Istanbul, July 06th 2003

EURO/INFORMS Joint International Meeting 2003

LOSS OF CUSTOMER GOODWILL IN THE SINGLE ITEM LOT-SIZING PROBLEM WITH IMMEDIATE LOST SALES

Deniz Aksen
College of Administrative Sciences and Economics
Koç University
Sarıyer, ISTANBUL

Kemal Altınkemer
Krannert Graduate School of Management
Purdue University
West Lafayette, INDIANA

Introduction: Lot-sizing

- Solution to the tradeoff between inventory holding costs and setup costs.
- Dynamic demand streams or rates bankrupt the nice formulae of EOQ and EOQ with backlogging: $\sqrt{\frac{2 \cdot S_{common} \cdot D_{average}}{h_{common}}}$

$$\sqrt{\frac{2 \cdot S_{common} \cdot D_{average}}{h_{common}} \times \frac{h_{common} + b_{common}}{b_{common}}} \qquad \sqrt{\frac{2 \cdot S_{common} \cdot D_{average}}{b_{common}} \times \frac{h_{common}}{h_{common} + b_{common}}}$$

- When also cost data is changing over time, then lot-sizing techniques become indispensable.

Introduction: Lot-sizing

- **Pioneering work: Wagner-Whitin Model W-W (1958)**
- **Single-item, deterministic, uncapacitated, finite planning horizon, time-variant demand and setup costs, zero production (procurement) costs.**
- **Also time-variant production and inventory holding costs.**

$$c_t, h_t, S_t, d_t$$

Lot-sizing Literature

- **Alternative Variants and Solution Techniques for the W-W Model <u>without backlogging</u> (w/o backordering) :**
  - **Faster Myopic Lot-sizing Heuristics**
    - **Silver** and **Meal** (1973)
    - **Groff** (1979
  - **Faster Optimal Algorithms for the W-W model with uniform costs**
    - **Federgruen** and **Tzur** (1990) : $O(T \cdot log\ T)$ or $O(T)$
    - **Wagelmans, Van Hoesel,** and **Kolen** (1991) $O(T \cdot log\ T)$ or $O(T)$
  - **Forecasting demand beyond the planning horizon**
    - **Stadtler** (2000) : rolling schedules

Lot-sizing Literature

- **The version with <u>backlogging</u>: satisfy demand in a later period of the planning horizon.**
  - **Zangwill** (1969, 1986)
  - **Blackburn** and **Kunreuther** (1974)
  - **Pochet** and **Wolsey** (1988)
  - **Webster** (1989)
  - **Choo** and **Chan** (1990)
  - **Gupta** and **Brennan** (1992)

Istanbul, 06th July 2003

Lot-sizing Literature

- **<u>Capacitated</u> <u>Multi-Item</u> Version of Dynamic Lot-Sizing:**
  - **Eisenhut** (1975)
  - **Lambrecht** and **Vanderveken** (1979)
  - **Dixon** and **Silver** (1981)
  - **Doğramacı**, **Panayiotopoulos**, and **Adam** (1981)
  - **Karni** and **Roll** (1982)
  - **Thizy** and **Van Wassenhove** (1985)
  - **Fleischmann** (1988)
  - **Kırca** and **Kökten** (1994)

Lot-sizing Literature

- **Dynamic Lot-Sizing with Lost Sales : demand is not backlogged, but it does not have to be satisfied either.**
  - **Sandbothe** and **Thompson** (1990, 1993)
  - **Loss of demand incurs a unit cost.**
  - **There are uniform or period-dependent capacity constraints.**
  - **All costs are time-invariant.**

p, c, h, S, d_t

Istanbul, 06th July 2003

Lot-sizing Literature

- **Excellent Surveys:**
  - **Maes** and **Van Wassenhove** (1988)
    - Multi-item single-level capacitated lot-sizing heuristics: a general review
      - *Journal of Operational Research Society* **39** (11) 991-1004.
  - **Baker** (1989)
    - Lot-sizing procedures and a standard data set. A reconciliation of the literature.
      - *Journal of Manufacturing Operations Management* **2** 199-221.
  - **Saydam** and **Evans** (1990)
    - A comparative performance analysis of the Wagner-Whitin algorithm and lot-sizing heuristics.
      - *Computers & Industrial Engineering* **18** (1) 91-93.

Lot-sizing Literature

- **Excellent Surveys:**
  - **Salomon, Kroon, Kuik**, and **Van Wassenhove** (1991)
    - Some extension of the discrete lotsizing and scheduling problem.
      - *Management Science* **37** (7) 801-812.
  - **Kuik, Salomon** and **Van Wassenhove** (1994)
    - Batching decisions: Structure and models.
      - *European Journal of Operational Research* **75** (2) 243-263.

