

Design of a Government-Subsidized Collection System for Incentive-Dependent Returns

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Abstract - We address the problem of locating collection centers for a company that aims to collect used products (cores) in order to capture their remaining value by recovery operations. A pick-up strategy is in place according to which vehicles are dispatched from collection centers to the locations of product holders to transport their returns. Each product holder has an inherent willingness to return a core, and decides on the basis of the quality-dependent financial incentive offered by the company. Since the company seeks only economic profitability, the collected amounts may not be aligned with the target collection ratio imposed by the government. In this case, the government may alleviate the under-collection issue through a subsidy paid to the company for each core collected. From the government's perspective the problem is to find the minimum subsidy level while meeting the target collection ratio. We propose a bilevel programming formulation for this collection system design problem. Since the problem is \mathcal{NP} -hard, a heuristic method is developed to solve medium and large size instances. This approach explicitly focuses on the relationship between government authorities and profit-oriented companies, and yields a frontier between the concurrent objectives of collection ratio satisfaction and subsidy minimization.

Keywords – Bilevel programming, facility location-allocation, heuristics, reverse logistics, subsidy

I. INTRODUCTION

Product recovery has been a popular business strategy within the context of sustainable development. Common activities in product recovery are (i) collection of used products from product holders, (ii) determining the condition of the returns by inspection and/or separation, (iii) reprocessing the returns to capture their remaining value, (iv) disposal of the returns which are found to be unrecoverable due to economic and/or technological reasons, and (v) redistribution of the recovered products. There are two main reasons that make product recovery attractive for many companies. The first one is economic. Remanufacturing can reduce the unit cost of production by 40 to 60% and the necessary effort by 80% by reutilizing the product components [1]. For example, Xerox Europe obtained over \$80 million savings by implementing an end-of-life equipment take-back and reprocessing program in 1997 [2]. However, accepting all end-of-use products in the waste stream is not a viable strategy for most companies since a considerable

percentage of these may have a poor quality, hence would not be recoverable. As a consequence, adopting a proactive approach and implementing a used product acquisition strategy by offering the appropriate incentive is crucial for a company engaged in product recovery. Some manufacturers have been able to influence the quantity of returns by using buy-back campaigns and offering financial incentives to product holders. A successful implementation of buy-back programs in the power tools industry is provided in [3]. A major issue here is that the number of collected used products (referred to as a core in the sequel) may be limited. The second reason to engage in product recovery is its environmental aspect. In many countries, there exist tightened environmental laws, and it is not uncommon that governments announce target recovery rates. The WEEE Directive of the European Parliament and of the Council (Directive 2002/96/EC), for example, establishes target component, material and substance reuse and recycling rates at 75% by weight for large household appliances such as refrigerators, washing machines, and dishwashers. For desktop/notebook computers as well as printers the target rate is set at 65% by weight. Such regulations, however, may cause the product recovery to be an unprofitable operation for the company.

The remaining value that can be captured in high quality cores can be regarded as revenue due to the savings in the production cost. There are a few works in the literature that tackle the core collection problem. Reference [4] focuses on locating collection centers under a drop-off scenario where product holders have to bring cores to the centers by themselves. They decide whether or not to do so based on the distance to the nearest collection center and the financial incentive offered. The authors formulate a mixed-integer nonlinear programming (MINLP) model to determine the optimal number and locations of the centers as well as the optimal incentive value for each quality class of the cores so as to maximize the net profit. A continuous model is developed in [5] for optimally designing a drop-off facility network and determining the sales price under deposit-refund requirements, where customers' purchasing and return decisions are incorporated by a stochastic utility choice model. More recently, [6] formulates a similar problem to that of [4] where cores are collected under a pickup policy from the premises of the product holders by homogeneous vehicles of limited capacity. Each vehicle departs from a

collection center, visits product holders at a given location, and drives back to the center. All collection related costs, i.e., cost of operating the vehicles and transportation cost of used products are incurred by the company. The product holders' willingness to return a core is determined by the amount of the offered financial incentive. This incentive is dependent on the condition of the core. A higher incentive reduces the unit cost savings from a return, but at the same time increases product holders' willingness to return their cores. Assuming that the number of collection centers is predetermined, the objective is to find the optimal locations of collection centers, the necessary number of vehicles, and the incentive values such that the net profit is maximized.

