

## Reminders from Lecture 4

# Gaussian Elimination

$$\begin{array}{rclcl} 2x_1 + & 3x_2 + & & x_3 = & 1 \\ -x_1 + & 2x_2 + & & 3x_3 = & 3 \\ & x_1 + & 5x_2 + & -2x_3 = & -1 \end{array}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 1 & 5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

# Gaussian Elimination

$$\begin{array}{rclcrcl} 2x_1 & + & 3x_2 & + & x_3 & = & 1 \\ -x_1 & + & 2x_2 & + & 3x_3 & = & 3 \\ x_1 & + & 5x_2 & + & -2x_3 & = & -1 \end{array}$$

$$\left[ \begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ -1 & 2 & 3 & 3 \\ 1 & 5 & -2 & -1 \end{array} \right]$$

# Gaussian Elimination

$$\left[ \begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ -1 & 2 & 3 & 3 \\ 1 & 5 & -2 & -1 \end{array} \right] \sim \mu_{21}=1/2, \mu_{31}=-1/2$$

$$\left[ \begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & 7/2 & 7/2 & 7/2 \\ 0 & 7/2 & -5/2 & -3/2 \end{array} \right] \sim \mu_{32}=-1$$

$$\left[ \begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & 7/2 & 7/2 & 7/2 \\ 0 & 0 & -6 & -5 \end{array} \right]$$

# Gaussian Elimination

$$\left[ \begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ -1 & 2 & 3 & 3 \\ 1 & 5 & -2 & -1 \end{array} \right] \sim \mu_{21}=1/2, \mu_{31}=-1/2$$

$$\left[ \begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & 7/2 & 7/2 & 7/2 \\ 0 & 7/2 & -5/2 & -3/2 \end{array} \right] \sim \mu_{32}=-1$$

$$\left[ \begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & 7/2 & 7/2 & 7/2 \\ 0 & 0 & -6 & -5 \end{array} \right]$$

$$(I + \mu_{32}E^{(32)})(I + \mu_{31}E^{(31)})(I + \mu_{21}E^{(21)})A = U$$

$E^{(ij)}$  - Matrix of 0s except  $(i, j)$  entry is 1

# Gaussian Elimination

$$A \sim U$$

$$(I + \mu_{n(n-1)}E^{(n(n-1))}) \dots (I + \mu_{31}E^{(31)})(I + \mu_{21}E^{(21)})A = U$$

# Gaussian Elimination

$$A \sim U$$

$$(I + \mu_{n(n-1)}E^{(n(n-1))}) \dots (I + \mu_{31}E^{(31)})(I + \mu_{21}E^{(21)})A = U$$

Since  $(I + \mu_{ij}E^{(ij)})^{-1} = I - \mu_{ij}E^{(ij)}$  for  $i > j$ ,

$$A = \underbrace{(I - \mu_{21}E^{(21)})(I - \mu_{31}E^{(31)}) \dots (I - \mu_{n(n-1)}E^{(n(n-1))})}_L U$$

►  $L$  is unit lower triangular.