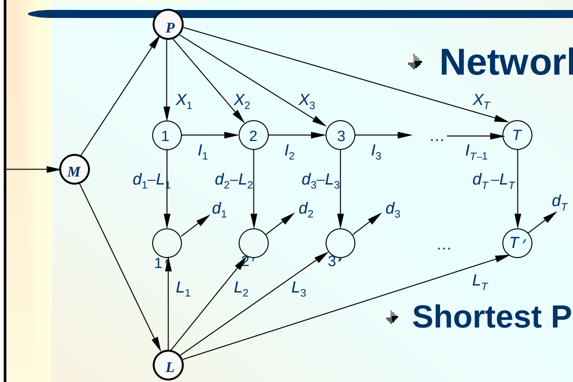
Solving Dynamic Lot-sizing Models

- **W-W model solved to optimality with a Forward-Recursive Dynamic Programming (DP) algorithm requiring $O(T^2)$ CPU time and memory storage.**
  - Efficient Implementation by **James R. Evans** (1985)

$$M_{jk} = S_j + c_j \sum_{t=j}^{k} d_t + \sum_{t=j}^{k-1} h_t \times \sum_{r=t+1}^{k} d_r$$

$$\boldsymbol{F}_k = \underset{1 \le j \le k}{minimum}\left[\boldsymbol{F}_{j-1} + M_{jk}\right], \quad j_k^* = \underset{1 \le j \le k}{argmin}\left[\boldsymbol{F}_{j-1} + M_{jk}\right]$$

Solving Single-Item Dynamic Lot-sizing Models



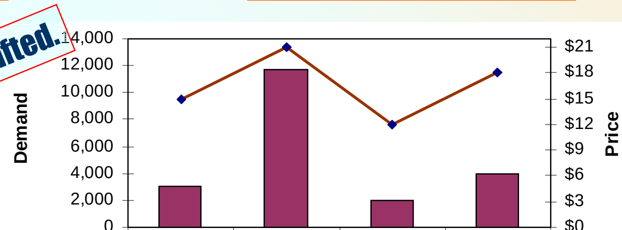
- Network Representations
- Shortest Path (SP) network and Dijkstra's Algorithm
  - For the W-W model w/o backlogging
- A concave cost network flow problem
  - For the W-W model w/ backlogging
  - For the W-W model w/ lost sales

Modifying the model of Sandbothe – Thompson (1990)

Aksen, Altınkemer, Chand (2003)

p, c, h, S, d_t → p_t, c_t, h_t, S_t, d_t

Capacity constraints lifted.



	Spring	Summer	Fall	Winter
Demand	3,000	11,750	2,000	4,000
Price	$15	$21	$12	$18

Season

Seasonal demand levels vs. prices

The AAC model of dynamic lot-sizing

- **Maximize the net profit from realized sales.**
- **An efficient algorithm with $O(T^2)$ CPU time and memory storage.**
- **Exploit the structural properties of an optimal solution.**
- **Inspiration from the *DP* Algorithm of the W-W Model.**
- **Gross Marginal Profit is never negative!**

Lemma 1. $L^*_t \cdot X^*_t = 0 \quad \forall t \in \{1, 2, \ldots, T\}$

Lemma 2. $I^*_{t-1} \cdot X^*_t = 0 \quad \forall t \in \{1, 2, \ldots, T\}$

Zero-Inventory Ordering Policy of W-W solution

Lemma 3. $L^*_t \cdot (d_t - L^*_t) = 0 \quad \forall t \in \{1, 2, \ldots, T\}$

The AAC model of dynamic lot-sizing

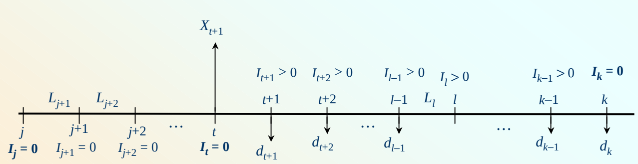
Definition 1a. *Period t is a* ***regeneration period*** *if* $(I_t = 0)$ *and* $(L_t = 0)$.

Definition 1b. *Period t is a* ***production period*** *if* $(X_t > 0)$.

Definition 1c. *Period t is a* ***loss period*** *if* $(I_t > 0)$.

Definition 1d. *Period t is a* ***stockout period*** *if* $(I_t > 0)$ *and* $(I_t = 0)$.

