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CUSTOMER SELECTION and PROFIT MAXIMIZATION in VEHICLE ROUTING PROBLEMS

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Intro: Capacitated Vehicle Routing

- **NP-hard** problem as a total distance minimization problem.
- Natural extension of the notorious traveling salesperson problem (**TSP**) with…
  - Delivery vehicles (*homogeneous or heterogeneous fleet*) have a limited capacity.
  - One or more nodes in the graph (*single depot vs. multi-depots*) are designated as origins of routes. Each route should emanate from such an origin node. The rest of the nodes are identified as customer nodes.
  - Each route terminates at its own origin (*closed route*), or either at a different origin *(depot)* or at a customer node *(open route)*.

Intro: Capacitated Vehicle Routing (cont.)

- Natural extension of the notorious (TSP) with… (*cont.*)
  - All customer nodes should be visited exactly once.
  - Customer nodes have each either a known demand that has to be delivered (*linehaul customers*) or a known supply that has to be picked up (*backhaul customers as in VRP with Backhauls*.)
  - A QoS (*quality of service*) guarantee in the routing plan can arise in the form of two-sided time windows (TW), time deadlines (TD), maximum route duration or maximum route length.

Intro: Capacitated Vehicle Routing (cont.)

- Natural extension of the notorious (TSP) with… (*cont.*)
  - The vehicle fleet size can be fixed (no costs associated with vehicles) or …
  - A vehicle acquisition cost (*daily rental cost* or *discounted vehicle purchasing price*) can be incurred.

OBJECTIVES *subject to* system constraints and QoS guarantees

- Minimization of total route length.
- Minimization of total operational costs which can be a function of both route lengths, route durations, and number of vehicles employed.

CVRP Applications

- A wide variety of real-world applications since its first introduction in **1959** (*G.B. Dantzig* and *J.H. Ramser* in *Mgmt Sci*) and since its most common solution method proposed in **1964** (*G. Clarke* and *J.V. Wright* in *Oper Res*)…

- Demand of customers can be probabilistic, travel times can be probabilistic, arrival of customer orders can be dynamic (*real-time vehicle routing*)…

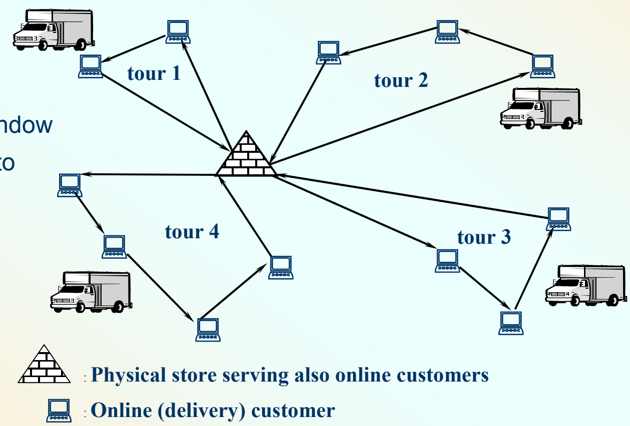
- Routing decisions can be bound up with scheduling plans.

- Routing decisions *(short-term tactical)* can be incorporated into facility location decisions (*long-term strategic*).

CVRP Applications (cont.)

- A **must** in e-commerce
  - the e-fulfillment of online orders for perishable goods
  - the realization of the last mile of delivery

A **quality of service** guarantee is time window restricted deliveries to online customers' residences.



CVRP Literature

To TSP with Profits

- A few *precious survey and milestone pearls*

1. **Computers & Operations Research**
   The entire Volume 10, No. 2, 1983 special issue:
   Routing and Scheduling of Vehicles and Crews – The State of the Art by *Lawrence Bodin, Bruce L. Golden, Arjang Assad and Michael Ball*

2. **American Journal of Mathematical and Management Sciences,** the entire Volume 13, No. 3 and No. 4, 1993
   Bruce L. Golden, Ulrich Derigs, Gregor Grabenbauer, Julien Bramel, Chung-Lun Li, David Simchi-Levi, Sam R. Thangiah, Ibrahim H. Osman, Rajini Vinayagamoorthy, Tong Sun, I-Ming Chao, Bruce L. Golden and Edward Wasil among many.

CVRP Literature (cont.)

3. **"Algorithms for the vehicle routing and scheduling problems with time window constraints"**
   by *Marius M. Solomon,*
   Operations Research, Vol. 35, No. 2, March-April 1987

4. **"Fleet Management and Logistics"**
   by *Teodor Gabriel Crainic and Gilbert Laporte (eds),*
   Centre for Research on Transportation,
   Copyright © 1998 by Kluwer Academic Publishers.

