	XXX | OX	360.580
	[1, 3, 2] – [8, 22]	23625.70	XOO | OX	350.260	23625.70	XOO | OX	68.060
	[1, 3, 2] – [8, 22]	6557.67	OCO | XX	720.010	6557.67	OCO | XX	207.120
	[1, 3, 2] – [9, 31]	4050.00	XCX | XX	105.180	4050.00	XCX | XX	711.610
	[1, 4, 3]– [10, 40]	45614.00	CCXX | XXX	1302.500	44876.00	CCXX | XXX	1713.540
	[1, 4, 2]– [11, 19]	3842.00	OCOX | XX	271.330	5171.45	XXCX | XO	18038.46



Benchmarking with the XA Solver of GAMS 2.50 (cont.)

		ANA-cmbm			XA Solver of GAMS		
	Problem Size [WH, BM, CM], [Online, Walk-in]	**Best Objective**	**Best Store Allocation Plan**	**CPU time [sec]**	**Best Objective**	**Plan for Best Objective**	**CPU time [sec]**
Rectilinear	[1, 3, 2] – [8, 22]	387125.50	XCC │ XX	223.770	380860.50	XCX │ OX	3658.650
	[1, 3, 2] – [8, 22]	51170.30	XCC │ XX	274.020	50831.90	XCC │ XX	3649.300
	[1, 3, 2] – [8, 22]	12249.41	XCC │ XO	994.870	11595.45	XCC │ XO	729.910
	[1, 3, 2] – [9, 31]	7658.00	CCC │ XX	785.050	6949.45	XXX │ OX	18382.470
	[1, 4, 3]– [10, 40]	59272.00	XCOX │ XXX	764.340	Infeasible	N/A	> 5 hr
	[1, 4, 2]– [11, 19]	4498.00	CCCX │ XX	202.570	Infeasible	N/A	> 5 hr



Concluding Remarks

- **A composite mathematical model for the logistics of a traditional Brick-and-Mortar retailer**
  - *Part of a business plan for the Click-and-Mortar retailer model.*
  - *Separate treatment of walk-in and online customers.*
  - *Combines facility location / customer allocation decisions with the time deadline constrained vehicle routing decisions.*
- **An approximate Efficient Frontier**
  - *to contrast total fixed + variable costs of operations with the cost of deliveries to online customers and traveling of walk-in customers.*
- **Decomposition by Lagrangian Relaxation into two independent subproblems.**
  - *Enhanced by heuristic methods for local optimality.*

Concluding Remarks (cont.)

- **An $\mathcal{NP}$-hard problem – even after LR.**
  - General-purpose MIP solvers cannot compete.
  - Large-size problems have unfavorably long solution times.
- **Trade-off between QoS guarantees and costs of operations.**
- **Location / Allocation plan of stores cost-sensitive.**
  - The higher *OC* and *FC* of a CM store, the less likely to open it.
  - The higher *CC* a BM store, the less likely to convert it.
- **Empirical finding in accordance with intuition:**
  - Delivering online orders from WHs is likely to be a cost-saving option over opening a new physical facility.

Problem Space and Entities of `AP50-100-10`