Math 20C (Lecture C) - Homework 3

(Due on November 14th, Wednesday by 3pm)

This homework covers sections 14.1-4 from Stewart's book. You need to return only the questions marked with (∗). The others are provided for practicing purposes.

1. (Section 14.1) Find the domains and the ranges of the following functions.

a. $f(x,y) = x^2 + y$ **b.** $g(x,y) = \ln(xy)$

c. $h(x,y) = ye^x - 2$ **d.** $u(x,y) = x^y$

2.(*) (Section 14.1) Plot the contour diagrams of the following functions for $z = 0, 1, 2, 3$. Describe in words in which directions the functions are becoming steeper.

a. $z = f(x,y) = \sqrt{4x^2 + y^2}$ **b.** $z = g(x,y) = 4x^2 + y^2$

c. $z = h(x,y) = (4x^2 + y^2)^{1/4}$ **d.** $z = u(x,y) = \sqrt{9 - 4x^2 - y^2}$

3.(*) (Section 14.2) In each part determine whether the limit exists or not. If the limit exists indicate which value the function is approaching. Justify your answers.

a. $\lim_{(x,y)\to(\sqrt{\pi/2},\sqrt{\pi/2})} \sin(x^2 + xy + y^2)$ **b.** $\lim_{(x,y)\to(0,0)} \frac{x^2}{x+y}$

c. $\lim_{(x,y)\to(\infty,\infty)} \frac{\sin(xy)}{xy}$ **d.** $\lim_{(x,y)\to(0,0)} \frac{y^2}{|x|}$

4.(*) (Section 14.2) Which of the following functions are continuous at $(x,y) = (2,1)$? Justify your answer.

a. $f(x,y) = \begin{cases} \frac{x-2}{y-1} & (x,y) \neq (2,1) \\ 0 & (x,y) = (2,1) \end{cases}$ **b.** $u(x,y) = \frac{x-2}{y}$

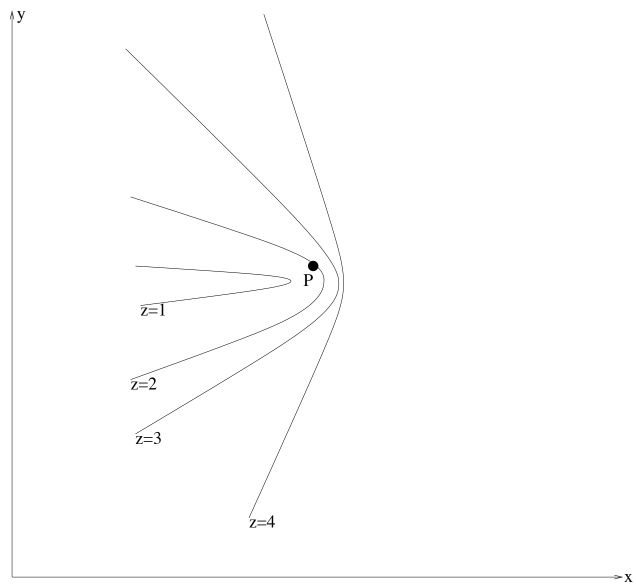
c. $g(x,y) = \frac{(x-2)^2}{|y-1|}$ **d.** $h(x,y) = \begin{cases} \frac{(x-2)^4}{(y-1)^2} & (x,y) \neq (2,1) \\ 0 & (x,y) = (2,1) \end{cases}$

5.(*) (Section 14.3) Calculate the first and second partial derivatives of the functions below. Verify in each case that $f_{xy} = f_{yx}$.

a. $f(x,y) = x^3 + \cos(y)$ **b.** $f(x,y) = x^3\cos(y)$

c. $f(x,y) = \cos(x^3 y)$ **d.** $f(x,y) = (\cos(x))^{3y}$

6. **(Sections 14.2-3)** Based on the contour diagram of a function $z = f(x, y)$ illustrated below determine the signs of the derivatives f_x, f_y, f_{xx}, f_{xy} and f_{yy} at the point P.



7. **(*)** **(Section 14.4)** Let $f(x, y) = x \ln(xy)$.

a. Find the equation of the tangent plane for $f(x, y)$ at $(x, y) = (1, e)$.

b. Find the parametric equation of the line passing through $(x, y) = (1, e)$ that is perpendicular to the tangent plane for $f(x, y)$ at $(x, y) = (1, e)$. This line is called the *normal line* to $f(x, y)$ at $(x, y) = (1, e)$.

c. Find the symmetric equation of the line tangent to $f(x, y)$ at $(x, y) = (1, e)$ and lying on the plane $x = 1$.

d. Find the symmetric equation of the line tangent to $f(x, y)$ at $(x, y) = (1, e)$ and lying on the plane $y = e$.

8. **(*)** **(Section 14.4 - Modified from exercise 14.4.37 from Stewart's book)** The total resistance R of two resistors with resistances R_1 and R_2 connected in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Suppose that the resistances R_1 and R_2 are measured in ohms as 20Ω and 60Ω, respectively.

a. Find a linear approximation for R in terms of R_1 and R_2 around $(R_1, R_2) = (20, 60)$.

b. Use the linear approximation from part **a.** to estimate the total resistance if the actual resistance of each of R_1 and R_2 is $\%1$ greater than the value indicated by the measurements.