5. **"A Computational Study of Vehicle Routing Applications"**
   doctoral thesis by *Jennifer L. Rich,* **RICE UNIVERSITY**,
   Houston, May 1999, USA (Advisor: William J. Cook)

CVRP Literature (cont.)

6. **"The Dynamic Vehicle Routing Problem" IMM-PHD-2000-73**, doctoral thesis by *Allan Larsen*, Department of Mathematical Modelling (IMM) at the **TECHNICAL UNIVERSITY OF DENMARK** (DTU), Lyngby June 2000, Denmark (Advisor: Oli B.G. Madsen)
7. **"A heuristic method for the open vehicle routing problem"** by *Sariklis D and S Powell* in **Journal of the Operational Research Society**, Vol. 51, No. 5, May 2000
8. **"The Vehicle Routing Problem"** by *Paolo Toth and Daniele Vigo (eds),* **SIAM Monographs on Discrete Mathematics and Applications**, Copyright © 2002, SIAM Publishing.
9. **Featured Issue: "New technologies in transportation systems,"** *Y. Siskos and E. Sambracos (eds),* **European Journal of Operational Research**, Vol. 152, No. 2, January 16th 2004, ELSEVIER B.V. Copyright © 2004.

CVRP Literature (cont.)

10. **"A guide to vehicle routing heuristics"** by *Jean-Françoise Cordeau, Michel Gendreau, Gilbert Laporte, Jean-Yves Potvin, Frédéric Semet* in **Journal of the Operational Research Society**, Vol. 53, No. 5, May 2002
11. **"Real-time vehicle routing: Solution concepts, algorithms and parallel computing strategies"** by *Ghiani G, Guerriero F, Laporte G, Musmanno R* in **European Journal of Operational Research**, Vol. 151, No. 1, November 2003
12. **The Ultimate List of Vehicle Routing References** at the **Center for Traffic and Transport Research (CTT)** of the **Technical University of Denmark** http://www.imm.dtu.dk/or/vrp_ref/vrp.html

CVRP Literature (cont.)

13. **The VRP Web and the Networking and Emerging Optimization Group** at the **University of Málaga in Spain**
http://neo.lcc.uma.es/radi-aeb/WebVRP/

14. **Branch Cut and Price Applications : Vehicle Routing Links** maintained by *Ted Ralphs* (ted@branchandcut.org) at
http://branchandcut.org/VRP/

15. **Canada Research Chair in Distribution Management** with a collection of standard benchmark problems
http://www.hec.ca/chairedistributique/

TSP: Profit Maximization Case

16. **"A tabu search heuristic for the undirected selective travelling salesman problem"** by *Michel Gendreau, Gilbert Laporte, Frédéric Semet* in **European Journal of Operational Research**, Vol. 106, No. 2-3, April 1998

17. **"A branch-and-cut algorithm for the undirected selective traveling salesman problem"** by *Michel Gendreau, Gilbert Laporte, Frédéric Semet* in **NETWORKS**, Vol. 32, No. 4, December 1998

18. **"An Exact Algorithm for the Elementary SPP with Resource Constraints: Application to Some VRPs"** by *Dominique Feillet, Pierre Dejax, Michel Gendreau and Cyrille Gueguen* in **NETWORKS**, Vol. 44, No. 3, October 2004

19. **"Traveling salesman problems with profits"** by *Dominique Feillet, Pierre Dejax and Michel Gendreau*, to appear in **Transportation Science - 2005**

TSP: Profit Maximization Case (cont.)

Customer Selection , Selective TSP , Lost Sales , Maximum Collection, Prize Collecting, Profitable Tour, Orienteering

Traveling Salesman Problems with Profits (TSPs with Profits) are a generalization of the Traveling Salesman Problem (TSP) where it is not necessary to visit all vertices. With each vertex is associated a profit. The overall goal pursued is the simultaneous optimization of the collected profit and the travel costs. These two optimization criteria appear either in the objective function or as a constraint. In this paper, a classification of TSPs with Profits is proposed and the existing literature is surveyed. Different classes of applications, modeling approaches and exact or heuristic solution techniques are identified and compared. Conclusions emphasize the interest of this class of problems, with respect to applications as well as theoretical results.

