Name:	PID:

Discussion Section		
No:	Time:	TA:

Midterm 1, Math 20F - Winter 2008

Duration: 100 minutes
This is an open book exam. Calculators are not allowed.
To get full credit you should support your answers.

1. Consider the system of linear equations

$$\begin{aligned} x_1 - 3x_2 &= 4 \\ 3x_1 - 6x_2 &= 3 \\ -4x_1 + hx_2 &= b \end{aligned}$$

where h and b are scalars.

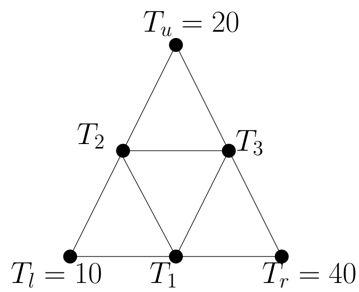
(a) (1 point) Explain why the system cannot have infinitely many solutions.

(b) (2 points) For what values of h and b is the system inconsistent?

(c) (1 point) For what values of h and b does the system have a unique solution?

#	Score
1 (4 points)	
2 (4 points)	
3 (4 points)	
4 (4 points)	
5 (4 points)	
Total (20 points)	

2. In this question for the triangular shape below we want to estimate the temperatures T_1, T_2 and T_3 at the internal nodes. The temperatures at the corners of the triangle are given as 10, 20 and 40 as shown in the figure below. The temperature at each internal node is estimated by the average of the temperatures at the four adjacent nodes. Two nodes are said to be adjacent if they are connected by an edge.



(a) (2 points) Set up a linear system of three equations with unknowns T_1, T_2 and T_3. Write down also the augmented matrix.

(b) (2 points) By reducing the augmented matrix into the reduced echelon form solve the linear system from part **(a)**.

3. Let

$$a_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}, \quad \text{and} \quad a_3 = \begin{bmatrix} -4 \\ -6 \\ 7 \end{bmatrix}.$$

(a) (1 point) Give geometric descriptions of the sets $\text{span}\{a_1\}$ and $\text{span}\{a_1, a_2\}$.

(b) (2 points) Is the set $\{a_1, a_2, a_3\}$ linearly independent?

(c) (1 point) Does the set $\{a_1, a_2, a_3\}$ span $\mathbb{R}^3$? Give a geometric description of $\text{span}\{a_1, a_2, a_3\}$?

4. The solution set of a linear system

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = b$$

of "three equations in four unknowns" is given as

$$\begin{aligned} x_1 &= 3x_2 - x_3 + 4 \\ x_2,\ x_3 &\text{ are free} \\ x_4 &= 3. \end{aligned}$$

(a) (1 point) Give a particular solution for the linear system.

(b) (1 point) Determine the pivot columns of the 3×4 coefficient matrix A for the linear system.

(c) (2 points) Write down the solution set for the associated homogeneous system as a span of set of vectors.

5. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$,

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ 0 \\ x_1 \end{bmatrix}.$$

(a) (2 points) Find the transformation matrix for T, that is find a matrix A such that

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = A\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 0 \\ x_1 \end{bmatrix}.$$

(b) (1 point) Is the transformation T one-to-one? Justify your answer.

(c) (1 point) Is the transformation T onto $\mathbb{R}^3$? Justify your answer.