Math 170A (Winter 2009) - Numerical Linear Algebra
Syllabus

Instructor
Emre Mengi
Applied Physics and Mathematics (APM) 5763
e-mail : emengi@math.ucsd.edu

Lecture Hours and Location
Monday, Wednesday, Friday at 10am at APM B412

Textbook
Fundamentals of Matrix Computations, 2nd Edition by David S. Watkins

Discussion Sections
Thursday at 3pm at APM 2402

Prerequisites
Math 20F with a grade of C- or better, or equivalent. Elementary level of programming knowledge is required. You will be performing computations in Matlab. The level of familiarity with Matlab that you gain in Math 20D and/or Math 20F should be sufficient. The knowledge of a high-level programming language such as C, C++, Fortran, Java would help, but is not required.

Course Webpage
http://www.math.ucsd.edu/~emengi/teaching/math170a/math170a.html

Grading
Your overall score will be determined based on your performance in the homeworks, midterms and the final using the scheme below.

Total Score = 0.2(Homework Score) + 0.15(Midterm 1) + 0.25(Midterm 2) + 0.40(Final)

Please keep in mind that there will be a curve in the end. Your letter grade will depend on your rank in the class.

Midterms
The midterms will be held during the regular lecture hours at B412 on the following dates

- Midterm 1 - January 28th, Wednesday between 10am and 10:50am
- Midterm 2 - February 25th, Wednesday between 10am and 10:50am

Typically all of the topics covered in class by the end of the week preceding the midterm are included in each midterm. The precise topics will be announced one week in advance. The midterms will be open-book exams.
Final
The final is scheduled on March 20th, Friday between 8am and 11am. All of the topics covered in class throughout the quarter are included. Location of the final will be announced later towards the end of the quarter. Final will be a closed-book exam.

Homeworks
The homeworks will be assigned weekly on Fridays and due following Fridays by 3pm. You can turn in your homeworks either to me or to the TA during the discussion section on Thursday. If you decide not to turn in your homework in class on Friday or during the discussion section on Thursday, please make sure to return it to my office (APM 5763) by 3pm on Friday. If I am not in my office, please slide your homework under my door. There will be eight homeworks in total. Your homework score will be the average of the eight homeworks.

Half of the homework questions will be conceptual. The remaining half will be computational and require performing computations in Matlab. You can reach a freely available Matlab manual from the website http://www.math.mtu.edu/~msgocken/intro/intro.html

Description
This course concerns the numerical solutions of linear systems of equations, least squares problems (the best approximate solution for an inconsistent linear system), eigenvalue problems and singular value problems. We will develop numerical algorithms for these four main-stream problems. The quality of a numerical algorithm is often judged based on two criteria namely efficiency (vaguely speaking number of arithmetic operations required) and accuracy. We will analyze the accuracy and efficiency of the numerical algorithms developed.

The second of these criteria, accuracy, may be unfamiliar. Only finitely many real numbers can be represented on a computer, consequently rounding errors are unavoidable in numerical computation. The usefulness of a numerical algorithm largely depends on how accurate results it yields in the presence of rounding errors.

Special emphasis will be put on physical examples. One major application where the numerical solutions of linear systems are required is the solution of differential equations numerically. Furthermore eigenvalues are essential to characterize the analytic solutions of linear differential equations. In the textbook differential equations are derived for the motion of mass-spring systems as well as the current on electrical circuits with resistors and inductors. Such examples will be discussed in the class as motivating examples for linear systems and eigenvalues. However, be aware of the fact that this is not a modeling class. Here we will take the approach that the models are provided to us and we will use our tools such as linear systems and eigenvalues to solve and analyze the equations that stem from these models.

Another important aspect of this class is the focus on matrix factorizations. You may recall some of the matrix factorizations such as the LU decomposition and eigenvalue decomposition from Math 20F. In this class we will further see the QR factorization, Cholesky factorization, Schur factorization and singular value decomposition. Numerical computation of all these factorizations as well as some of their practical uses will be elaborated.
Specifically the topics that will be covered and the corresponding sections in the textbook are listed below. We will usually follow the textbook closely. However, on certain occasions (such as “fundamentals of numerical computation” below) we will depart from the textbook.

- **Fundamentals of Numerical Computation**: Counting # of floating point operations, IEEE floating point arithmetic, vector and matrix norms, sensitivity analysis and condition numbers, forward and backward errors, backward error analysis (Sections 1.1, 2.1-5)

- **Numerical Solution of Linear Systems**: Positive definite systems, Cholesky factorization, Gaussian elimination, LU decomposition, pivoting, error analysis of Gaussian elimination (Sections 1.2-4, 1.7-8, 2.6-7)

- **Numerical Solution of Least Squares**: QR factorization with Householder transformations, QR factorization with Gram-Schmidt, modified Gram-Schmidt (Sections 3.1-4)

- **Numerical Computation of Eigenvalues and Eigenvectors**: Power method, inverse iteration, Schur factorization, QR algorithm, sensitivity of eigenvalues and eigenvectors (Sections 5.1-6, 5.8, 6.1-2, 6.5)

- **Singular Value Decomposition (SVD) and its Computation**: Definition and geometric interpretation, applications, QR algorithm for SVD (Sections 4.1-2, 5.9)