Dominique Feillet, Pierre Dejax and Michel Gendreau

VRP : Profit Maximization Case

Alternative Objectives

- Minimize total vehicle distance *whilst collecting a specified minimum amount of total profit from visited customers.*
- Minimize total vehicle distance *whilst serving a specified minimum number of customers.*
- Maximize total profit collected from customers *whilst a specified maximum total distance is traveled.*
- JOINT OBJECTIVE FUNCTION

MAXIMIZE Net Profit = Total Profit Collected — Total Cost of Traveling

VRPTD: Profit Maximization Case

Our Problem of Interest

- Single-depot capacitated vehicle routing problem with a flexible size fleet of homogeneous vehicles.
- Demand of all customers + Profit from all customers + coordinates of all customers and the depot known in advance with certainty.
- Profit from each customer is linearly proportional to that customer's demand volume.
- Time deadline + maximum route duration/length constraints are to be satisfied.
- JOINT OBJECTIVE FUNCTION

MAXIMIZE Net Profit = Total Profit Collected – Total Cost of Deliveries

VRPTD: Profit Maximization Case (cont.)

Mathematical Formulation (No TDs) – Alternative I

$$\max.\ \Pi = \sum_{i \in N \setminus \{0\}} p_i y_i \; - \; UNITCOST \times \sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} c_{ij} x_{ij}$$

$$s.t. \quad \sum_{\substack{j \in N \\ j \neq i}} x_{ij} = \sum_{\substack{j \in N \\ j \neq i}} x_{ji} \qquad \forall i \in N$$

$$\sum_{\substack{j \in N \\ j \neq i}} x_{ij} = y_i \qquad \forall i \in N \setminus \{0\}$$

$$\sum_{i \in N \setminus \{0\}} x_{0i} \geq \sum_{i \in N \setminus \{0\}} d_i y_i$$

$$\sum_{i \in S} \sum_{\substack{j \in S \\ j \neq i}} x_{ij} \leq |S| - L_S \qquad \forall S \subseteq N \setminus \{0\} \wedge |S| \geq 2$$

$$L_S = \left\lceil \frac{1}{Q} \sum_{i \in S} d_i \right\rceil$$

$$y_i, x_{ij} \in \{0,1\}$$

VRPTD: Profit Maximization Case (cont.)

Mathematical Formulation (No TDs) – Alternative II

$$\max.\ \Pi = \sum_{i \in N \setminus \{0\}} p_i y_i \; - \; UNITCOST \times \sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} c_{ij} x_{ij}$$

$$s.t. \quad \sum_{\substack{j \in N \\ j \neq i}} x_{ij} = \sum_{\substack{j \in N \\ j \neq i}} x_{ji} \qquad \forall i \in N$$

$$\sum_{\substack{j \in N \\ j \neq i}} x_{ij} = y_i$$

$$\sum_{i \in N \setminus \{0\}} x_{0i} \geq \sum_{i \in N \setminus \{0\}} d_i y_i$$

$$u_i - u_j + Q \times x_{ij} \leq Q - d_j \qquad \forall (i,j) \in N \setminus \{0\} \wedge i \neq j$$

$$d_i \leq u_i \leq Q \qquad \forall i \in N \setminus \{0\}$$

$$y_i, x_{ij} \in \{0,1\}$$

Miller-Tucker-Zemlin valid inequalities extended to the VRP by Kulkarni and Bhave ***EJOR***, Vol. **20** (1985)

VRPTD: Maximize the NET PROFIT

Solving CSVRP and its time constrained variants

- **Classical Heuristics** preferred when solution time is more critical than solution quality.

- (Parallel) Savings Algorithm by Clarke and Wright (1964) ✓
- Sweep Algorithm by *Gillett and Miller* (1974)
- Push-Forward-Insertion by *Solomon* (1987), by *Thangiah et al.* (1993)
- Nearest Neighbourhood Search by *Rosenkratz, Stearns and Lewis* (1977), *Solomon* (1987), *Fisher* (1994).

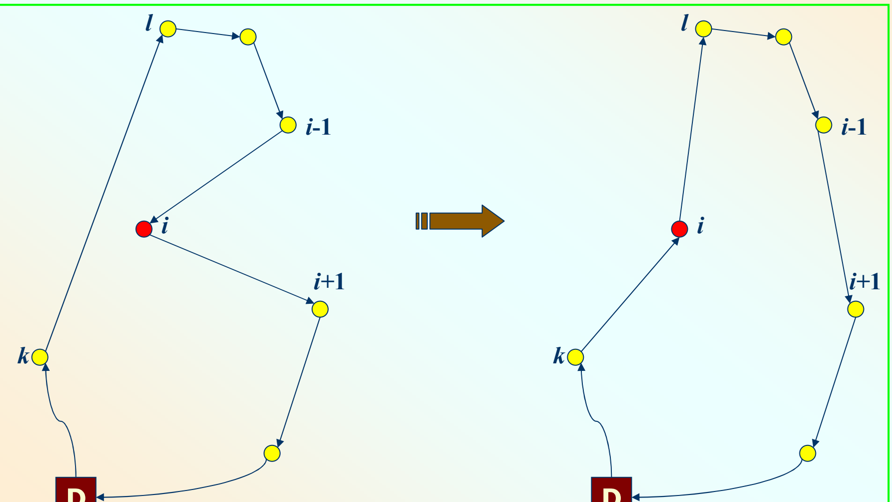
VRPTD: Maximize the NET PROFIT (cont.)

