Jacobi's Method

November 8, 2018

Given $A \in \mathbb{R}^{n\times n}$, $A^T = A$, compute all eigenvalues of A

$A^{(0)} \leftarrow A$, $k \leftarrow 0$

while $\sum_{j,\ell,j\neq\ell} \left[a_{j\ell}^{(k)}\right]^2 > \varepsilon$ **do**

Find $p, q \in \{1, 2, \ldots, n\},\ p \neq q$ such that

$$\left|a_{pq}^{(k)}\right| = \max_{j,\ell,j\neq\ell} \left|a_{j\ell}^{(k)}\right|$$

$A^{(k+1)} \leftarrow R^{(pq)}(\theta_k)^T A^{(k)} R^{(pq)}(\theta_k)$, $k \leftarrow k + 1$

end while

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- Choose θ_k so that $a_{pq}^{(k+1)} = 0$
- $\theta_k = (1/2)\text{arctan}\left(\frac{2a_{pq}}{a_{qq}-a_{pp}}\right) \in [-\pi/4, \pi/4]$

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 $\theta_k \leftarrow (1/2)\text{arctan}\left(\frac{2a_{pq}}{a_{qq}-a_{pp}}\right)$, $c \leftarrow \cos(\theta_k)$, $s \leftarrow \sin(\theta_k)$
 $A^{(k+1)} \leftarrow A^{(k)}$
 % Right Multiply by $R^{(pq)}(\theta_k)$
 $A^{(k+1)}(:,p) \leftarrow cA^{(k)}(:,p) - sA^{(k)}(:,q)$
 $A^{(k+1)}(:,q) \leftarrow sA^{(k)}(:,p) + cA^{(k)}(:,q)$
 % Left Multiply by $R^{(pq)}(\theta_k)^T$
 $r_p \leftarrow cA^{(k+1)}(p,:) - sA^{(k+1)}(q,:)$
 $A^{(k+1)}(q,:) \leftarrow sA^{(k+1)}(p,:) + cA^{(k+1)}(q,:)$
 $A^{(k+1)}(p,:) \leftarrow r_p$.
 $k \leftarrow k+1$
end while
