Math 20C (Lecture C) - Homework 1

(Due on October 12 at 3pm)

This homework covers the most fundamental concepts in Chapter 12. You need to return only the questions marked with ($*$). The others are provided for practicing purposes. If you can solve these questions on your own, you most likely have understood the basics of vectors and 3D space. It may help to solve some of the suggested questions in Chapter 12 before attempting these ones.

1.(*) (Section 12.1) Express each of the following surfaces in terms of an equation or the regions in terms of one or more inequalities in 3D space.

a. The plane parallel to the xz-plane with y-intercept equal to -2.

b. The vertical plane with x-intercept and y-intercept equal to 1.

c. The surface of the cylinder extending infinitely parallel to the x-axis and whose projection onto the yz-plane is the circle of radius 2 centered at $(y, z) = (3, 1)$.

d. The set of points inside or on the unit cube with faces parallel to the xy, yz or xz-plane and opposite vertices along the diagonal located at the origin and $(x, y, z) = (1, 1, 1)$.

2.(*) (Section 12.2) You need to cross a lake 100 miles wide from south to north on a ship. The flow in the lake points to the east at a speed of 4 miles/hour. The velocity of the ship in still water is 20 miles/hour to the north.

a) Let the vectors $\vec{i}$, $\vec{j}$ denote the velocity of one mile/hour to the north and one mile/hour to the east, respectively. Represent your ground velocity (with respect to the shore) in terms of $\vec{i}$ and $\vec{j}$.

b) Find your ground speed (the magnitude of the ground velocity) and the unit vector in the direction of your ground velocity.

c) How many miles would you be away from the pier where you boarded the ship initially when you reach the shore on the north? (Assume that the lake is rectangle-shaped 100 miles wide from south to north.)

3. (Section 12.2) Consider the trapezoid with vertices A, B, C, D and edges from A to B, B to C, C to D and D to A.

a) Find $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$.

b) Suppose $A = (0, 0, 0), B = (1, 1, 0), C = (0, 1, 1)$, $|\overrightarrow{AD}| = 2$ and the edges from A to D and B to C are parallel. Find $\overrightarrow{AD} + 2\overrightarrow{AB}$.

4. **(Section 12.3)** Let $\mathbf{v}$ and $\mathbf{w}$ be vectors perpendicular to each other. Simplify the following expressions. (Each of these expressions is either a constant or can be written in terms of $|\mathbf{v}|$ and $|\mathbf{w}|$.)

a. $\mathbf{v}\cdot 2\mathbf{v}$

b. $\mathbf{v}\cdot 2\mathbf{w}$

c. $(\mathbf{v}+\mathbf{w})\cdot(\mathbf{v}-\mathbf{w})$

5.(*) **(Section 12.3)** Let $\mathbf{u} = \vec{\imath} + 2\vec{\jmath}$ and $\mathbf{v} = -\vec{\imath} + \vec{\jmath} - 2\vec{k}$.

a. Find the *cosine* of the angle between $\mathbf{u}$ and $\mathbf{v}$.

b. Find the unit vector $\mathbf{w}$ so that the dot product $\mathbf{u}\cdot\mathbf{w}$ is as large as possible.

c. Find the unit vector $\mathbf{w}$ so that the dot product $\mathbf{v}\cdot\mathbf{w}$ is as small as possible.

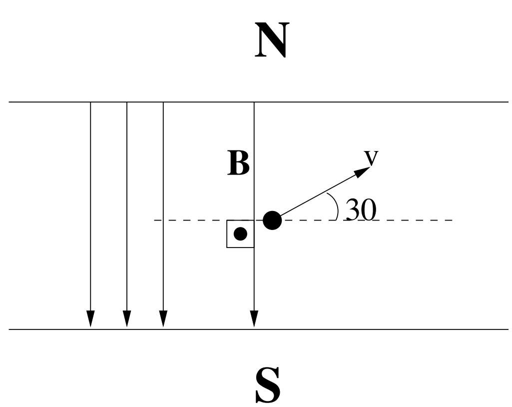
d. Find two vectors that are perpendicular to $\mathbf{u}$.

6.(*) **(Section 12.4)** Let $\mathbf{u} = \vec{\imath} + 2\vec{\jmath}$ and $\mathbf{v} = \vec{\jmath} - \vec{k}$.

a. Find a vector that is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

b. Perform the vector operations $(\mathbf{u}+\mathbf{v}) \times (\mathbf{u}-\mathbf{v})$ and $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}\times\mathbf{v})$. (Hint: Using the properties of the cross-product and dot-product, and your answer from part **a.** would simplify the calculations.)

7.(*) **(Section 12.4)** A magnetic field is a vector field (essentially a vector field consists of a set of vectors in a region; corresponding to each point in the region there is an associated vector) that exerts force on moving charged particles. In the magnetic field in the figure the



vectors point from the north pole (labeled **N** in the figure) to the south pole (labeled **S** in the figure), that is at any point the magnetic field vector $\mathbf{B}$ is perpendicular to the horizontal plates. The moving particle is subject to the magnetic force given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}.$$

If the speed of the moving particle is $|\mathbf{v}| = 5$ meters/sec with charge $q = -2$ Coulombs and the magnitude of $\mathbf{B}$ is 15 Tesla ((Newton.sec)/(meter.Coulomb)),

a. what is the magnitude of the magnetic force on the particle?

b. what is the direction of the magnetic force?

8.(*) (Section 12.5) Consider the lines

- $\mathcal{L}_1$ that passes through the point $(-1, -2, -1)$ and parallel to the vector $\vec{i} + 2\vec{j} + \vec{k}$,
- $\mathcal{L}_2$ that passes through the points $(-1, 1, -1)$ and $(3, 3, 3)$.

a. Find the parametric equations of the lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

b. Find the symmetric equations of the lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

c. Find the point at which the lines $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect each other.

d. Find the angle between the lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

9. (Section 12.5) Consider the planes

- $\mathcal{P}_1$ that passes through the point $(1, 1, 1)$ with the normal vector $n_1 = \vec{i} + \vec{j} + \vec{k}$,
- $\mathcal{P}_2$ that passes through the point $(1, 1, 1)$ with the normal vector $n_2 = \vec{i} - 2\vec{j} + \vec{k}$.

a. Find the linear equations of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$

b. Find a vector that is parallel to both of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.

c. Find the line of intersection of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.