Solving CSVRP and its time constrained variants

- **Local Post Optimization (LPO) Procedures**
  a.k.a. *Local Improvement Heuristics (Parallel or First)*
  - 1-0 move (1-Opt) of *Golden, Magnanti and Nguyen* (1977) ☑
  - 1-1 exchange of *Waters* (1987) ☑
  - 2-opt of *Croes* (1958), *Lin* (1965) ☑
  - 3-opt of *Lin* (1965), of *Lin and Kernighan* (1973)
  - 4-opt* of *Renaud, Laporte and Boctor* (1996)
  - Or-opt of *Or* (1976)

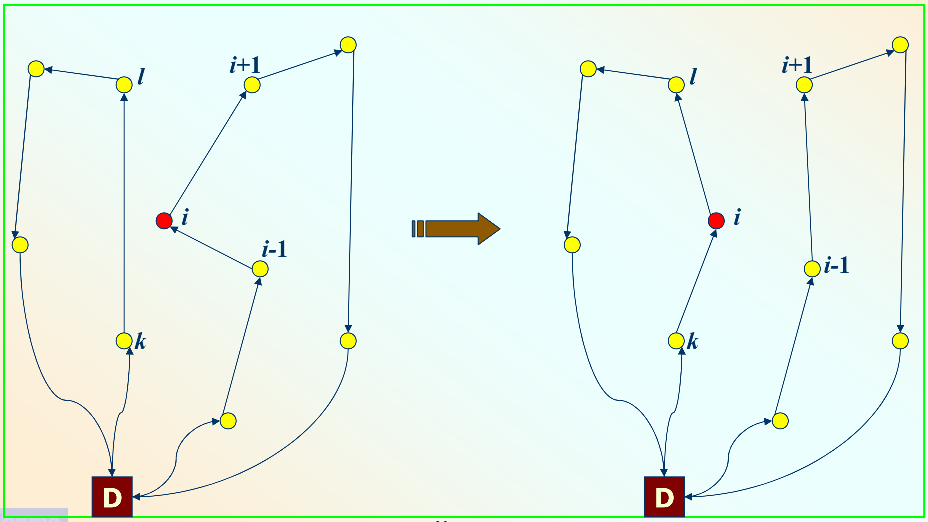
VRPTD: Maximize the NET PROFIT (cont.)

1-0 move LPO on the same route



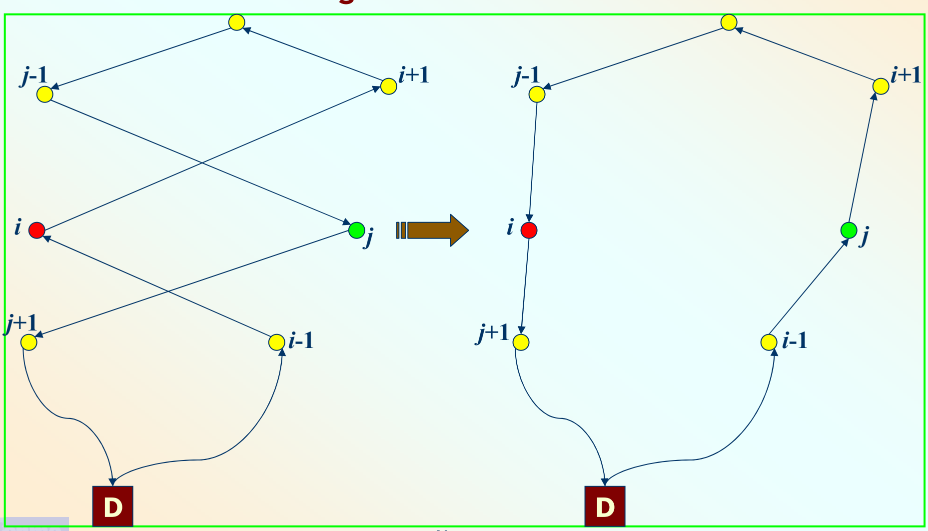
VRPTD: Maximize the NET PROFIT (cont.)

