Math 20F - Lecture C (Winter 2008)
Homework 4
(Due on Friday, March 14th by 3pm)

**Please return your homeworks to the homework box on the 6th floor in the APM building.*

**This homework covers sections 3.1-2, 5.1-3 and 6.1-5 from your textbook.*

1. (Section 3.1-2) Find the determinants of the following matrices.

(a) $A_1 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

(b) An orthogonal $n \times n$ matrix Q satisfying $Q^T Q = I_n$ (Note : The answer is not unique; there are two possible answers. Give both of the possible answers.)

(c) $A_2 = \begin{bmatrix} 3 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 2 & 4 & -5 & 0 \\ 1 & -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) The $n \times n$ matrix $R_n = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & & 1 & 0 \\ & & \iddots & & \\ 0 & 1 & & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{bmatrix}$

2. (Section 3.1-2) Given a 3×3 matrix A with $\det(A) = 4$. What are the determinants of $2A$, A^2, A^T and A^{-1}?

3. (Section 5.1-3) Let

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

(a) Find the characteristic polynomial and eigenvalues of A.

(b) Find the eigenspace associated with each each eigenvalue.

(c) Is A diagonalizable? If yes, find an eigenvalue decomposition of the form $P\Lambda P^{-1}$. If no, explain why not.

(d) What happens to the entries of A^k as $k \to \infty$? In particular do all of the entries of A^k approach zero or do some of the entries of A^k diverge as $k \to \infty$? Explain.

4. (Section 5.1-3) Show that the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

is not diagonalizable.

5. (Section 6.1-2) Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

(a) Find bases for the null and column space of A and show that these subspaces are orthogonal complements of each other.

(b) Write the vector $v = [1\ 1\ 1]^T$ of the form $v = v_r + v_n$ so that $v_r \in \text{Col}(A)$ and $v_n \in \text{Null}(A)$.

6. (Section 6.2-4) Given the set of vectors

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

(a) Verify that $\mathcal{B}$ is a basis for $\mathbb{R}^3$, but it is not orthogonal.

(b) Find an orthonormal basis by applying the Gram-Schmidt process to the basis $\mathcal{B}$.

7. (Section 6.5) Given

$$A = \begin{bmatrix} 1 & 1 & 5 \\ 1 & -1 & 1 \\ 1 & 1 & 5 \end{bmatrix}, \text{ and } b = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

(a) Show that the system $Ax = b$ is inconsistent.

(b) Find a least-squares solution $\hat{x}$ such that $\|b - A\hat{x}\|$ is as small as possible. Is this least-squares solution unique? (*i.e.* is there another $\bar{x}$ such that $\|b - A\hat{x}\| = \|b - A\bar{x}\|$?)