Math 20C (Lecture C) - Homework 5
(Due on December 10th, Monday by 3pm)

This homework covers sections 15.1-4 from Stewart's book. You need to return only the questions marked with (∗). The others are provided for practicing purposes.

1.(*) (Section 15.1) Consider the functions

$$f(x,y) = \sqrt{4 - x^2 - y^2} \quad \text{and} \quad g(x,y) = \sqrt{x^2 + y^2}$$

defined over the domain

$$D = \{(x,y) : 0 \leq x \leq 1, \quad 0 \leq y \leq 1\}.$$

a. Approximate the volumes under the functions over the domain D by using the midpoint sum and dividing the intervals $[0,1]$ along the x and y axes into two subintervals of length 0.5 or equivalently dividing D into four rectangles with side-lengths $\Delta x = \Delta y = 0.5$.

b. The left and right sum are the Riemann sums such that the function values are evaluated at the lower-left and upper-right end points of each rectangle. Without calculating the left and right sum order the left, midpoint and right sum with $\Delta x = \Delta y = 0.5$ from the smallest to the largest for the functions $f(x,y)$ and $g(x,y)$ over the domain D.

2.(*) (Section 15.2-3) Evaluate the following integrals over the domains indicated. The order of integration (whether you integrate with respect to x or y first) may simplify some of the integrals.

a. The integral

$$\int\int_D \frac{y}{x}\, dx\, dy$$

over $D = \{(x,y) : 0 \leq y \leq 1, \quad 1 \leq x \leq 2\}$.

b. The integral

$$\int\int_D ye^{xy}\, dx\, dy$$

over $D = \{(x,y) : -1 \leq y \leq 1, \quad 0 \leq x \leq 1\}$.

c. The integral

$$\int\int_D ye^{xy}\, dx\, dy$$

over $D = \{(x,y) : 1 \leq x \leq 2, \quad \frac{1}{x} \leq y \leq 1\}$.

d. The integral

$$\int\int_D \frac{e^{yx}}{x}\, dx\, dy$$

over $D = \{(x,y) : 1 \leq x \leq 2, \quad \frac{1}{x} \leq y \leq \frac{2}{x^2}\}$.

3.(*) (Section 15.3) Rewrite the integrals below by changing the order of integration with respect to x and y.

a.

$$\int_0^2 \int_{-y/2}^{y/2} f(x,y)\, dx\, dy.$$

b.

$$\int_{-3}^{3} \int_{-\frac{\sqrt{9-x^2}}{3}}^{0} f(x,y)\, dy\, dx$$

4.(*) (Section 15.4) The equations $x^2 + y^2 = 1$ and $z = \sqrt{x^2 + y^2}$ represent a cylinder extending along the z- axis and a cone above the z-axis, respectively. Find the volumes of the solids described below by first writing an integral and then evaluating the integral in the polar coordinates.

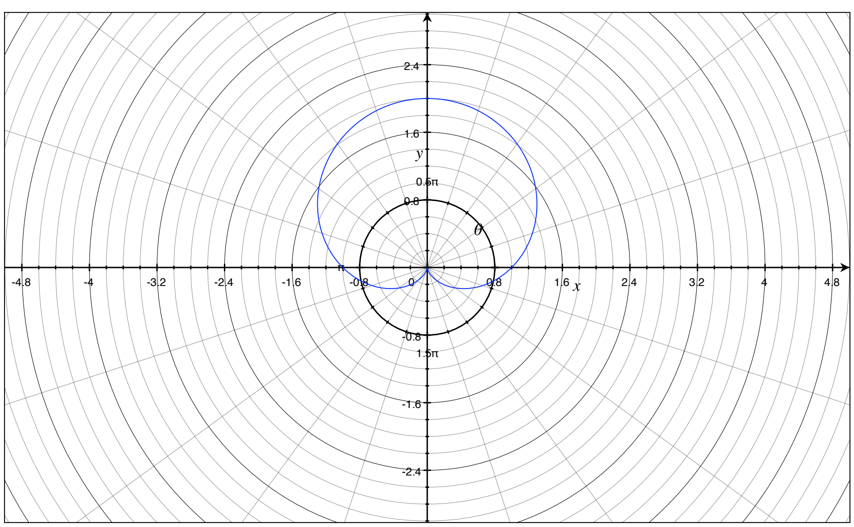
a. The solid bounded by $x^2 + y^2 = 1$, $z = \sqrt{x^2 + y^2}$ and $z = 0$.

b. The solid above $z = \sqrt{x^2 + y^2}$ and below the horizontal plane $z = 1$.

5. (Section 15.4) In polar coordinates the equation

$$r(\theta) = 1 + \sin(\theta)$$

represents a cardioid. The volume of the cylindrical shape with height equal to one and



base formed by this cardioid is equal to the area enclosed by the cardioid. Set up a double integral in polar coordinates for the volume of the cylindrical shape or equivalently for the area enclosed by the cardioid. Find the area enclosed by the cardioid by evaluating the integral.

6. (Section 15.1-2) Determine the height of the box whose base is given by the area

$$D = \{(x, y) : -1 \leq x \leq 1, \quad -1 \leq y \leq 1\}$$

and whose volume is equal to the volume of the solid lying under $f(x, y) = x^2 + x + y + y^2$ and above D. This is the average value of $f(x, y)$ over D.