1-0 move LPO on 2 routes



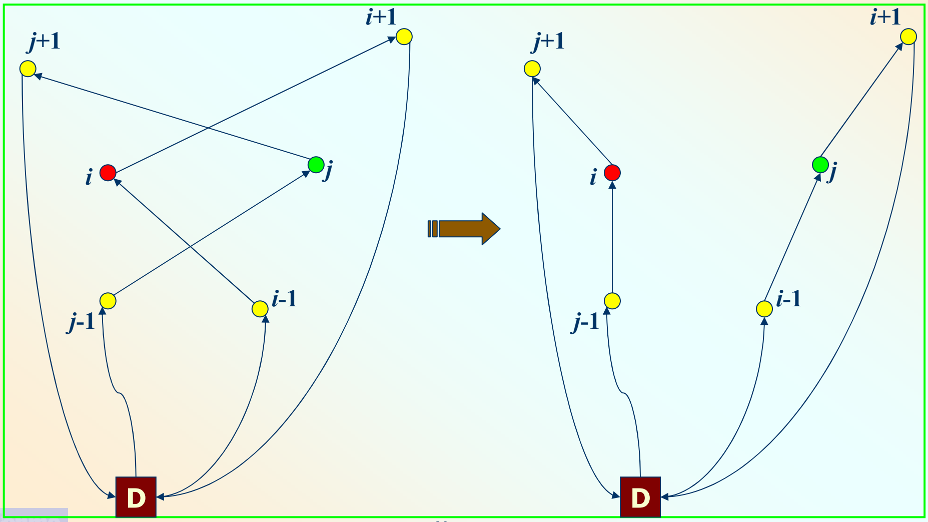
VRPTD: Maximize the NET PROFIT (cont.)

1-1 exchange LPO on the same route



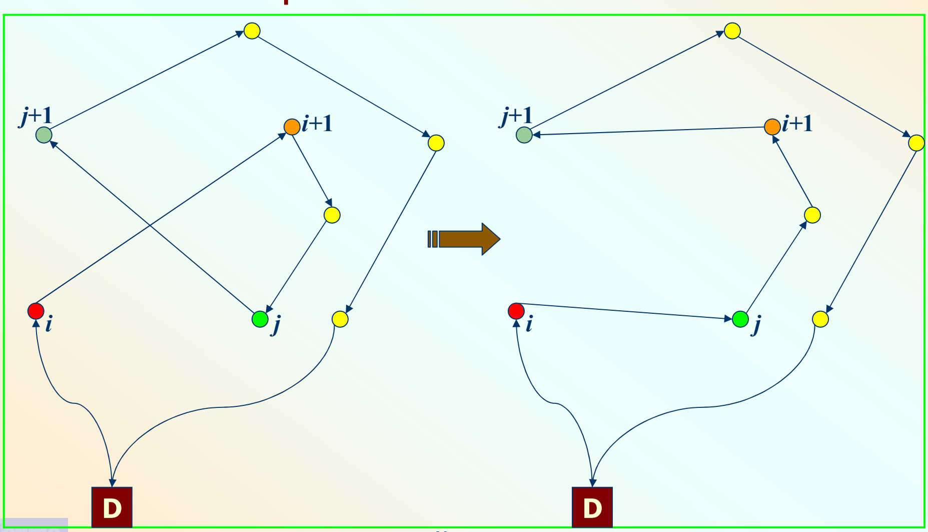
VRPTD: Maximize the NET PROFIT (cont.)

1-1 exchange LPO on 2 routes



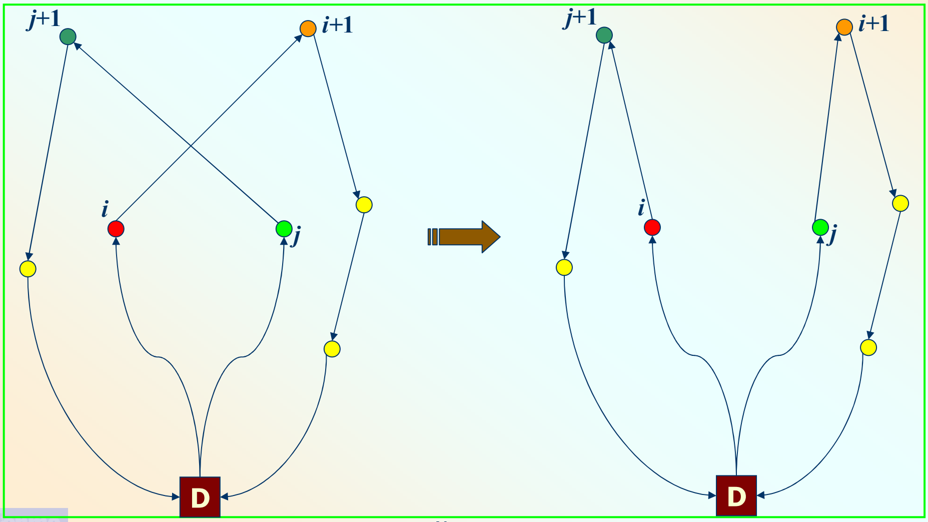
VRPTD: Maximize the NET PROFIT (cont.)