Definition 1e. *Period t is a* ***conservation period*** *if* $(L_t > 0)$ *and* $(I_t > 0)$.



Optimally solving the AAC lot-sizing model

- **Forward Recursive *DP* Formulation with cumulative "best cost of handling" definitions**

Tradeoff $MC_{jt} = minimum\{ p_t ,\ c_j + H_{t-1} - H_{j-1}\}$

Linked List $\mathcal{K}_j = \{k \in \langle j+1, T \rangle : \ c_j + H_{k-1} - H_{j-1} > p_k\}$

Minimum Cost of handling periods $\langle 1, t\rangle$ *with a last production made in period j*

$$\boldsymbol{C}_j(t) = \boldsymbol{C}_j(t-1) + MC_{jt} \cdot d_t$$
$$\boldsymbol{C}_t(t) = \boldsymbol{F}_{t-1} + S_t + c_t \cdot d_t$$

$$\boldsymbol{F}_t = \underset{j \in \langle 0, t\rangle}{minimum}\{\boldsymbol{C}_j(t)\}, \quad j_t^* = \underset{j \in \langle 0, t\rangle}{argmin}\{\boldsymbol{C}_j(t)\}$$

Expanding the AAC model: Loss of Customer Goodwill

- **Cost of Losing Demand $>$ Gross Profit of Lost Demand**
- **Diminishing impact on the next demand : Goodwill Loss**
- **Extends to exactly one period following the current**
- **Effective Demand = Original Demand – Diminishing Impact**
- $e_t = d_t - LG_t$
- **Diminishing impact in period *t* depends *ONLY* on the amount of effective demand *PURPOSELY NOT SATISFIED* in period $(t-1)$**
- $e_t = max\{0, d_t - \beta \cdot LU_{t-1}\}$ ⟸ **Needs a pretty good reformulation**
- **Total Lost Demand in period t :** $(LG_t + LU_t)$
- **Total Sales in period t :** $(e_t - LU_t) = d_t - (LG_t + LU_t)$

The AAC-GWL model: Loss of Customer Goodwill

- **The intentional loss of demand in period t cannot exceed the effective demand of that period.**
- $LU_t \leq e_t$
- **A logical constraint:** $e_1 = d_1$
- **A new inventory balance constraint in period t**
- $I_{t-1} + X_t - (e_t - LU_t) = I_t$
- **Indication of procurement activity in period t**
- $X_t \leq \sum_{r=t}^{T} d_r y_t$

The AAC-GWL model: Loss of Customer Goodwill

Lost goodwill effects of different number of loss periods

Period	Original Demand	Effective Demand	Satisfied Demand	Lost Effective Demand	Total Lost Demand
May	2,000	2,000	0	2,000	2,000
June	2,000	1,000	0	1,000	2,000
July	3,000	2,500	0	2,500	3,000
August	4,000	**2,750**	2,750	0	1,250

Solving the AAC-GWL model

- *NP-complete* problem?
- An optimal procedure to solve?
- PROPOSAL
  - Use the optimal procurement / loss schedule of the associated AAC model w/o goodwill considerations.
  - Exhaustively "Search and Restore" the pseudo-optimality of that solution for the Goodwill Loss (GWL) case.
  - Update the 1st and 3rd lemmae as follows (use Lemma 2 "as is.")

Lemma 1. $LU^*_t \cdot X^*_t = 0 \qquad \forall t \in \{1, 2, \ldots, T\}$

Lemma 3. $LU^*_t \cdot (e_t - LU^*_t) = 0 \qquad \forall t \in \{1, 2, \ldots, T\}$

Istanbul, 06th July 2003

Solving the AAC-GWL model

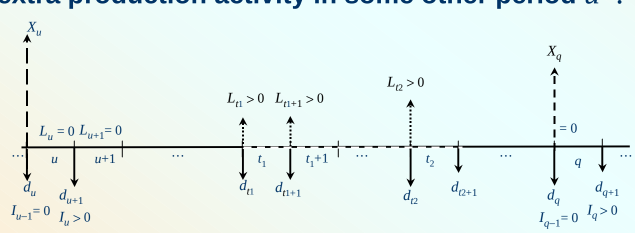
- Outline of the SEARCH-AND-RESTORATION Procedure
  - From $t = 1$ towards $t = T$ locate each (block of) loss period(s).
  - Let $\langle t_1, t_2 \rangle$ denote such a period. When introduced, the goodwill loss hits the demand in period (t_2+1) unless e_{t_2} is zero.
  - We either satisfy at least e_{t_2} to prevent goodwill loss in period (t_2+1) or lose at least LG_{t_2+1} therein.

