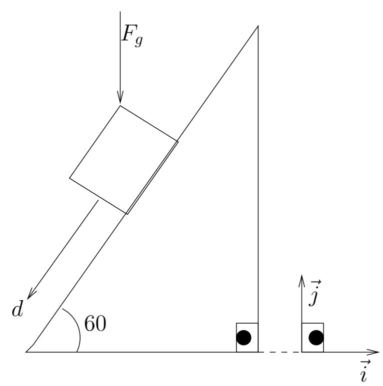
Name: **PID:**
Discussion Section - No: **Time:**

Midterm 1, Math 20C (Lecture C)
November 2nd, 2007

Duration: 50 minutes
This is an open-book exam. Calculators and computing devices are not allowed.
To get full credit you should support your answers.

1. An object slides 5 meters over the inclined surface with slope 60° due to the vertical gravitational force $\mathbf{F_g}$ with magnitude $|\mathbf{F_g}| = 50$ Newton as illustrated in the figure below.



a) (1 point) Express the displacement vector $\mathbf{d}$ (note that $|\mathbf{d}| = 5$ meters) in terms of the standard basis vectors $\vec{i}$ and $\vec{j}$.

b) (2 points) Find the projection of $\mathbf{F_g}$ onto $\mathbf{d}$.

#	Score
1 (5 points)	
2 (7 points)	
3 (6 points)	
4 (7 points)	
Total (25 points)	

c) (2 points) Find the work done by the gravitational force in Newton $\cdot$ meters.

2. The position of a particle in 3D space as a function of time t is given by

$$\mathbf{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}, \quad t \geq 0.$$

a) (2 points) Find the velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ of the particle as functions of time.

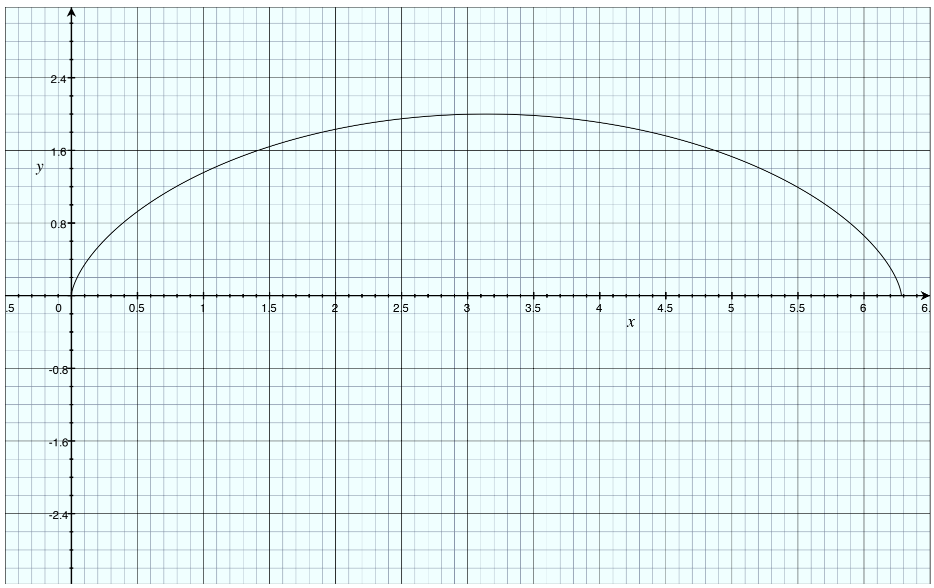
b) (2 points) Find the *cosine* of the angle between $\mathbf{a}(1)$ and $\mathbf{v}(1)$.

c) (3 points) Find the equation of the plane passing through $\mathbf{r}(1)$ and containing $\mathbf{a}(1)$ and $\mathbf{v}(1)$. This is the *osculating* plane to which the particle stays close around $t = 1$.

3. The graph of the cycloid generated by the parametric equation

$$x(\theta) = (\theta - \sin(\theta)) \quad \text{and} \quad y(\theta) = (1 - \cos(\theta)), \qquad 0 \le \theta < 2\pi$$

is illustrated below. The curve intersects the x-axis at $\theta = 0$ and $\theta = 2\pi$.



a) (3 points) Write down a definite integral for the area between the cycloid and the x-axis. Do not evaluate the integral.

b) (3 points) Find the equation of the line tangent to the cycloid at $\theta = \pi/2$.

4. Consider the parametric curve $\mathcal{C}$ in 3D space

$$x(t) = 3\sin(t), \quad y(t) = 5\cos(t) \quad \text{and} \quad z(t) = 4\sin(t), \quad t \geq 0$$

a) (3 points) Find the unit tangent vector $\mathbf{T}(t)$ to $\mathcal{C}$.

b) (2 points) Find the unit tangent vector $\hat{\mathbf{T}}(s)$ parametrized in terms of the arc-length s, that is $\hat{\mathbf{T}}(s) = \mathbf{T}(t')$ where s is the arc-length of the curve $\mathcal{C}$ from $t = 0$ to $t = t'$.

c) (2 points) Show that the curvature $\kappa(s)$ of $\mathcal{C}$ is constant and equal to $1/5$.