2-Opt LPO on the same route



VRPTD: Maximize the NET PROFIT (cont.)

2-Opt LPO on 2 routes



VRPTD: Maximize the NET PROFIT (cont.)

MARGINAL PROFIT ANALYSIS

Is customer *i* worth keeping (inserting) between nodes *k* and *l* ?

During the heuristic method

Applicable with constructive heuristics such as nearest insertion, nearest neighbourhood search, and push-forward insertion heuristics

After the heuristic method

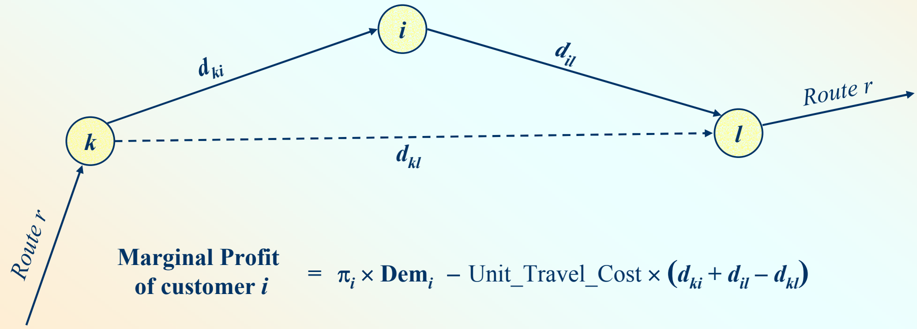
Applicable **ALSO** with the Clarke-Wright Parallel Savings Method in addition to other constructive heuristics

VRPTD: Maximize the NET PROFIT (cont.)

Is customer *i* worth keeping (inserting) between nodes *k* and *l* ?

Triangular Inequality holds true even if distances are not perfectly Euclidean.

Predecessor of customer node *i* : node *k*
Successor of customer node *i* : node *l*



Marginal Profit of customer *i* $= \pi_i \times \mathbf{Dem}_i - \text{Unit\_Travel\_Cost} \times (d_{ki} + d_{il} - d_{kl})$

Algorithm Discard_Nonpositive_Marginal_Profits

Initialization

Solve the given problem instance including the local post optimization procedure of choice. In the resulting solution all customers are visited.

Step 1

For the current set of routes ***R*** compute each customer's marginal profit value $\mathbf{M}\pi_i$. Sort the $\mathbf{M}\pi_i$ values in ***nondecreasing order*** and obtain a sorted stack $\mathbf{M}\pi_{[i]}$.

Step 2

If $\mathbf{M}\pi_{[0]}$, the 1st (*thus*: **LOWEST**) marginal profit in the stack, which belongs to customer $i_{[0]}$ is positive then go to ***Step 5***! O/w go to ***Step 3.***

Step 3

Let $succ_i$ and $pred_i$ be the successor and predecessor nodes of customer $i_{[0]}$ on its current route r_i, resp. Delete $i_{[0]}$ from r_i. Update the $\mathbf{M}\pi$ values of $succ_i$ and $pred_i$ if they are customer nodes. Cancel route r_i if $i_{[0]}$ was the only customer on it.

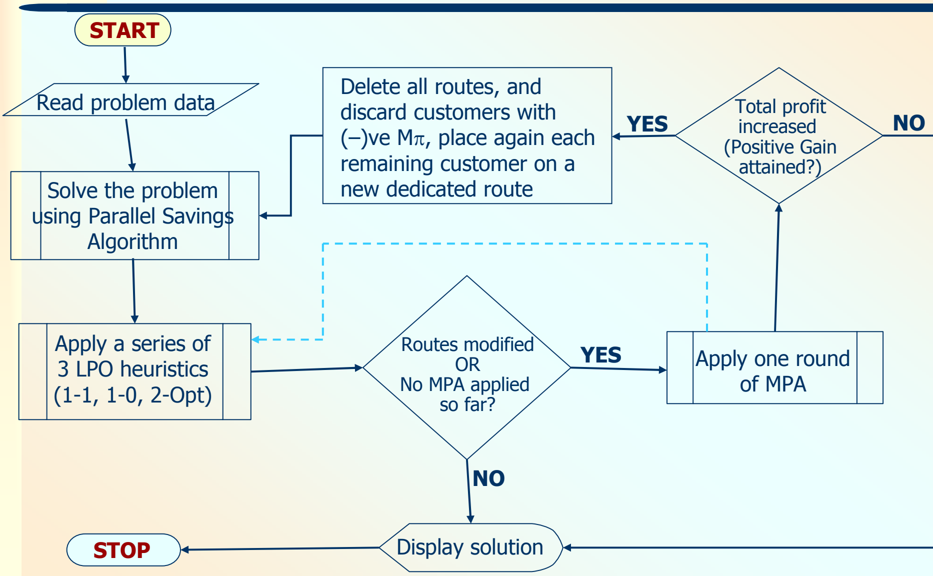
Step 4

Set $\mathbf{M}\pi$ value for $i_{[0]}$ to INFINITY such that it is put at the end of the stack.
Restore the nondecreasing order of $\mathbf{M}\pi$ values in the stack and go to ***Step 2***.