All of these works stem from a company standpoint, and formulate models to increase the welfare of the company. Thus, the best solutions obtained may not always satisfy the target collection ratios imposed by governments. In other words, the profit maximization objective of the company does not match the objective of the government to reach a desired level of collected cores. To ensure that this level is met, the government may choose to offer a subsidy for each core collected. Certainly, they would like to keep the amount of this subsidy as low as possible. The role of government subsidies is analyzed in the literature in different contexts. For example, [7] considers a case where the government gives a subsidy to a firm involved in recycling of product packages. Reference [8] presents a multi-objective optimization model to deal with integrated logistics problems of green-supply chain management and take into account subsidies from governmental organizations.

In this paper, we propose a bilevel programming formulation that takes into account the relationship between the government and the companies involved in product recovery. To the best of our knowledge, this problem has not been addressed before in the literature. Section II presents the mathematical formulation, while the solution procedure is described in Section III. Experimental results on 24 test instances are provided in Section IV. Finally, Section V concludes the paper with a short summary and suggests future research directions.

II. MODEL DEVELOPMENT

A. Preliminaries

We assume that cores can be grouped with respect to their quality conditions referred to as types. In fact, usage rate and the duration at each use determine the type of the cores. Without loss of generality, we assume that there are K core types. The proportion of product holders in zone j having used products of type k ($k = 1, \dots, K$) is denoted by γ_{jk} and assumed to be known. The number of product holders in zone j who own cores of type k is then given by $h_{jk} = \gamma_{jk} h_j$ where h_j is the total number of product holders in zone j . We suppose that Type 1 products have

the highest quality and Type K products have the lowest quality. We denote by g_k the unit cost savings from a core of type k . It represents the positive difference between the production cost of a new product and the sum of the handling and recovery cost (remanufacturing or material recycling) of a core. When the company offers incentive R_k for a core of type k , the profit associated with that core becomes $(g_k - R_k)$. Therefore, in order to ensure the economic viability of the collection operation $R_k < g_k$ must hold true.

We model product holders' return decisions using the notion of consumer surplus. Each product holder having a core of type k (referred to as product holder of type k) would be willing to return if the company offered an incentive that is at least as large as a certain reservation incentive R_{0k} . In fact, product holders are likely to be heterogeneous in terms of their willingness. We assume that the probability density function of R_{0k} is given by

$$f(R_{0k}) = 2R_{0k}/a_k^2. \quad (1)$$

The proportion of product holders of type k that are willing to return their products is then written as

$$\Pr(R_k \geq R_{0k}) = F(R_k) = R_k^2/a_k^2. \quad (2)$$

Note that R_{0k} takes on values in the interval $[0, a_k]$ where $a_k > 0$ represents the maximum incentive level of product holder of type k . This means that if $R_k = a_k$, then every product holder of type k will return his/her used product. Therefore it is not viable for the company to offer an incentive $R_k > a_k$ for a return of type k . This R_{0k} distribution implies that the change in the number of potential returns occurs at an increasing rate per unit increase in R_k (see Fig. 1). The total number of potential returns of type k from zone j is expressed as $h_{jk} R_k^2/a_k^2$. Hence the maximum possible profit can be computed as $h_{jk} (g_k - R_k) R_k^2/a_k^2$ which is realized if all returns are collected.

