Name: **PID:**
Discussion Section - No: **Time:**

Final, Math 20C (Lecture C)
December 10th, 2007

Duration: 180 minutes
This is a closed-book exam. Calculators and computing devices are not allowed.
To get full credit you should support your answers.

1. Consider the lines

- $\mathcal{L}_1$ which passes through the point $(1, 1, 1)$ and is parallel to the vector $\vec{i} + \vec{k}$.
- $\mathcal{L}_2$ which passes through the points $(1, 1, 1)$ and $(1, 2, 1)$.

a) (3 points) Find the parametric equations of the lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

b) (2 points) Find the angle between $\mathcal{L}_1$ and $\mathcal{L}_2$.

#	Score
1 (8 points)	
2 (6 points)	
3 (6 points)	
4 (8 points)	
5 (6 points)	
6 (7 points)	
7 (9 points)	
8 (5 points)	
Total (55 points)	

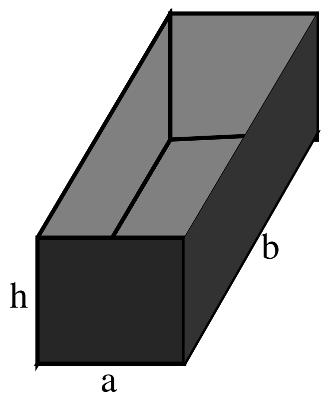
c) (3 points) Find a vector that is perpendicular to both of the lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

2. Use the chain rule to find the indicated derivatives below.

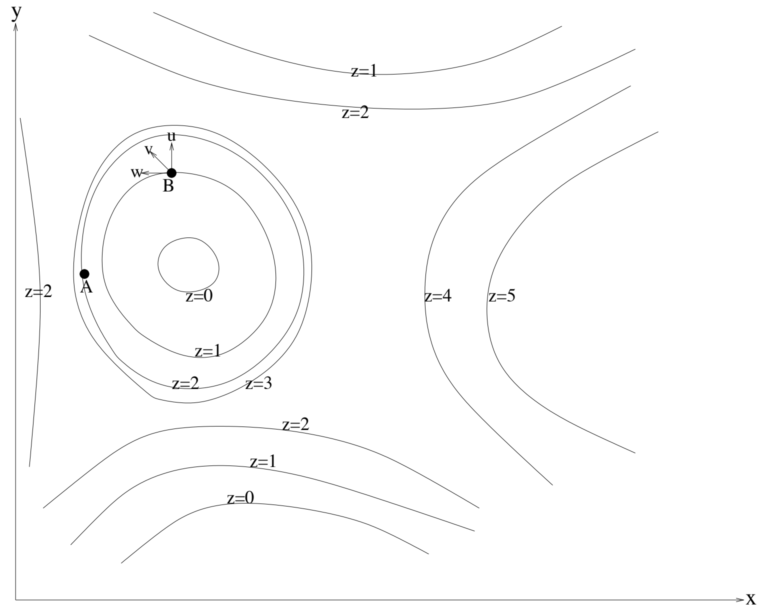
a) (3 points) Let $g(t) = f(u(t), v(t)) = ue^v$ where $u(t) = \cos(t)$ and $v(t) = \sin(t)$. Find $g'(t)$.

b) (3 points) Let $h(x, y) = f(u(y), v(x)) = \frac{\sin u}{\cos v}$ where $u(y) = \frac{1}{y^2}$ and $v(x) = x^2$. Find $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$.

3. (6 points) Determine the dimensions of the box (the dimensions are the side-lengths of the rectangular base a,b and the height h) without a lid and with volume equal to $500cm^3$ whose surface area is *as small as possible* by posing an *unconstrained optimization problem* over two variables.



4. Answers the questions below for the function $z = f(x, y)$ whose contour diagram is illustrated below for various z.



a) (3 points) Determine the signs of the derivatives $f_x(x, y)$, $f_y(x, y)$ and $f_{xx}(x, y)$ at the point A.

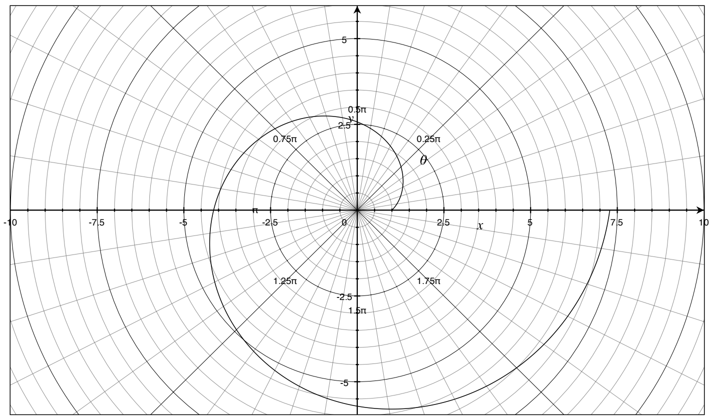
b) (3 points) Order the directional derivatives $D_{\mathbf{u}}f(x, y)$, $D_{\mathbf{v}}f(x, y)$ and $D_{\mathbf{w}}f(x, y)$ at the point B from the smallest to the largest.

c) (2 points) Mark two points on the contour diagram that are possibly a local minimum and a saddle point with the letters M and S, respectively.

5. (6 points) The equation

$$r(\theta) = 1 + \theta$$

in polar coordinates represents an Archimedean spiral whose graph is provided below.



The double integral

$$\int\int_D 1 \; dx \; dy$$

over a closed-region D is equal to the area of D. Using this fact set up a double integral in polar coordinates for the area in between the Archimedean spiral and the horizontal axis. Solve the double integral to find this area.

6. Consider the integral

$$\int\int_D 9 - x^2 - y^2 \; dx \, dy$$

over the domain

$$D = \{(x, y) : \; 0 \leq x \leq 2, \;\; 0 \leq y \leq 2\}.$$

a) (4 points) Write a left and right Riemann sum for the integral above by dividing D into four sub-rectangles of equal size with side-lengths $\Delta x = \Delta y = 1$ and evaluating the function values at the lower-left and upper-right corners of the rectangles, respectively.

b) (3 points) Without evaluating the integral order the left, right Riemann sums and the exact value of the integral from the smallest to the largest.

7. Evaluate the integrals below.

a) (4 points)

$$\int_1^3 \int_0^\pi x \sin(y) \; dy \, dx$$

b) (5 points)

$$\int_0^{\sqrt[4]{\pi}} \int_{x^2}^{\sqrt{\pi}} x \sin(y^2) \; dy \, dx$$

(Hint: You need to change the order of integration)

8. (5 points) The density of a substance within the region

$$V = \{(x, y, z) : 0 \le x \le \pi/2,\ 0 \le y \le 1,\ 0 \le z \le 1\}$$

in 3D space is given by

$$\rho(x, y, z) = y \cos(x) e^z.$$

Find the mass M of the substance defined as

$$M = \int\int\int_V \rho(x, y, z)\ \ dz\ dy\ dx.$$