Solving the AAC-GWL model

- Outline of the SEARCH-AND-RESTORATION Procedure
  - Let periods u and q be the last production period preceding and the first such period following $\langle t_1, t_2\rangle$, respectively.
  - The solution space of handling loss periods $\langle t_1, t_2\rangle$ is primarily characterized by these production periods
  - There are four alternatives of meeting e_{t_2} .
  - We choose the cheapest way of meeting e_{t_2} and contrast it with losing LG_{t_2+1} in period (t_2+1).

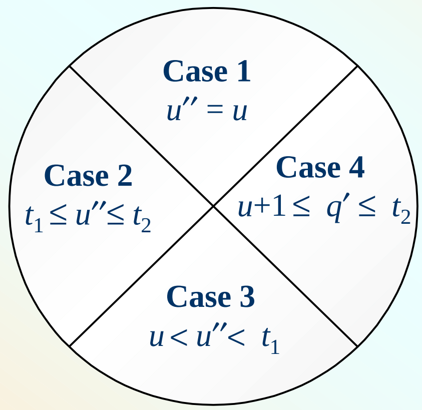
Solving the AAC-GWL model

- Outline of the SEARCH-AND-RESTORATION Procedure
  - Periods u' and q' which replace original production periods u and q.
  - Original productions in u and q are entirely shifted to u' and q'.
  - An extra production activity in some other period u''.



Solving the AAC-GWL model

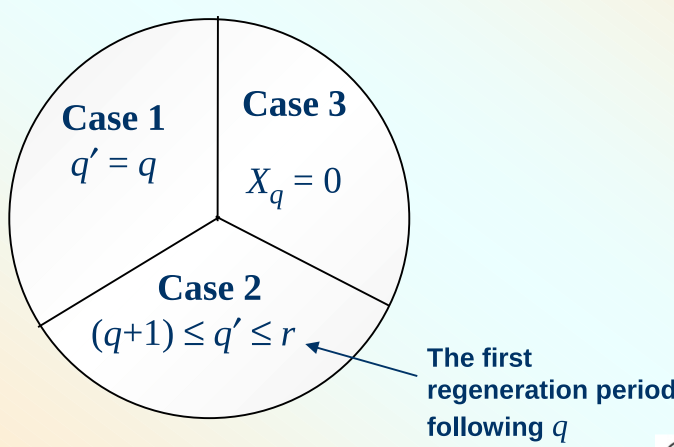
- Mutually exclusive and exhaustive cases of meeting the effective demand in t_2



Istanbul, 06th July 2003

Solving the AAC-GWL model

- Mutually exclusive and exhaustive cases of losing the effective demand in t_2



A Numerical Example

- Data of the sample lot-sizing problem with lost goodwill

Period *t*	1	2	3	4	5	6	7	8
Demand	9	12	9	5	9	20	20	25
Production Cost	10	10	10	10	10	10	10	10
Holding Cost	5	5	5	5	5	5	5	5
Selling Price	20	12	21	14	61	32	32	40
Setup Cost	100	100	100	100	100	100	100	100

A Numerical Example solved

β = 50%	Period *t*	1	2	3	4	5	6	7	8	Profit Π
Optimal Solution w/o Goodwill Loss	X_t	0	0	18	0	0	40	0	25	
	e_t	*9*	*12*	*9*	*5*	*9*	*20*	*20*	*25*	$1,698
	L_t *(total)*	9	12	0	5	0	0	0	0	
Solution with Goodwill Loss	X_t	0	0	11	0	0	40	0	25	
	e_t	*9*	*7*	*5*	*5*	*6*	*20*	*20*	*25*	$1,531
	L_t *(total)*	9	12	4	5	3	0	0	0	
Actual Optimal Solution with Goodwill Loss	X_t	0	0	19	0	0	40	0	25	
	e_t	*9*	*7*	*5*	*5*	*9*	*20*	*20*	*25*	$1,649
	L_t *(total)*	9	12	4	0	0	0	0	0	

Remarks on AA-SR Heuristic

- Runs in $O(T^2)$ CPU time with $O(T^2)$ memory storage.
- Preprocessing via the $O(T^2)$ algorithm AAC
- Insignificant rise in the CPU time of the AAC algorithm.
- Though…
  - NOT OPTIMAL!
  - How come?
  - Proof by a Counterexample… PRESTO!

Istanbul, 06th July 2003

Questions & comments?