Jacobi's Method
(Reminders from Lecture 14)

The Algorithm

$A^{(0)} \leftarrow A$, $k \leftarrow 0$

while $\sum_{j,\ell,j\neq\ell} \left[a_{j\ell}^{(k)}\right]^2 \quad > \varepsilon$ **do**

Find $p, q \in \{1, 2, \ldots, n\},\ p \neq q$ such that

$$\left|a_{pq}^{(k)}\right| \quad = \quad \max_{j,\ell,j\neq\ell} \left|a_{j\ell}^{(k)}\right|$$

$A^{(k+1)} \leftarrow R^{(pq)}(\theta_k)^T A^{(k)} R^{(pq)}(\theta_k)$, $k \leftarrow k + 1$

end while

- $A, A^{(k)}$ are similar for all k with the same eigenvalues

Plane Rotation Matrices

$$R^{(pq)}(\theta) = \begin{bmatrix} I & & & & \\ & c & & s & \\ & & I & & \\ & -s & & c & \\ & & & & I \end{bmatrix}, \qquad \begin{aligned} c &:= \cos(\theta) \\ s &:= \sin(\theta) \end{aligned}$$

c - (p, p) entry $\qquad$ c - (q, q) entry

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- $R^{(pq)}(\theta)v$ rotates the pth, qth components of v by θ in clock-wise direction.
- $R^{(pq)}(\theta) \in \mathbb{R}^{n \times n}$ is orthogonal.

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Task to do - find θ such that (p, q) entry of $B := R^{(pq)}(\theta)^T A R^{(pq)}(\theta)$ is 0.