Step 5

Apply to the new routes the local post optimization procedure of choice.

Iterative Marginal Profit Analysis (IMPA) for VRPTD-CS



A Numerical Example solved: test0.txt

CUST#	XCOORD.	YCOORD.	DEMAND	READY	DUE	SERVICE	UNIT PROFIT
0	0	0	0	0	100000	0	0
1	8	6	1	0	100000	3	20
2	16	12	1	0	100000	3	20
3	24	18	1	0	100000	3	20
4	-8	6	1	0	100000	3	20
5	-16	12	1	0	100000	3	20
6	-24	18	1	0	100000	3	20
7	-8	-6	1	0	100000	3	20
8	-16	-12	1	0	100000	3	20
9	-24	-18	1	0	100000	3	20
10	8	-6	1	0	100000	3	20
11	16	-12	1	0	100000	3	20
12	24	-18	1	0	100000	3	20
13	40	30	1	0	100000	3	45
14	-40	30	1	0	100000	3	45
15	-40	-30	1	0	100000	3	45
16	40	-30	1	0	100000	3	45
17	0	30	1	0	100000	3	15
18	0	-30	1	0	100000	3	15
19	30	0	1	0	100000	3	15
20	-30	0	1	0	100000	3	15

Uniform Vehicle Capacity: 6

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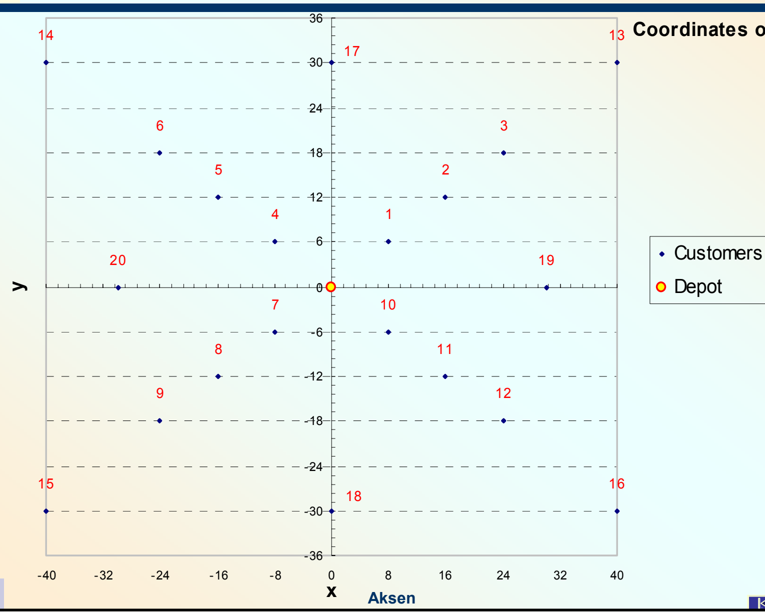
A Numerical Example: test0.txt (cont.)