B. Model Formulation

We formulate the problem as a bilevel programming model where the government is the leader and the company is the follower. A similar leader-follower relationship is captured for example in [9] between the government (regulator) and the carriers by the use of a bilevel framework where the problem is to select a road network for dangerous-goods shipments from an existing transportation infrastructure. In our formulation, the government's problem (GP) is to determine the minimum value of the unit subsidy S to be paid for each collected core while making sure that the target collection ratio t is

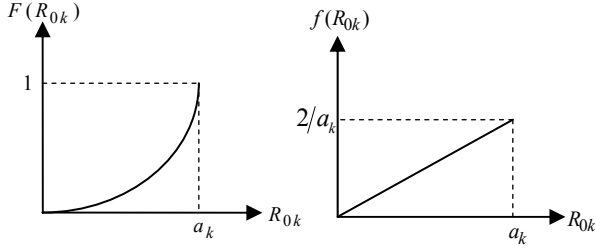


Fig. 1. Reservation incentive distribution

reached. The government is indeed not interested in which type of cores are actually collected by the company. The company has a different objective. It wants to maximize its total net profit by determining the optimal number and locations of the collection centers (CCs), the number of vehicles to be dispatched from each CC to pick up the cores, the amount of the incentives to be paid to product holders for each core returned, and the amount of each core type to be collected. The company's problem (CP) is a variant of the discrete facility location-allocation problem in which there are n customer zones indexed by j and m candidate sites to open collection centers indexed by i . We refer to the whole problem as the government-subsidized collection system design problem (GSCSDP). Below we define all the parameters and decision variables followed by the objective functions and constraints of this bilevel programming problem.

Parameters:

- d_{ij} = road distance between customer zone j and candidate site i
- c_1 = unit vehicle operating cost
- c_2 = cost per unit distance traveled
- q = uniform vehicle capacity
- fc_i = fixed cost of opening a collection center at candidate site i
- h_j = number of product holders in zone j
- γ_{jk} = proportion of type k product holders in zone j
- h_{jk} = number of product holders of type k in zone j
- g_k = unit cost savings from a used product of type k
- a_k = the max. incentive level of product holder of type k
- t = target collection ratio

Decision Variables:

- S = unit subsidy offered for each collected core
- R_k = unit incentive offered for a core of type k
- X_{ijk} = fraction of potential returns of type k collected at zone j and transported to the CC at site i
- $Y_i = \begin{cases} 1 & \text{if a CC is located at site } i \\ 0 & \text{otherwise} \end{cases}$
- V_{ij} = number of vehicles required to transport returns from zone j to the CC at site i

$$\text{GSCSDP: } \min S \quad (3)$$

$$S \geq 0$$

s. t.

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K X_{ijk} h_{jk} R_k^2 / a_k^2 \geq t \sum_{j=1}^n h_j \quad (4)$$

where X_{ijk}, R_k solves:

$$\max \Pi_{net} = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K X_{ijk} h_{jk} R_k^2 (g_k + S - R_k) / a_k^2$$

$$- \sum_{i=1}^m \sum_{j=1}^n (c_1 + 2c_2 d_{ij}) V_{ij} - \sum_{i=1}^m fc_i Y_i \quad (5)$$

s. t.

$$\sum_{i=1}^m X_{ijk} \leq 1 \quad j = 1, \dots, n; k = 1, \dots, K \quad (6)$$

$$X_{ijk} \leq Y_i \quad i = 1, \dots, m; j = 1, \dots, n; k = 1, \dots, K \quad (7)$$

$$V_{ij} \geq \frac{\sum_{k=1}^K X_{ijk} h_{jk} R_k^2 / a_k^2}{q} \quad i = 1, \dots, m; j = 1, \dots, n \quad (8)$$

$$R_k \leq a_k \quad k = 1, \dots, K \quad (9)$$

$$R_k \leq S + g_k \quad k = 1, \dots, K \quad (10)$$

$$R_k \geq 0 \quad k = 1, \dots, K \quad (11)$$

$$X_{ijk} \geq 0 \quad i = 1, \dots, m; j = 1, \dots, n; k = 1, \dots, K \quad (12)$$

