Name: **PID:**
Discussion Section - No: **Time:**

Midterm 2, Math 20C (Lecture C)
November 28th, 2007

Duration: 50 minutes
This is an open-book exam. Calculators and computing devices are not allowed.
To get full credit you should support your answers.

1. Consider the function

$$f(x,y) = \begin{cases} \frac{4x^2+y^2}{2xy} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

a) (3 points) Plot the contour diagrams of $z = f(x,y)$ for $z = -2, 0, 2$.

b) (3 points) Is $f(x,y)$ continuous at $(x,y) = (0,0)$? Justify your answer.

#	Score
1 (6 points)	
2 (6 points)	
3 (8 points)	
4 (5 points)	
Total (25 points)	

2. The kinetic energy of an object in $Joules$ (a unit for work and energy) with mass m in kg and velocity v in m/sec is given by the equation

$$E(m, v) = \frac{1}{2}mv^2.$$

a) (3 points) Find the linearization of $E(m, v)$ at $(m, v) = (1, 2)$.

b) (3 points) The mass and velocity of an object are measured as 1 kg and 2 m/sec, respectively. If each of the actual values of the mass and velocity is %1 greater than the corresponding measured value, estimate the kinetic energy of the object using the linearization from part **a)**.

3. Let

$$g(x, y) = e^{x^2-(y-2)^2}$$

a) (2 points) Find the gradient vector $\nabla g(1, 3)$.

b) (3 points) Find the directional derivative $D_{\vec{u}}g(1, 3)$ in the direction of $\vec{u} = -\frac{3}{5}\vec{i} - \frac{4}{5}\vec{j}$.

c) (3 points) Find the unit vector $\vec{u}$ so that the directional derivative $D_{\vec{u}}g(1, 3)$ in the direction of $\vec{u}$ at $(x, y) = (1, 3)$ is as small as possible, that is for any unit vector $\vec{w}$, $D_{\vec{u}}g(1, 3) \leq D_{\vec{w}}g(1, 3)$.

4. (5 points) Find the points on the curve $yx = 4$ that are closest to the origin by posing a constrained optimization problem and solving it using Lagrange multipliers. (Remark : The global minima of $x^2 + y^2$ and $\sqrt{x^2 + y^2}$ subject to any constraint are the same.)

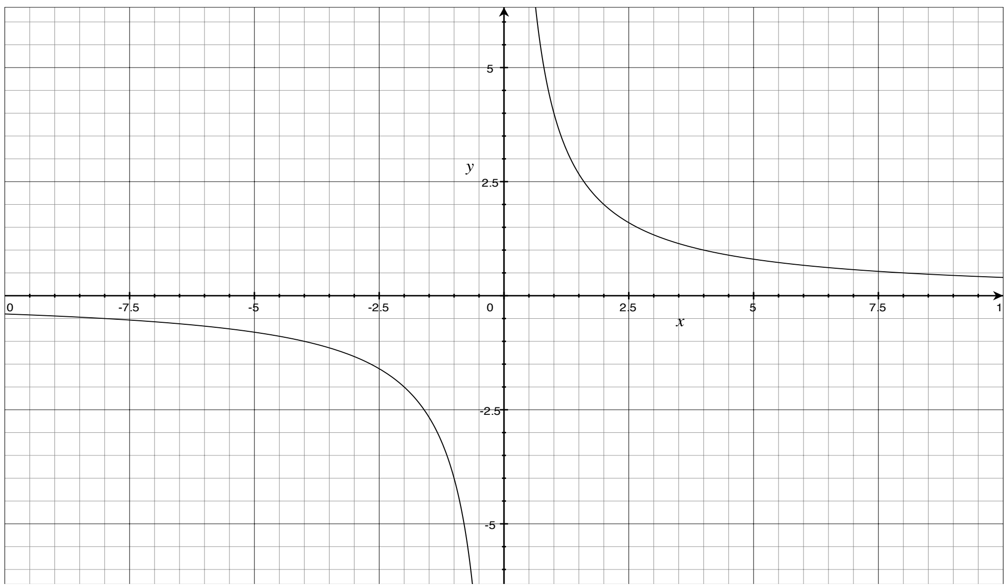

Figure 1: The graph of the curve $yx = 4$ is illustrated above.