Reminders from Lecture 2

September 25, 2018

Fixed point sequence $\{x_k\}$, $x_{k+1} = g(x_k)$

Suppose $\lim_{k\to\infty} x_k = x_*$ s.t. $g(x_*) = x_*$.

$$\lim_{k\to\infty} \frac{|x_{k+1} - x_*|}{|x_k - x_*|} = |g'(x_*)|$$

Newton's method generates a sequence $\{x_k\}$ such that

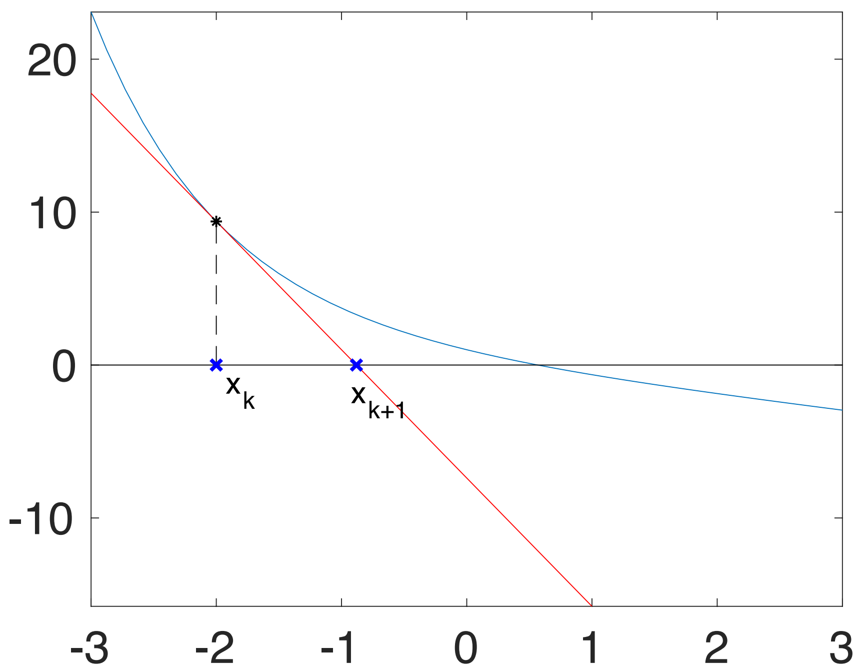
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$

- Associated fixed point function $g(x) = x - \frac{f(x)}{f'(x)}$

- If $\lim_{k\to\infty} x_k = x_*$ is s.t. $f'(x_*) \neq 0$, then

$$g(x_*) = x_* \iff f(x_*) = 0.$$

- $g'(x_*) = 0$ (i.e., superlinear convergence)



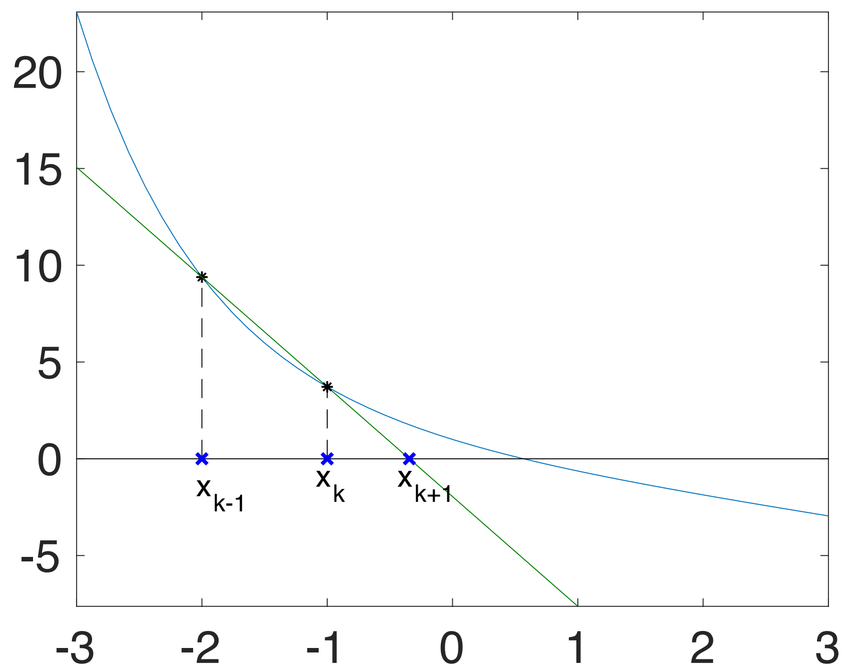
$$\ell(x_{k+1}) = 0, \quad \textbf{where} \quad \ell(x) = f(x_k) + f'(x_k)(x - x_k)$$

Secant method is like Newton's method,
but uses the approximation

$$f'(x_k) \quad = \quad \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}.$$

Generates a sequence $\{x_k\}$ s.t. (given x_0, x_1)

$$x_{k+1} \quad = \quad x_k - f(x_k) \cdot \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}.$$



$$s(x_{k+1}) = 0, \quad \text{where} \quad s(x) = f(x_k) + \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}(x - x_k)$$