$$V_{ij} \geq 0 \text{ and integer} \quad i = 1, \dots, m; j = 1, \dots, n \quad (13)$$

$$Y_i \in \{0, 1\} \quad i = 1, \dots, m \quad (14)$$

The inner problem CP is represented by (5)–(14). The unit subsidy S of the outer problem GP constitutes a parameter for the inner problem. The objective of the GP is given in (3). Constraints (4) guarantee that the total number of cores is greater than or equal to the target collection amount which is obtained by multiplying the target collection ratio t by the total number of available cores. The objective of the CP given in (5) is obtained by subtracting the total vehicle operating cost and the cost of opening CCs from the total profit. The total profit is calculated by first multiplying the total number of type k cores collected from zone j and transported to CC at site i ($X_{ijk} h_{jk} R_k^2 / a_k^2$) by the unit profit of those cores ($g_k + S - R_k$) and then summing up these profits over all core types, collection centers, and zones. The fraction of potential returns collected should be less than or equal to one as shown in (6). Constraints (7) ensure that this fraction is zero if there is no CC at site i . The number of vehicles V_{ij} required to transport the returns from zone j to

CC at site i is given in (8). Constraints (9) and (10) are upper bound constraints for R_k . Finally, (11)–(14) are nonnegativity and integrality constraints.

III. SOLUTION PROCEDURE

Bilevel programs are shown to be \mathcal{NP} -hard in [10]. In the case of GSCSDP, the inner problem CP has to be solved using the subsidy value S determined by the outer problem GP. Once the value of S is set, the inner problem CP becomes a MINLP problem which itself is difficult to solve. An efficient heuristic solution method for this problem is described in [6]. It capitalizes on the following characteristic: given the locations of CCs, the problem reduces to a nonlinear problem in variables R_k and V_{ij} only. In order to explore the space for all possible CC locations, a tabu search is implemented which calls simplex search procedure as a subroutine. In other words, for a possible configuration of opened CCs simplex search finds the best values for R_k and V_{ij} as well as the corresponding profits.

Since the inner problem CP is solvable for a given value of S , we may adopt the following approach. We can start with the smallest possible value of S , which is zero, and solve the CP. If this solution satisfies the minimum collection ratio constraint (4) of the outer problem GP, then we are done. Otherwise, we slightly increase S and solve the CP again. This iterative procedure is continued until constraint (4) is satisfied. Instead of this exhaustive search, we suggest in this paper to use Brent's search [11] for the minimization of S . Brent's search is a root finding method that combines bisection method, secant method and inverse quadratic interpolation. It can be employed when the function in question has a unique root. It has been shown that this method often converges superlinearly and is never slower than bisection method [11]. We adapt Brent's search to the GP by defining a penalty function which measures the infeasibility of the solution of the inner problem CP with respect to constraint (4), i.e., the discrepancy between the target collection ratio t and the realized collection ratio

$T = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K X_{ijk} h_{jk} R_k^2 / a_k^2 / \sum_{j=1}^n h_j$. Equation (15) shows the function fed into Brent's search as input where M is a very big number.

$$f(S) = \begin{cases} S & \text{if } T \geq t \\ -M(t - T) & \text{otherwise} \end{cases} \quad (15)$$

The function $f(S)$ is a non-decreasing function which has positive values for feasible solutions and negative values for infeasible ones. When finished, Brent's search finds that value of S at which $f(S)$ changes its sign. As a matter of fact, this point is the minimum unit subsidy the company receives from the

government such that the realized collection ratio is equal to at least t .