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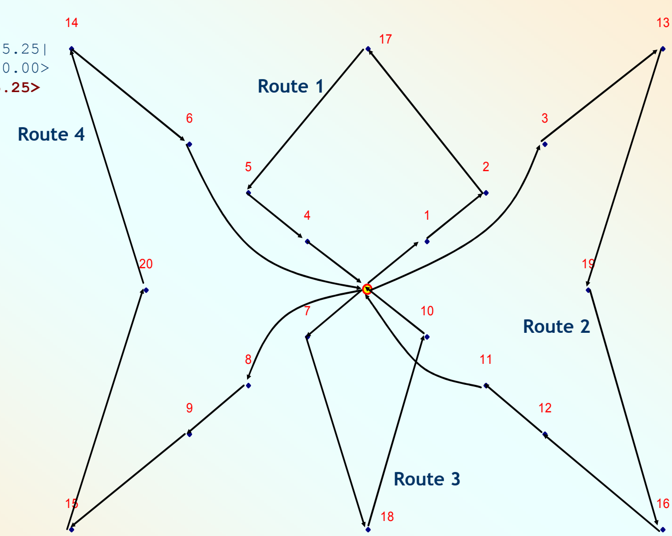
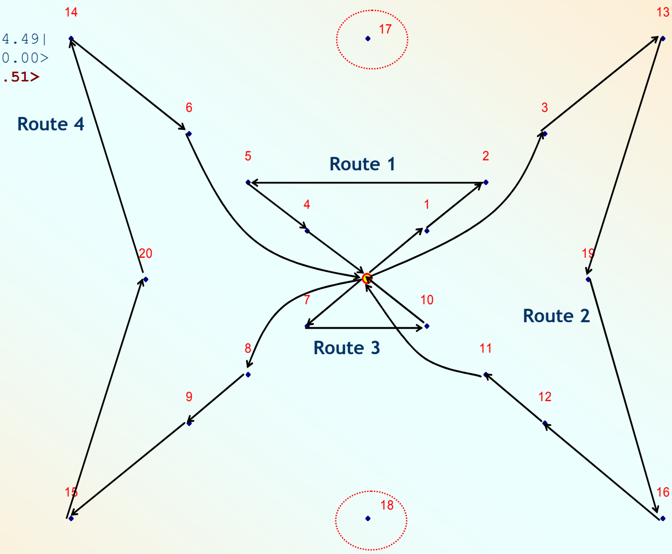
Before the Marginal Profit Analysis (MPA)

```
No customers lost  : (0)
No.required veh's  : 4
Total Distance     : |485.25|
Total Gross Profit : <480.00>
Total Net Profit   : <-5.25>
```



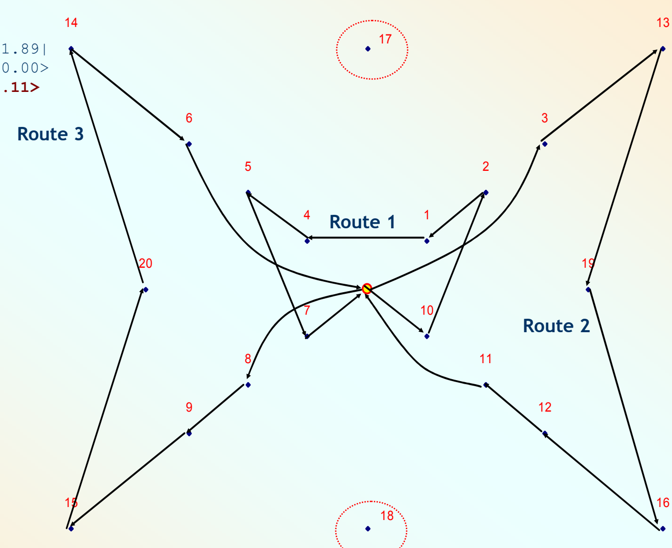
After one sweep of the MPA

```
No customers lost  : (2)
No.required veh's  : 4
Total Distance     : |434.49|
Total Gross Profit : <450.00>
Total Net Profit   : <15.51>
```



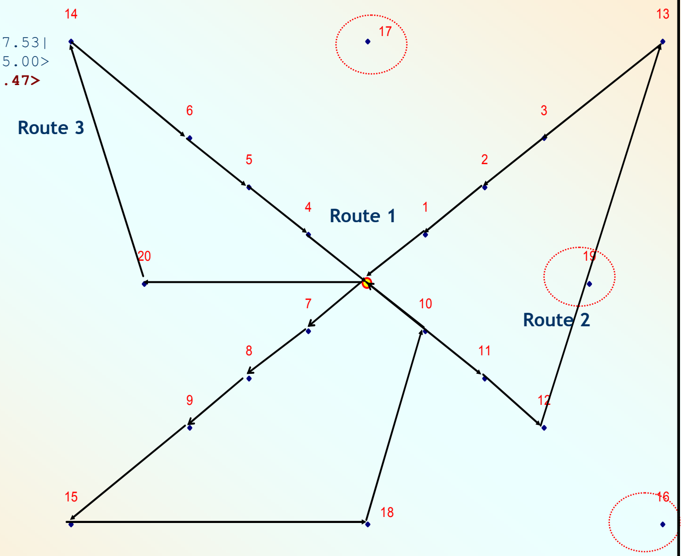
At the end of the MPA (2 sweeps)

```
No customers lost  : (2)
No.required veh's  : 3
Total Distance     : |421.89|
Total Gross Profit : <450.00>
Total Net Profit   : <28.11>
```



CPLEX 8.0 Optimal Solution (148.9 sec on Pentium 4 3.40 GHz with 2 GB RAM)

```
No customers lost   : (2)
No.required veh's   : 3
Total Distance      : |367.53|
Total Gross Profit  : <405.00>
Total Net Profit    : <37.47>
```



Questions & Comments?

TJPP : traveling junior professor problem

N P-hard in the strong sense