Name: **PID:**
Discussion Section - No: **Time:**

Midterm 1, Math 20F - Lecture B (Spring 2007)

Duration: 50 minutes
This is a closed-book exam. Calculators are not allowed. You can use one page of notes.
To get full credit you should support your answers.
There are four questions in this exam. All of the questions have two parts.

1. Consider the system of linear equations

$$2x_1 + 4x_2 - x_3 = 2$$
$$x_1 + 2x_2 + hx_3 = b$$

where h and b are scalars.

a) (2.5 points) Explain why the system cannot have a unique solution.

b) (2.5 points) For what values of h and b the system is inconsistent.

#	Score
1	
2	
3	
4	
Total	

2. The solution set of the homogeneous system $Ax = 0$ where A is a 3 by 3 matrix (corresponding to a system of three equations in three unknowns) is given by

$$\text{span}\left(\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right).$$

Let

$$p = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

be a particular solution satisfying $Ax = b$ for some given vector b of size 3.

a) (2 points) Give a solution of the system $Ax = b$ not equal to the particular solution p.

b) (3 points) Based on the solution for the homogeneous system given above provide a reduced echelon matrix that is possibly row-equivalent to A.

3. Let

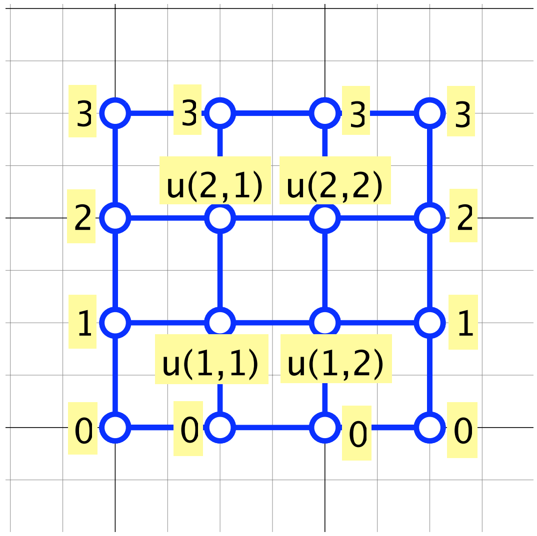
$$v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \quad \text{and} \quad v_3 = \begin{bmatrix} 0 \\ -1 \\ h \end{bmatrix}$$

where h is a scalar.

a) (2 points) Do v_1 and v_2 span $\mathbb{R}^3$? (or equivalently can any vector in $\mathbb{R}^3$ be expressed as a linear combination of v_1 and v_2?) If it spans, explain why. If not, provide a vector of size 3 that cannot be expressed as a linear combination of v_1 and v_2.

b) (3 points) For what values of h do the vectors v_1, v_2 and v_3 span $\mathbb{R}^3$?

4. Consider the 4 by 4 grid illustrated below that will be used to approximate the density of a fluid inside a container at various positions.



For the nodes on the boundary of the container the density values are given on the figure. At any node inside the container the density is approximated by the average of the surrounding nodes.

a) (2.5 points) Set up a system of 4 linear equations (one for each of the nodes not on the boundary) with unknowns $u(1,1)$ (the density at $(1,1)$), $u(1,2)$ (the density at $(1,2)$), $u(2,1)$ (the density at $(2,1)$) and $u(2,2)$ (the density at $(2,2)$).

b) (2.5 points) Solve the system of linear equations in part **a)** by reducing it to the reduced echelon form using the row operations. (Hint: sometimes performing the row-swap operation, when it is not essential, may avoid the entries of the matrix becoming very small or large and simplify the calculations.)