IV. RESULTS

By assigning four distinct values to n ($n=10, 20, 50, 100$), three different values to t ($t=0.2, 0.4, 0.6$), and two distinct values to K ($K=2,3$), we obtain $4 \times 2 \times 3 = 24$ test instances where each instance is labeled as a triplet (n, t, K) . In all instances, the candidate sites for collection centers coincide with the customer zones, i.e., $m = n$. The x - and y -coordinates of customer zones (collection centers) are sampled independently from a discrete uniform distribution in the interval $[0,100]$. The number of product holders in each zone is also generated from the same distribution supported on $[1,100]$. The travel distances between candidate sites and customer zones are calculated using the Euclidean distance giving rise to a symmetric distance matrix. We assume that the proportion of core types is the same across all customer zones, and all K core types are distributed equally (i.e., $\gamma_{jk} = \gamma_k = 1/K \quad \forall j=1, \dots, n$). The rest of parameters used in the generation of the test instances have been adopted directly from [6] where $c_1 = 75$, $c_2 = 1$, $q = 10$ and $f c_i = 1000 \quad \forall i=1, \dots, m$. Parameters related to the core types are given in Table I. Tables II and III present the results for two and three product types, respectively.

TABLE I
PARAMETER VALUES FOR THE PROBLEMS

	$K=2$		$K=3$		
k	1	2	1	2	3
g_k	25	15	25	20	15
a_k	15	15	15	10	5
γ_k	0.5	0.5	1/3	1/3	1/3

TABLE II
RESULTS FOR TWO CORE TYPES

Instance (n, t, K)	Number of CCs opened	Subsidy S	Total Profit Π_{net}	Realized Collection Ratio T
(10, 0.2, 2)	1	1.968	0.01	0.38
(10, 0.4, 2)	1	2.784	187.67	0.41
(10, 0.6, 2)	1	6.135	1195.88	0.64
(20, 0.2, 2)	1	1.773	0.01	0.26
(20, 0.4, 2)	2	4.105	755.68	0.57
(20, 0.6, 2)	2	4.443	944.69	0.61
(50, 0.2, 2)	2	1.126	687.92	0.31
(50, 0.4, 2)	2	1.724	1195.46	0.40
(50, 0.6, 2)	4	3.629	3760.17	0.70
(100, 0.2, 2)	3	0	2389.526	0.31
(100, 0.4, 2)	4	0.442	3213.902	0.42
(100, 0.6, 2)	5	1.461	6012.459	0.62

TABLE III
RESULTS FOR THREE CORE TYPES

Instance (n, t, K)	Number of CCs opened	Subsidy S	Total Profit Π_{net}	Realized Collection Ratio T
(10, 0.2, 3)	1	2.051	0.003	0.38
(10, 0.4, 3)	1	2.917	200.36	0.41
(10, 0.6, 3)	1	6.135	1157.24	0.62
(20, 0.2, 3)	1	1.88	0.001	0.27
(20, 0.4, 3)	2	4.153	718.74	0.58
(20, 0.6, 3)	2	4.443	883.654	0.61
(50, 0.2, 3)	2	1.133	683.42	0.30
(50, 0.4, 3)	3	2.246	1654.96	0.54
(50, 0.6, 3)	3	3.873	4047.09	0.60
(100, 0.2, 3)	4	0	2352.37	0.35
(100, 0.4, 3)	5	0.505	3294.38	0.49
(100, 0.6, 3)	5	1.508	5966.03	0.60

We also solved the company's collection system design problem for given values of unit subsidy S , and reported the resulting realized collection ratio T . This way, we are able to observe the monotonically increasing trend of T with respect to S as depicted in Fig. 2.

V. CONCLUSION

In this study, we elaborate on a bilevel programming formulation to model the relationship between the government (leader) and a company (follower) engaged in core collection operations. On the one hand, the government tries to keep a subsidy paid to the company for each collected core at the minimum possible level while achieving a target collection ratio. On the other hand, the company seeks to maximize its net profit by adding that subsidy to the unit cost savings attained from each and every core. More precisely, the problem of the company under consideration is to determine the best locations of collection centers (CCs) among a set of candidate sites and to offer the best financial incentives for returns of different quality types. An incentive offered by the company to product holders determines the willingness of product holders to return their cores. We focus on a pick-up scenario in which a homogeneous fleet of vehicles is sent from CCs to customer zones in order to collect and bring the returns. The number of vehicles operated and the amount of each return type collected are also decision variables.

For the government's problem we propose a solution procedure derived from Brent's search, while we adapt to the company's problem a tabu and simplex search based methodology mentioned in [6]. Experimental results demonstrate that as the target collection ratio gets more and more restraining, both the unit subsidy and the realized collection ratio move upward. This, however, benefits the company rather than the government since the profit maximization objective of the company is not bound by any minimum collection constraint.

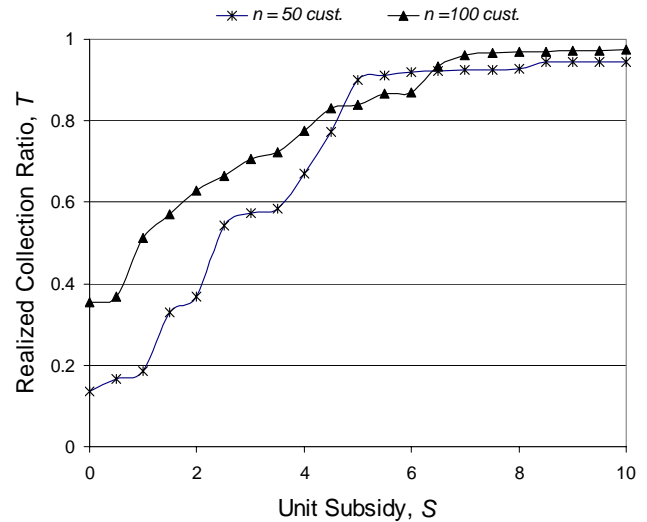


Fig. 2. Frontier curves for $n=50$ and $n=100$ with $K=3$

In the extension of this study, the regulation about the collection ratio could be shifted from the government's (leader's) problem to the company's (follower's) problem. This way the government can demand more legal responsibility from the company while guaranteeing a minimum level of profitability on behalf of the company.

REFERENCES

- [1] S. Dowlatshahi, "Developing a theory of reverse logistics," *Interfaces*, vol. 30, pp. 143–155, 2000.
- [2] I. Maslennikova and D. Foley, "Xerox's approach to sustainability," *Interfaces*, vol. 30, pp. 226–233, 2000.
- [3] M. Klausner and C. T. Hendrickson, "Reverse logistics strategy for product take-back," *Interfaces*, vol. 30, pp. 156–165, 2000.
- [4] N. Aras and D. Aksent, "Locating collection centers for distance and incentive dependent returns," *Int. J. Prod. Econ.*, in press.
- [5] R. Wojanowski, V. Verter, and T. Boyacı, "Retail collection network design under deposit-refund," *Comput. Oper. Res.*, vol. 34, pp. 324–345, 2007.
- [6] N. Aras, D. Aksent, and A. G. Tanuğur, "Locating collection centers with incentive dependent returns," Research Papers FBE-IE-09/2006-11, 2006.
- [7] P. Kulshreshtha and S. Sarangi, "No return, no refund: an analysis of deposit-refund systems," *J. Econ. Behav. Organ.*, vol. 46, pp. 379–394, 2001.
- [8] J.-B. Sheu, Y.-H. Chou, and C.-C. Hu, "An integrated logistics operational model for green-supply chain management," *Transport. Res. E-Log.*, vol. 41, pp. 287–313, 2005.
- [9] B. Y. Kara and V. Verter, "Designing a road network for hazardous materials transportation," *Transport. Sci.*, vol. 38, no. 2, pp. 188–196, 2004.
- [10] O. Ben-Ayed, C. E. Blair, "Computational difficulties of bilevel linear programming," *Oper. Res.*, vol. 38, no. 3, pp. 556–560, 1990.
- [11] R. P. Brent, "An algorithm with guaranteed convergence for finding a zero of a function," *Comput. J.*, vol. 14, pp. 422–425, 1971.